Survey of India.

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THE EARTH'S AXES AND TRIANGULATION

BY

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TABLE OF CONTENTS.

ERRATA ET ADDENDA.

Page

3 line 4 from top, column (Old value) *insert* (=6,377,276 metres) *after* 20,922,840·95 feet

,, ,, 5 ,, ,, ,, ,, (=6,356,075 metres) ,, 20,853,284·03 feet

13 ,, 23 ,, *for* known *read* know.

15 After equation Nos. (18), (19), (20) *add* A

16 line 7 from top *for* (20) *read* (20) A

30 For value of u under Long. 82° ,, 0·977 ,, 0·997

39 In equation (47) ,, 6 cos 4λ ,, 6 $\cos^4\lambda$

,, At the bottom *add a footnote*:—In equations (42) to (47) dL, $d\lambda$ are expressed in radians.

40 At the bottom *add* an alternative proof of (2):—

For the curve whose osculating plane is normal to the spheroid we have for a small element

$$d\lambda = -\frac{ds}{\rho}\cos A$$

$$dA = -\frac{ds}{\nu}\sin A\tan\lambda$$

Hence

$$\cot A.\,dA = \frac{\rho}{\nu}\tan\lambda\cdot d\lambda = \frac{1-e^2}{1-e^2\sin^2\lambda}\cdot\tan\lambda\cdot d\lambda = \frac{1-e^2}{2}\cdot\frac{d\eta}{(1-\eta)(1-e^2\eta)},$$

where $\eta = \sin^2\lambda$

$$\therefore\ d\log\sin A = \tfrac{1}{2}\left(\frac{1}{1-\eta} - \frac{e^2}{1-e^2\eta}\right)d\eta = \tfrac{1}{2}\,d\log\frac{1-e^2\eta}{1-\eta} = -\tfrac{1}{2}\,d\log\cos^2\phi = -d\log\cos\phi$$

$$\therefore\ \sin A\cos\phi = \text{constant on a geodesic.}$$

52 line 5 from top *for* ϕ *read* Q

54 For equation (5) *read*

$$s = -a\sqrt{1-e^2}\left[\chi(1+\tfrac{1}{4}h^2 - \frac{3}{64}h^4) - \frac{h^2}{8}\sin 2\chi\left(1-\frac{3}{16}h^2\right) + \frac{h^4}{32}\cos\chi\sin^3\chi\right]_{\chi'}^{\chi}$$

54 For equation (12) *add a note*—The complete term to next power of e^2 is—

$$-\frac{\delta h^2}{8}\left[(2\chi - \sin 2\chi)\left(1-\frac{3h^2}{8}\right) + \frac{h^2}{4}\sin 2\chi\sin^2\chi\right]$$

55 In equations (16) and (17) *for* $\sin 2\phi$, $\sin 2\phi'$ *read* $\sin 2\lambda$, $\sin 2\lambda'$

59 Against 10°λ in the table in col. L—L′ *for* $+v$ *read* $\pm v$

68 At the end *add a para*—The values found from tables XXIX—XXXIV for the central meridian (L=77° 40′ approximately) differ slightly from those found in Chapter I. This is due to an approximation in Chapter I (*vide* footnote on page 4).

ERRATA ET ADDENDA—(*Continued*).

Page					
70	line 6 from top	*after*	$d\phi = u_1$	*insert*	(*vide* p. 54).
84	„ 13 from bottom	*for*	about	*read*	above.
96	„ 9 from top	„	$w_r - w_{r-1}$	„	$w_r - w_{r-1} - \eta_{r-1}$
„	„ 13 „ „	„	E_0	„	E
105	„ 13 from bottom	„	10^7	„	10^{-7}
115	„ 7 from bottom, col. 12	„	+2·20	„	+2·02
120	„ 17 „ „	„	whence	„	where
148	Against r=10, u=20, in col. ${}_rk_u$	„	−·004	„	−·0041.
149	In table LXXI against r =16, in col. 23	„	−·2399	„	−·0399.

INTRODUCTION.

The preparation of this work has extended over some four years and has been delayed by press of other work resulting mainly from a shortage of officers in the department due to the war. Its final completion has been very hurried as the author has been ordered to Mesopotamia. The end of Chapter VIII has been much abbreviated owing to there not being time to work out a sufficient number of cases to form the basis for a proper discussion. It has been decided however to publish the work at the stage it has reached rather than to wait an indefinite period for its amplification.

The origin of the research was the need which had arisen of converting geodetic results, obtained in India and referred to the now obsolete Everest spheroid, into terms of the best determined spheroid. The work soon showed that certain inconsistencies arose and that multiple values of the changes were found according to the route followed. The reason of this is that the observations of triangulation in India have been *adjusted* to fit the Everest spheroid and will not fit any other without *readjustment*. Had the spheroid of Everest been regarded merely as a reference figure and not, for purposes of triangulation, as identical with the geoid this difficulty would not have arisen: but I believe a similar method of treatment has been followed in all other countries. It would generally be impossible to avoid this treatment, as in the early stages of survey work values of deflections are not usually available. There is little reason why these deflections should not be determined roughly as the triangulation proceeds: and were this done all computations could be made quite correctly, and the deflection results would also be useful. The discrepancies involved, however, are not as large as the probable errors and offer what is more of a computation difficulty than a practical difficulty: and it is believed that the conclusions reached in the first four chapters overcome the difficulty quite satisfactorily for all practical geodetic purposes. The explanation of the discrepancy was by no means discovered at once and it is in Chapter V that the fundamental inconsistency is discussed.

Meanwhile other related subjects come under consideration and in Chapter VI reference is made to some of these. Here a complete analogy between adjustment of errors and small strains in a mechanical framework is shown to exist. This has led to the idea of "strength" of triangulation and gives a less abstract view of the adjustments by the method of least squares than was hitherto available. It enables one to picture in a tangible way the changes necessary, which will also be those most probable. A new method of adjustment of chains of triangulation has been developed, and applied to a number of Indian series.

In this connection a quantity M has been introduced as a criterion of the strength of triangulation. It is based on General Ferrero's quantity "m", but also takes cognisance of the length of sides and general formation of a series of triangulation. This quantity has been taken out numerically for all the Indian series.

The quantity M permits of probable errors not only of side and azimuth but also of latitude and longitude being expressed at any point of the triangulation. Application has been made to all the circuits of India and the closing errors actually met with are found in good accordance with the theoretical probable errors based on M.

The case of probable errors after adjustment has also been considered. This is much more troublesome and involves very heavy work in the form of solution of numerous equations. The value of the enquiry, however, will be considerable, as an answer can be found to questions such as,

how often should extra base lines or Laplace stations be introduced? It appears that Laplace stations should be just as numerous as extra bases: and that the observation of extra bases alone is only a half measure not likely to improve matters much. It may be roughly compared to closing a traverse circuit for northing and omitting to do so for easting. The improvement due to adjustment is but briefly considered owing to force of circumstances; but the necessary equations have been solved for the cases of the N.W. Quadrilateral of the Indian triangulation and this question will be resumed when it is possible to do so. As has been mentioned the process involves the solution of a large number of simultaneous linear equations; and this has to be done for a large number of values of the absolute term in these equations. It is believed that several novelties have been introduced in this connection which may be of general interest. This question is treated on pages 126-153.

In Chapter IX the results of all the deflection observations are expressed in terms of Helmert's spheroid; the azimuth observations all having been adjusted on the Laplace conditions, none of which had been made use of during the simultaneous adjustment of the triangulation. The quantities given also permit of easy reference to any other spheroid which may at any future time be adopted. Certain observations in Russian Turkistan have also been added, as by means of the recent Indo-Russian connection these can now be stated in terms of the Indian survey. It has also been considered convenient to give a tabular statement of all determinations of g, so as to make a complete statement of gravity, not only its direction, but also its intensity. Full details of the pendulum operations are to be found in Professional Papers Nos. 10, 15.

These quantities are of immediate interest when the question of the form of the geoid and the underlying reasons for that form are considered. A start had been made with this question, which I had hoped to include in this work, but which must now be held over. An approximate method of finding the underground density anomalies by means of Poisson's equations seems possible. This is independent of the usual isostatic hypotheses, and may throw light on the whole question of isostasy.

For convenience of reference a certain number of various determinations of the figure of the earth have been included.

The computations incidental to the preparation of this work have been very heavy. Those of the earlier chapters I to IV have been performed mostly by Babu Mukundananda Acharya and Babu Hem Chandra Banerji, B.A. The solution of equations in Chapter VIII has been entirely carried out by Babu Diwan Chand Nanda, to whom I am especially indebted for his industry and accuracy in a troublesome and monotonous piece of work. The data in Chapter IX have been compiled by Babu Surendranath Mitra, M.R.A.S.

Mr. Sarat Kumar Mukerji has been responsible for the printing of the whole letter press.

DEHRA DUN,
15th Sept. 1917.

J. DE GRAAFF HUNTER.

CHAPTER I.

First method of finding the changes of coordinates of triangulated points due to changes in axes of the terrestrial spheroid and coordinates of origin.

1. The subject of the first 16 sections of this chapter was printed in abstracted form in 1912 to draw attention to some of the difficulties of the problem, and with the hope that some light might be thrown on it at the Triennial Geodetic Conference held at Hamburg in that year.

The spheroid on which the triangulation of India has been adjusted is now believed to be considerably in error, as from the nature of the case was inevitable. Owing to possible deflection of the plumb line at the origin of the survey at Kalianpur, the values of latitude and azimuth at that point are somewhat in doubt. The problem for solution was to find the changes in latitude, longitude and azimuth of all triangulated points in India due to changes in the adopted values of the axes of the terrestrial spheroid and in the adopted coordinates of the origin of the Survey.

2. The new spheroid which is adopted in the first 16 sections of this chapter is defined by

$$a = \text{semi major axis} = 6378200 \text{ metres}$$

$$\epsilon = \text{compression or flattening} = \frac{1}{298 \cdot 3} = \frac{a-b}{a}.$$

These values are given on page 173 of *"The Figure of the Earth and Isostasy, from Measurements in U.S.A."* Washington 1909, where they are said to be Dr. Helmert's latest values.

Heretofore the axes used in the Survey of India are those due to Everest, known as "Everest's constants, first set". The numerical values are—

$$a = 20{,}922{,}931 \cdot 80 \text{ feet}$$

$$\epsilon = \frac{1}{300 \cdot 8}$$

All base lines of the Survey of India have been expressed in terms of the Indian ten-foot standard, known as bar *A*. The base lines were not reduced to standard British feet but were

given as some number of times $\frac{A}{10}$ feet. In making use of Everest's constants we have accordingly been taking the semi major axis as

$$20,922,931\cdot80\ \frac{A}{10}\ \text{British feet.}$$

The value of A is given* as $3\cdot333,318,86\ Y$, Y being the British standard yard.

We accordingly have $\frac{A}{10} = 1 - \cdot000,004,342$ from which it follows that the semi major axis which has actually been used in India is

$$a = 20,922,840\cdot95 \text{ British feet.}$$

Similarly $$b = 20,853,284\cdot03 \quad ,, \quad ,,$$

3. Converting 6,378,200 metres into feet by means of the relation

$$1 \text{ metre} = 39\cdot370113 \text{ inches}$$

deduced by Benoit (*see "Raport du Yard au mètre, Paris 1896*) we get as our new semi major axis 20,925,871·23 British feet, and denoting by δa and δb the corrections which have to be applied to the values used in the Survey of India; we have

$$\delta a = +3030\cdot28 \text{ feet} = 923\cdot63 \text{ metres}$$

and $$\delta b = +2436\cdot78 \text{ feet} = 742\cdot73 \text{ metres}$$

Also since $e^2 = \epsilon\,(2-\epsilon)$ and $\delta\epsilon = \frac{1}{298\cdot3} - \frac{1}{300\cdot8} = \cdot000,027,86$ it follows that,

$$\delta e^2 = \cdot000,055,54\dagger$$

where e is the eccentricity.

4. It is now considered that the coordinates of the origin of the survey at Kalianpur require modification. Captain G. P. Lenox Conyngham R. E. observed a group of azimuths and latitudes round Kalianpur. His results gave the mean value reduced to Kalianpur

Latitude 24° 7′ 11″·57 Azimuth of Surantal 190° 27′ 6″·39.

The values heretofore adopted in the triangulation are,

Latitude 24° 7′ 11″·26 Azimuth of Surantal 190° 27′ 5″·10.

We have to apply corrections to the origin of $+0''\cdot31$ in latitude and $+1\cdot''29$ in azimuth. As regards the old value of azimuth a correction of $-1''\cdot1$ was applied to the observed value by General Walker in order to make azimuths observed at other parts of the triangulation agree with geodetic values. We are now annulling this by reverting to an observed value of azimuth.

5. Accordingly it is necessary to investigate equations giving the change in coordinates due to the changes of both axes of the spheroid and of the latitude of the origin and of the azimuth of a ray through it, as exhibited in the following table

---

* Account of the Operations of the G.T. Survey of India Vol. I. p. 28

† This corresponds to $\epsilon = \frac{1}{300\cdot8}$. Everest's actual value was $\frac{1}{300\cdot8017}$ and the corresponding value of δe^2 is ·000,055,58.

TABLE I.

	Old value	New value
Longitude of Kalianpur ...	... ...	77° 39′ 17″·57
Latitude of Kalianpur ...	24° 7′ 11″·26	24° 7′ 11″·57
Azimuth at Kalianpur of Surantal ...	190° 27′ 5″·10	190° 27′ 6″·39
Length of semi major axis ...	20,922,840·95 feet	20,925,871·23 feet (=6,378,200 metres)
Length of semi minor axis ...	20,853,284·03 feet	20,855,720·81 feet (=6,356,818, metres)
Compression	$\frac{1}{300 \cdot 8_{017}}$	$\frac{1}{298 \cdot 3}$

The latest value of longitude* is merely given for convenience of reference. Any change in longitude of origin is of course immediately applicable to the whole of the triangulation by simple addition (or subtraction).

6. The old triangulation was adjusted, that is to say its apparent errors were distributed, by a process following the method of least squares as closely as was thought to be practicable in view of the great number of observed angles involved. Owing to the errors in the chosen values of the axes, the equations which the errors were made to satisfy were not quite correct. In the first place the spherical excesses of the several triangles were computed with uncorrect values of the axes: but, owing to the smallness of these spherical excesses, the change on this account is not appreciable to 0.″01—the accuracy to which they were computed. None the less the error on this account being of a systematic kind—always of the same sign—will have had some small effect. With the "circuit equations" the case is less favourable. In following series of triangulation, which embrace much larger areas, the spherical excess becomes much more appreciable, and its value on the new spheroid differs from the old value by about one second in an area of 75 square degrees in Indian latitudes. This difference modifies the circuit equations. It is a smaller error than the errors generated in the triangulation, but is systematic.

The only theoretically accurate course would be to readjust all the triangulation. This would be a very large piece of work, and one object of tbe present paper is to avoid this labour by putting forward alternative methods, which will give the desired changes, with a departure from strict theoretical accuracy smaller than the errors due to fallible observations. The methods will also be applicable to any further changes that may be found desirable at any subsequent date.

7. The following notation is used

a = semi major axis

b = semi minor axis

ϵ = ellipticity or compression

e = eccentricity

ρ = radius of curvature to meridian $= a\,(1-e^2)\,(1-e^2\sin^2\lambda)^{-\frac{3}{2}}$

ν = normal terminated by the minor axis $= a\,(1-e^2\sin^2\lambda)^{-\frac{1}{2}}$

which is the other principal radius of curvature

* Account of the Operations of the G.T. Survey of India Vol. XVII p. xv.

$\beta^2 = \nu/\rho$
λ = latitude
L = longitude
A = azimuth, measured from South by West
u = change in latitude due to change of origin and axes
v = ,, longitude ,, ,, ,,
w = ,, azimuth ,, ,, ,,
c = distance between points whose coordinates are λ, L and $\lambda + \Delta\lambda$, L + Δ L.

As only very small values of c will be considered it is unnecessary to specify whether this distance is measured along a normal plain section or a geodesic line.

8. For small values of c

$$\left.\begin{aligned} \Delta\lambda &= -\frac{c}{\rho}\cos A \\ \Delta L &= -\frac{c}{\nu}\frac{\sin A}{\cos\lambda} \\ \Delta A &= -\frac{c}{\nu}\sin A\tan\lambda \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Differentiating* these equations with respect to A, λ, ρ, ν the corresponding changes of $\Delta\lambda$, Δ L, Δ A are obtained: and remembering that $\delta\,\Delta\lambda = \delta u$ and $\delta\lambda = u$ etc., we obtain

$$\left.\begin{aligned} \delta u &= \frac{c}{\rho}\cos A.\frac{\delta\rho}{\rho} + \frac{c}{\rho}\sin A.w \\ \delta v &= \frac{c}{\nu}\frac{\sin A}{\cos\lambda}.\frac{\delta\nu}{\nu} - \frac{c}{\nu}\frac{\cos A}{\cos\lambda}.w - \frac{c}{\nu}\frac{\sin A}{\cos^2\lambda}\sin\lambda.u \\ \delta w &= \frac{c}{\nu}\sin A\tan\lambda.\frac{\delta\nu}{\nu} - \frac{c}{\nu}\cos A\tan\lambda.w - \frac{c}{\nu}\sin A\sec^2\lambda.u \end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

These are three simultaneous partial equations from which u, v, w are to be determined. They express the small changes in u, v, w developed along a short (elementary) line in direction of azimuth A. Before they can be integrated it is necessary to define the route along which to travel. It might be supposed at first that the only important matter was the terminal points of the route: but it will be seen later that a different result is found from each route followed. The equations are not integrable in finite terms for all routes, and two special cases are now considered, firstly along a parallel of latitude and secondly along a meridian. These cases correspond to $A = 90°$ and $A = 0$ respectively. A means of dealing with the general case of an oblique curvilinear ray is given later, §13 *et seq.*

9. *Case I, when $A = 90°$.* In the case of a route along a parallel of latitude it is clear that $\dfrac{c}{\nu\cos\lambda} = dL$ and equations (2) can accordingly be written

$$\left.\begin{aligned} -\frac{du}{dL} &= \frac{\nu}{\rho}\cos\lambda.\,w \\ -\frac{dv}{dL} &= \frac{\delta\nu}{\nu} - \tan\lambda.\,u \\ -\frac{dw}{dL} &= \sin\lambda.\frac{\delta\nu}{\nu} - \sec\lambda.\,u \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3)$$

---

* The quantities $\frac{d\rho}{d\lambda}$, $\frac{d\nu}{d\lambda}$ were neglected as they contain the factor e^2.

Putting $\beta^2 = \dfrac{\nu}{\rho}$ it follows at once from (3) that

$$\frac{du^2}{dL^2} = -\frac{\nu}{\rho}\cos\lambda.\frac{dw}{dL}$$

$$= \beta^2 \sin\lambda\cos\lambda.\frac{\delta\nu}{\nu} - \beta^2 u$$

or $$\frac{d^2u}{dL^2} + \beta^2 u = \tfrac{1}{2}.\beta^2 \sin 2\lambda\,\frac{\delta\nu}{\nu} \qquad (4)$$

The solution of this is

$$u = \tfrac{1}{2}\sin 2\lambda\frac{\delta\nu}{\nu} + P\cos(\beta L) + Q\sin(\beta L) \qquad (5)$$

where P and Q are constants. Using (3) and differentiating (5) it follows that

$$-\beta^2\cos\lambda.\, w = \frac{du}{dL} = \beta\left\{-P\sin(\beta L) + Q\cos(\beta L)\right\}$$

$$w = \frac{1}{\beta\cos\lambda}\left\{P\sin(\beta L) - Q\cos(\beta L)\right\} \qquad (6)$$

To determine P and Q put $L = 0$ in (5) and (6)

$$\left.\begin{aligned} u_0 &= \tfrac{1}{2}\sin 2\lambda\,\frac{\delta\nu}{\nu} + P \\ w_0 &= -\frac{Q}{\beta\cos\lambda} \end{aligned}\right\} \qquad (7)$$

where the suffix zero indicates values at the beginning of the line.

Further, using (3) and (5)

$$-\frac{dv}{dL} = \frac{\delta\nu}{\nu} - \tan\lambda\left\{\tfrac{1}{2}\sin 2\lambda.\frac{\delta\nu}{\nu} + P\cos(\beta L) + Q\sin(\beta L)\right\}$$

whence

$$v - v_0 = -\frac{\delta\nu}{\nu}.\cos^2\lambda\, L + \frac{\tan\lambda}{\beta}\left\{P\sin(\beta L) + Q\left(1-\cos(\beta L)\right)\right\} \qquad (8)$$

longitude being measured from the starting point.

Expressing these equations in terms of seconds—they are at present in radian units—we write

$$\left.\begin{aligned} R'' &= \tfrac{1}{2}\sin 2\lambda.\frac{\delta\nu}{\nu}\operatorname{cosec} 1'' \\ P'' &= u''_0 - R'' \\ Q'' &= -\beta w''_0\cos\lambda \end{aligned}\right\} \qquad (9)$$

and

$$\left.\begin{aligned} u'' &= R'' + P''\cos(\beta L) + Q''\sin(\beta L) \\ v'' &= v''_0 - R''\cot\lambda.\frac{L^\circ}{57\cdot 3} + \frac{\tan\lambda}{\beta}\left\{P''\sin(\beta L) + Q''\left(1-\cos(\beta L)\right)\right\} \\ w'' &= \frac{1}{\beta\cos\lambda}\left\{P''\sin(\beta L) - Q''\cos(\beta L)\right\} \end{aligned}\right\} \qquad (10)$$

Since $\nu = a\,(1-e^2 \sin^2 \lambda)^{-\frac{1}{2}}$ it follows from logarithmic differentiation that

$$\frac{\delta\nu}{\nu} = \frac{\delta a}{a} + \frac{\sin^2\lambda}{1-e^2\sin^2\lambda}\,\frac{\delta e^2}{2}$$

Putting in the values of a, e, δa, δe^2 from §§3, 4 and expanding

$$\frac{\delta\nu}{\nu} = \cdot 000,\,144,\,83 + \cdot 000,\,027,\,77\,\sin^2\lambda + \cdot 000,\,000,\,18\,\sin^4\lambda + \quad . \; . \; . \qquad (11)$$

10. *Case, II, when $A = 0$.* In the case of a route along a meridian it is clear that $\frac{c}{\rho} = -\,d\lambda$ and equation (2) can accordingly be written

$$\left.\begin{aligned} \frac{du}{d\lambda} &= -\,\frac{\delta\rho}{\rho} \\ \frac{dv}{d\lambda} &= \frac{\rho}{\nu}\sec\lambda .\, w \\ \frac{dw}{d\lambda} &= \frac{\rho}{\nu}\tan\lambda .\, w \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (12)$$

Differentiating $\rho = a\,(1-e^2)\,(1-e^2\sin^2\lambda)^{-\frac{3}{2}}$ logarithmically

$$\frac{\delta\rho}{\rho} = \frac{\delta a}{a} - \delta e^2\left(\frac{1}{1-e^2} - \tfrac{3}{2}\cdot\frac{\sin^2\lambda}{1-e^2\sin^2\lambda}\right)$$

whence, expanding and putting in numerical values

$$\frac{\delta\rho}{\rho} = \cdot 000{,}144{,}83 - \cdot 000{,}055{,}54\left(1\cdot 00668 - \tfrac{3}{2}\sin^2\lambda - \tfrac{3}{2}e^2\sin^4\lambda \; . \; . \; . \; . \; . \; .\right)$$

$$= \cdot 000{,}088{,}92 + \cdot 000{,}083{,}31\,\sin^2\lambda + \cdot 000{,}000{,}55\,\sin^4\lambda$$

$$= \cdot 000{,}130{,}78 - \cdot 000{,}041{,}93\,\cos 2\lambda + \cdot 000{,}000{,}07\,\cos 4\lambda \; . \; . \; . \; . \; . \; . \; . \; . \qquad (13)$$

Integrating the first equation of (12)

$$u - u_0 = -\int\frac{\delta\rho}{\rho}.\,d\lambda$$

$$= -\,\cdot 000{,}130{,}78\;(\lambda-\lambda_0) + \Big[\cdot\cdot 000{,}020{,}97\,\sin 2\lambda - \cdot 000{,}000{,}02\,\sin 4\lambda \; . \; . \Big]_{\lambda_0}^{\lambda} \quad . \qquad (14)$$

From last equation of (12)

$$\frac{1}{w}\cdot\frac{dw}{d\lambda} = \frac{\rho}{\nu}\,\tan\lambda = \frac{1-e^2}{1-e^2\sin^2\lambda}\cdot\tan\lambda$$

Put $y = \sin^2\lambda$ and $dy = 2\,\sin\lambda\,\cos\lambda\,d\lambda$

Then

$$d\log w = \frac{1-e^2}{1-e^2y}\tan\lambda\cdot\frac{dy}{2\sin\lambda\cos\lambda}$$

$$= \tfrac{1}{2}\cdot\frac{1-e^2}{1-e^2y}\cdot\frac{dy}{1-y}$$

$$= \tfrac{1}{2}\left(\frac{1}{1-y} - \frac{e^2}{1-e^2y}\right)dy$$

Integrating

$$\log w = -\,\tfrac{1}{2}\log(1-y) + \tfrac{1}{2}\,(1-e^2y) + \text{constant}$$

$$w = K\sqrt{\frac{1-e^2y}{1-y}} \quad \text{where K is a constant}$$

$$w = \frac{\mathrm{K}\sqrt{1-e^2\sin^2\lambda}}{\cos\lambda} = \frac{\mathrm{K}a}{\nu\cos\lambda}$$

$$\therefore\quad w = w_0 \cdot \frac{\nu_0\cos\lambda_0}{\nu\cos\lambda} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

Again $$\frac{dv}{d\lambda} = \frac{1-e^2}{1-e^2\sin\lambda}\sec\lambda . \mathrm{K}\frac{\sqrt{1-e^2\sin^2\lambda}}{\cos\lambda}$$

$$\therefore\quad v = \mathrm{K}\left(1-e^2\right)\int\frac{d\lambda}{\cos^2\lambda\sqrt{1-e^2\sin^2\lambda}}$$

Putting $x = \sin\lambda$ then $dx = \cos\lambda\, d\lambda$ and

$$v = \mathrm{K}\left(1-e^2\right)\int\frac{dx}{(1-x^2)^{\frac{3}{2}}\sqrt{1-e^2x^2}}$$

Now $$\frac{1}{(1-x^2)^{\frac{3}{2}}\sqrt{1-e^2x^2}} = \frac{1}{(1-x^2)^{\frac{3}{2}}\sqrt{1-e^2+e^2(1-x^2)}}$$

$$= \frac{1}{\sqrt{1-e^2}} \cdot \frac{1}{(1-x^2)^{\frac{3}{2}}}\left\{1-\tfrac{1}{2}k(1-x^2)+\tfrac{3}{8}k^2(1-x^2)^2 \; . \; . \; .\right\} \quad . \; . \; . \quad (16)$$

where $k = \dfrac{e^2}{1-e^2} = 0 \cdot 006,682,2$

$$\therefore\quad v = K\sqrt{1-e^2}\int\left\{\frac{dx}{(1-x^2)^{\frac{3}{2}}} - \tfrac{1}{2}k . \frac{1}{\sqrt{1-x^2}} + \tfrac{3}{8}x^2\sqrt{1-x^2} \; . \; . \; . \; . \; .\right\}dx$$

$$= K\sqrt{1-e^2}\left[\frac{x}{\sqrt{1-x^2}} - \tfrac{1}{2}k\sin^{-1}x + \tfrac{3}{8}k^2\left(\tfrac{1}{2}x\sqrt{1-x^2}+\tfrac{1}{2}\sin^{-1}x\right) \; . \; .\right]$$

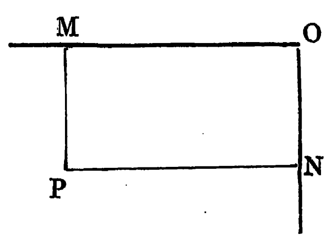
$$= K\sqrt{1-e^2}\left[\tan\lambda - \tfrac{1}{2}k\lambda + \tfrac{3}{16}k^2\left(\sin\lambda\cos\lambda+\lambda\right) \; . \; . \; . \; . \; . \; . \; .\right]$$

Collecting results and expressing them in terms of seconds we get

$$\left.\begin{aligned}
u''-u_0'' &= -\cdot 000,130,78\,(\lambda''-\lambda_0'') + 4\cdot 325\,(\sin 2\lambda - \sin 2\lambda_0) - \cdot 0035\,(\sin 4\lambda - \sin 4\lambda_0)\\
v''-v_0'' &= w_0'' . \frac{\nu_0\cos\lambda_0}{a}\sqrt{1-e^2}\Big[\tan\lambda - \cdot 003,332,7\lambda''\sin 1'' + \cdot 000,004,2\sin 2\lambda\Big]_{\lambda_0}^{\lambda}\\
w'' &= w_0'' . \frac{\nu_0\cos\lambda_0}{\nu\cos\lambda} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; .
\end{aligned}\right\} \quad . \quad (17)$$

The value of $\log\sqrt{1-e^2}$ is $\bar{1}\cdot 9985538$.

11. With the equations (10) and (17) we can now deduce the values of u, v, w for any point P. Starting from the origin O, we may compute along the parallel OM and find the values at M. Using these as initial values we can then proceed along the meridian MP and get values for P.



Or we may first proceed along meridian ON and then along the parallel NP.

The values arrived at by the two routes are not identical. This is inevitable. The discrepancy in azimuth is the change in spherical excess on the given area from the old to the new spheroid. We shall proceed to consider the discrepancies which occur. To study these

discrepancies the values of the changes were computed to five places of decimals. In the first place it was considered convenient to take as origin for this computation the point whose latitude and longitude were 24°, 78° on the old spheroid. The double values of u, v, w, for this point differ by a small amount and in view of what follows the following mean value of w was taken :—

$$w = \frac{\dfrac{w_x}{x} + \dfrac{w_y}{y}}{\dfrac{1}{x} + \dfrac{1}{y}} = \frac{y w_x + x w_y}{x + y}$$

The suffix x indicates the value found by route ONP
and the suffix y „ „ „ „ „ „ OMP
and $x = \mathrm{PN}$, $y = \mathrm{PM}$, but x and y are always treated as positive. The values of u_x, u_y and v_x, v_y were practically identical.

Starting from this origin by means of our equations the values exhibited in the following three tables are obtained :—

TABLE II.

LATITUDE (u'').

Lat.	Long.	58°	63°	68°	73°	78°	83°	88°	93°	98°
34°	u_x	−*2·07723*	−*2·65069*	−*3·09166*	−*3·39675*	−*3·56361*				
	u_y	−2·50858	−2·89432	−3·20019	−3·42385					
	$u_x - u_y$	+0·43135	+0·24363	+0·10853	+0·02710	0·00000				
29°	u_x	−*0·27462*	−*0·75738*	−*1·13306*	−*1·39881*	−*1·55258*				
	u_y	−0·49755	−0·88329	−1·18916	−1·41282					
	$u_x - u_y$	+0·22293	+0·12591	+0·05610	+0·01401	0·00000				
24°	u_x, u_y	+*1·40241*	+*1·01667*	+*0·71080*	+*0·48714*	+*0·34738*	+*0·29261*	+*0·32323*	+*0·43903*	+*0·63911*
	$u_x - u_y$	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000
19°	u_x	+*2·97072*	+*2·68699*	+*2·45452*	+*2·27510*	+*2·15009*	+*2·08045*	+*2·06672*	+*2·10900*	+*2·20697*
	u_y	+3·20512	+2·81938	+2·51351	+2·28985		+2·09532	+2·12594	+2·24174	+2·44182
	$u_x - u_y$	−0·23440	−0·13239	−0·05899	−0·01475	−0·00000	−0·01487	−0·05922	−0·13274	−0·23485
14°	u_x	+*4·44968*	+*4·27179*	+*4·11549*	+*3·98199*	+*3·87299*	+*3·78724*	+*3·72751*	+*3·69353*	+*3·68557*
	u_y	+4·92732	+4·54158	+4·23571	+4·01205		+3·81752	+3·84814	+3·96394	+4·16402
	$u_x - u_y$	−0·47764	−0·26979	−0·12022	−0·03006	−0·00000	−0·03028	−0·12063	−0·27041	−0·47845

TABLE III.

LONGITUDE (v).

		58°	63°	68°	73°	78°	83°	88°	93°	98°
	v_x	+11·82474	+8·92142	+5·98821	+3·03302	+0·06397				
34°	v_y	+11·55396	+8·70715	+5·84003	+2·95734					
	v_x-v_y	+ 0·27078	+0·21427	+0·14818	+0·07568	0·00000				
	v_x	+11·03498	+8·28493	+5·51411	+2·72776	−0·06874				
29°	v_y	+10·97521	+8·23535	+5·47878	+2·70942					
	v_x-v_y	+0·05977	+0·04958	+0·03533	+0·01834	0·00000				
24°	v_x, v_y	+10·45012	+7·80729	+5·15103	+2·48448	−0·18914	−2·86654	−5·54441	−8·21943	−10·88833
	v_x-v_y	0·00000	0·00000	0·00000	0 00000	0·00000	0·00000	0·00000	0·00000	0·00000
	v_x	+10·03969	+7·46470	+4·83195	+2·29303	−0·30049	−2·89693	−5·49462	−8·09189	−10·68705
19°	v_y	+ 9·96451	+7·41142	+4·84792	+2·27645		−2·88036	−5·46060	−8·03863	−10·61192
	v_x-v_y	+0·07518	+0·05328	+0·03405	+0·01658	0·00000	−0·01657	−0·03402	−0·05326	− 0·07513
	v_x	+9·78031	+7·23887	+4·69380	+2·14557	−0·40531	−2·95831	−5·51288	−8·06847	−10·62452
14°	v_y	+9·50738	+7·03876	+4·56259	+2·08063		−2·89337	−5·38170	−7·86848	−10·35171
	v_x-v_y	+0·27293	+0·20011	+0·13121	+0·06494	+0·00000	−0·06494	−0·13118	−0·20004	− 0·27281

TABLE IV.

AZIMUTH (w).

		58°	63°	68°	73°	78°	83°	88°	93°	98°
	w_x	+8·78306	+6·98775	+5·13898	+3·25090	+1·33803				
34°	w_y	+5·83531	+4·75699	+3·64230	+2·49973					
	w_x-w_y	+2·94775	+2·23076	+1·49668	+0·75117	0·00000				
	w_x	+6·97539	+5·60208	+4·18593	+2·73775	+1·26862				
29°	w_y	+5·53262	+4·51023	+3·45336	+2·37006					
	w_x-w_y	+1·44277	+1·09185	+0·73257	+0·36769	0·00000				
24°	w_x, w_y	+5·29810	+4·31905	+3·30698	+2·26960	+1·21485	+0·15080	−0·91440	−1·97260	−3·01574
	w_x-w_y	0·00000	0·00000	0·00000	0 00000	0·00000	0·00000	0·00000	0·00000	0·00000
	w_x	+3·71867	+3·11339	+2·48428	+1·83619	+1·17400	+0·50282	−0·17217	−0·84588	−1·51311
19°	w_y	+5·11995	+4·17383	+3·19579	+2·19329		+0·14573	−0·88365	−1·90627	−2·91483
	w_x-w_y	−1·40128	−1·06044	−0·71151	−0·35710	0 00000	+0·35709	+0·71148	+1·06039	+1·40122
	w_x	+2·20928	+1·96355	+1·70277	+1·42897	+1·14420	+0·85068	+0·55065	+0·24640	−0·05974
14°	w_y	+4·99000	+4·06789	+3·11467	+2·13762		+0·14203	−0·86122	−1·85789	−2·84036
	w_x-w_y	−2·78072	−2·10434	−1·41190	−0·70865	0·00000	+0·70865	+1·41187	+2·10429	+2·78062

12. Denote the distances PN, PM (in any linear unit, not in angular units) by x and y then.

$$x = (L - L_0)\, \nu\cos\lambda$$

$$y = \int_{\lambda_0}^{\lambda} \rho d\lambda.$$

By inspection of the numbers shown in tables *II, III, IV* the following equations are found to be approximately true:

$$\left.\begin{aligned} u_x - u_y &= Ax^2y \\ v_x - v_y &= Bxy^2 \\ w_x - w_y &= Cxy \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (18)$$

where *A, B, C* are quantities varying slightly with the latitude, but which may be treated as constants with their mean values over any area with which we shall need to deal. The last equation simply expresses that the closing error in azimuth is equal to the change in spherical excess.

Now $u_x - u_y$ is what we will call the "closing error in latitude" in proceeding round the circuit OMPN; $v_x - v_y$ and $w_x - w_y$ being corresponding quantities for longitude and azimuth. Over any elementary area

$$\left.\begin{aligned} dU &= d\ (u_x - u_y) = 2A\, x\ dxdy \\ dV &= d\ (v_x - v_y) = 2B\, y\ dxdy \\ dW &= d\ (w_x - w_y) = C\ dxdy \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (19)$$

By integrating over any area the closing error of the circuit enclosing that area is found.

To find the values of u, v, w then which would be obtained by proceeding along any route it is only necessary to find the values of u_x, v_x, w_x (or u_y, v_y, w_y) and apply the closing error with the correct sign. Integrating (18) it follows for *moderate areas,*

$$\left.\begin{aligned} U &= 2A\, \bar{x}a \\ V &= 2B\, \bar{y}a \\ W &= Ca \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (20)$$

where a is the area of the circuit and $\bar{x}\ \bar{y}$ are the coordinates of its centre of gravity. We say for moderate areas because the coordinates x and y are curvilinear: but for the areas we shall require to apply the formulæ to, x and y may be treated as rectilinear coordinates.

13. By means of the above equations it is possible to find the result of the change of axes and origin as computed along any line of any curvature or along any route whatever, by computing first along a parallel and then along a meridian (or in the reverse order) and then applying the "closing errors" of the circuit formed by the line in question and the parallel and meridian.

This, then, would solve the problem as for as solitary lines were concerned. When we come to a network of lines the case is different, for several values of the changes which occur at a point can be found corresponding to the several possible routes by which the point can be reached. In view of the fact that most of the triangulation of India is along meridian or parallel (see triangulation chart at end), the following procedure is suggested:—

(1). Select central meridian and parallel for India (Burma will be dealt with separately). The selected meridian is 78° and the selected parallel 24°N.

(2). Assume the values of u, v, w found by the forumlæ on these lines, which we will call axes, to be correct. We have then to distribute the closing errors in PM and PN. (see fig. §11).

(3). If PM is a meridional series the computations fixing the *length* PM depend only in a small measure on the size of the earth's axes. The way in which these axes have come in is through the spherical excess. In nearly all triangulation in the Survey of India, the spherical excess is such a small quantity that the change of axes proposed will not appreciably affect it (to $0''\cdot 01$). There is reason then for assuming that the length PM is correct. In the same way the length PN may be regarded as correct. If then u_y and v_x are taken for the changes in coordinates of P there should be no error to the first order: and as the values of u, v are so small the second order quantities may surely be neglected.

(4). This process would hold for the corners of circuits formed by meridian and longitudinal series, though some modification would be more correct for oblique series. In the Indian triangulation meridional and longitudinal series are the rule. Oblique series occur practically only along the coast of the Bay of Bengal and along the first range of the Himalayas. (See index chart of triangulation at end). As far as latitude and longitude are concerned we should not be committing much error in accepting the values of latitude and longitude, u_y and v_x.

(5). Now consider the azimuth change. This can be found from the change in position of two contiguous points. If we take two points originally on the same latitude whose changes are u_y and $u_y + \frac{\delta u_y}{\delta L} dL$ the azimuth change on the line joining them is

$$\rho \frac{\delta u_y}{\delta L} dL \frac{1}{\nu \cos \lambda . dL} = \frac{\rho}{\nu \cos \lambda} \cdot \frac{\delta u_y}{\delta L} = w_y$$

whereas the azimuth change deduced from two points originally on the same longitude is

$$\frac{\nu \cos \lambda}{\rho} \frac{\delta v_x}{\delta \lambda} = w_x$$

It has been seen that the azimuth closing errors is $C. x\, y$ where OM $= x$ ON $= y$. and C is a quantity which varies slightly with the latitude. Treating C as a constant and equal to its mean value over the area in consideration is permissible. This will be satisfied if the azimuth error is put into the lines PN and PM to amount proportional to their lengths.

This gives
$$\frac{\frac{w_x}{y} + \frac{w_x}{x}}{\frac{1}{y} + \frac{1}{x}} = \frac{x w_y + y w_x}{x + y}$$

as the best correction to the azimuth at P.

(6). The difference $w_y - w_x$ is not to be regarded as an error contained in the value of the angle MPN. Its effects is to alter the curvature of the lines PM and PN.

(7). In the case then of a gridiron system of meridional and longitudinal series at regular intervals all of equal weight, it seems that the best values we could assign to the changes are u_y, v_x, $\frac{xw_y + yw_x}{x+y}$. In this case the next step in correcting the triangulation would be to find the changes of intermediate points on the series as follows:—

$$\begin{array}{c} u_y \\ v_x \end{array} \text{ for change in } \begin{array}{c} \text{latitude} \\ \text{longitude} \end{array} \text{ along a } \begin{array}{c} \text{meridian} \\ \text{parallel} \end{array}$$

and for the other coordinate and azimuth to simply interpolate between the terminal values.

(8). A difficulty arises when the actual points of the triangulation series are considered. For if the above formula for two adjacent points A and B is used, the difference of coordinates will not exactly give the correct change of azimuth. Adopting the rule of computing the azimuth from the coordinates, a different azimuth change on a ray going east from that found on a ray going south is arrived at. That is to say the actual change of $w_x - w_y$ would be forced into the single angle formed by these rays. To avoid this it appears better to take the azimuth change to be $\frac{xw_y + yw_x}{x+y}$ and the change in coordinates of *one point only* to be given by u_y, v_x, and compute the coordinates of second adjacent point from this with the corrected azimuth (*i. e.* old azimuth $+ \frac{xw_y + yw_x}{x+y}$) and the old value of the distance c.

The computation alluded to in (8) may be performed with tables such as are given in the "Auxiliary Tables"* prepared for the new values of the axes: or we may at once deduce the changes in the position of the second point B by differentiation of the equations for $\Delta\lambda$, ΔL and ΔA.

The equations are:—

$$\Delta\lambda = -\frac{c}{\rho}\cos A - \tfrac{1}{2}\frac{c^2}{\rho\nu}\sin^2 A \tan\lambda = \delta_1\lambda + \delta_2\lambda$$

$$\Delta L = -\frac{c}{\nu}\cdot\frac{\sin A}{\cos\lambda} + \tfrac{1}{2}\frac{c^2}{\nu^2}\cdot\frac{\sin 2A\tan\lambda}{\cos\lambda} = \delta_1 L + \delta_2 L$$

$$\Delta A = -\frac{c}{\nu}\sin A\tan\lambda + \tfrac{1}{4}\frac{c^2}{\nu^2}(1 + 2\tan^2\lambda) = \delta_1 A + \delta_2 A$$

being equations (1) carried to an extra term in consideration of the larger value of c now contemplated.

Differentiating we have at once

$$\delta\Delta\lambda = \delta_1\lambda\left(-\frac{\delta\rho}{\rho} - \tan A.w\right) + \delta_2\lambda\left(-\frac{\delta\rho}{\delta} - \frac{\delta\nu}{\nu} + 2\cot A.w + \frac{2}{\sin 2\lambda}u\right)$$

$$\delta\Delta L = \delta_1 L\left(-\frac{\delta\nu}{\nu} + \cot A.w + \tan\lambda\, u\right) + \delta_2 L\left\{-\frac{2\delta\nu}{\nu} + 2\cot 2A.w + (\cot\lambda + 2\tan\lambda)\, u\right\}$$

* Auxiliary Tables of the Survey of India, Dehra Dūn, 1906.

$$\delta\Delta A = \delta_1 A\left(-\frac{\delta v}{v} + \cot A.w + \frac{2}{\sin 2\lambda}u\right) + \delta_2 A\left(-\frac{2\delta v}{v} + \frac{4\tan\lambda\sec^2\lambda}{1+2\tan^2\lambda}u + 2\cot 2A.w\right)$$

In above u, v, w are the values found for one end of the base A: the values for the other end B are then $u + \delta\Delta\lambda$, $v + \delta\Delta L$ $w + \delta\Delta A$.

A third method is to reach B by proceeding first along the parallel AC through A and then down the meridian CB through B, by means of the formulæ (or tables) already given: and then by applying the closing error of the area ACB.

It appears, then, that the expressions u_y, v_x, $\frac{xw_y + yw_x}{x+y}$ may be taken to represent the changes in latitude, longitude and azimuth respectively of any point in India (excluding Burma) with the restriction that adjacent points must be treated differently, the changes for the second point being deduced by one of the three methods just explained. On this basis the results may be given in convenient tabular form. They will represent the changes with accuracy considerably greater than the accuracy with which the points can be considered to be fixed in space by triangulation.

14. These values are believed to be satisfactory for all the purposes for which they can be used. As far as map producing goes the discrepancies are negligible. For geodetic purposes we require to know the absolute corrections to latitudes, longitudes and azimuths of a base where a junction is to be made with another survey—such as the Russian survey, or the Burma survey. We can do this as described in § 13 for one end of the base and then compute the coordinates of the other end of the base from a knowledge of its length. In the case of Burma the triangulation has not yet been adjusted. It will perhaps be adjusted with the new values of the axes and made to fit on to the most eastern series of the North-East Quadrilateral, *viz.*, the Shillong Meridional Series, after this has been corrected for change of axes.

We also wish to known corrections to triangulated latitudes or azimuths at stations where these quantities have also been observed astronomically, so as to know the actual plumb-line deflections. As regards latitude we have uncertainty of perhaps $0''\cdot 1$ on account of axes change after leaving the central latitude by $10°$, *i. e.* one part in 360,000 which is of the order of accuracy of our base-lines in India. The error generated in the triangulation must eventually be greater than this. The same argument holds as regards the azimuth, where the uncertainty of change due to change of axes, and due to error generated in triangulation are necessarily larger numbers when expressed in seconds of arc than occur in the latitude. The astronomic observations for azimuth are less precise, considered from point of view of plumb-line deflection, than the latitude observations. Apart from these considerations an error in plumb-line deflection in latitude of $0''\cdot 1$ is of little account. In India we have plume-line deflections of over $50''$ and, at least at present, tenths of second are too minute to be taken account of in any discussion of deflections.

15. It seems then that the method sketched above is sufficiently precise for the geodetic uses to which the results can be put, and higher accuracy could not be applied with advantage to the results of triangulation. The method of § 13 is applicable to points which can be reached by either route (meridian or parallel) without the route departing out of the region of triangulation. Thus while it applies to all the triangulation in India which has been adjusted, it could not be fairly applied without modification to Burma, for this would imply the existence of triangulation across the Bay of Bengal. As the Burma triangulation remains to be adjusted, this does not matter and it will only be necessary to apply the method as far as the Shillong Meridional Series, which can be done very satisfactorily, the more so as our selected central latitude crosses this series.

16. The Survey of India was asked in 1912 by the Siamese Survey Department to furnish the best possible values of the coordinates of Bangkok. The way in which this has been done will serve as a good illustration of the method of using the closing error to determine the changes which occur along a route which is neither meridional nor longitudinal. As far as longitude 90° the route may be taken to follow the central parallel, latitude 24° (*see* triangulation chart at end). From these it proceeds along the Burma Coast Series down to latitude 13° 45′, and thence to Bangkok along latitude 13° 45′. In this case then we first compute along parallel 24° up to longitude 98°: we then proceed along meridian 98° down to latitude 13° 45′. The result at this point is found from tables II, III, IV by extrapolation to be

$$u_y = +\ 4\cdot248$$
$$v_y = -10\cdot339$$
$$w_y = -\ 2\cdot837$$

Now *treating longitude 98° as axis* from which x is measured, we evaluate the closing errors over the area between the Coast series and latitude 24° and meridian 98° and get

$$\Sigma\, Ax^2y = U = +\cdot017$$
$$\Sigma\, Bxy^2 = V = -\cdot026$$
$$\Sigma\, Cxy = W = +\cdot426$$

Hence the changes at latitude 13° 45′, longitude 98°, as *determined by the route following the Burma Coast Series,* are $u_y + U,\ v_y + V,\ w_y + W$.

One further correction remains. The Bangkok Series which emanates from this point is expressed in "preliminary terms"—it was computed from preliminary values of the side from which it emanates. Later values of this side, found after the Coast Series had been computed from the preliminary value of the side, require the following changes to be applied to the beginning of the Bangkok Series, *viz.*

in	latitude	...	$-1''\cdot80$
	longitude	...	$-0''\cdot17$
	azimuth	...	$+5''\cdot50$

Combining these we arrive at the changes to be made at latitude 13° 45°, longitude 98°.

	Latitude	Longitude	Azimuth
By parallel and meridian route	+4·248	−10·339	−2·837
Correction to bring into terms of Burma Coast Series route ...	+0·017	− 0·026	+0·426
Correction from "preliminary terms"	−1·80	− 0·17	+5·50
Total ...	+2·47	−10·54	+3·09

With these initial values by computing along parallel 13° 45′ up to longitude 100° 33′ 3″·5 the old value of the longitude of Phukhao Thong Station* in Bangkok we get the changes

$$u = +\ 2''\cdot34$$
$$v = -\ 11''\cdot82$$
$$w = +\ 2''\cdot9$$

To bring into Greenwich terms the further correction − 2′ 27″·18 is required to the longitude, the final corrections being

+ 2″·34	in latitude
− 2′ 39″·00	in longitude
+2″·9	in azimuth

* Phukhao Thong Station is the most easterly triangulated point shown on the triangulation chart.

Owing to an unfortunate confusion of the quantities $u_y\ v_y\ w_y$ with $u_x\ v_x\ w_x$ the following corrections were wrongly supplied to the Royal Survey Department Siam in 1912

$$+1''\cdot73 \quad \text{in latitude}$$
$$-2'\ 39''\cdot32 \quad \text{longitude}$$
$$+5''\cdot7 \quad \text{azimuth}$$

17. So far the particular case of definite numerical values of $\frac{\delta a}{a}$, δe^2, $u_0\ w_0$ has been considered. It is desirable to put the solution in a form in which the results of any desired change can be calculated rapidly.

Let $u_1,\ u_2,\ u_3,\ u_4,$ be the changes in latitude respectively due to $\delta a = 1000$ metres, $\delta b = 1000$ metres, $u_0 = 1''$, $w_0 = 1''$ with corresponding notation for v and w. Since the quantities involved are small it is clear that

$$u = \delta a.\ u_1 + \delta b.\ u_2 + u_0.\ u_3 + w_0 u_4 \quad \ldots\ldots\ldots \quad (18)$$

with similar equations for v and w: where da, db are expressed in kilometers and u_0, w_0 in seconds. If values of $u_1,\ u_2,\ u_3,\ u_4$ are tabulated, equation (18) will enable the quantities $u,\ v,\ w$ to be evaluated for any desired case. Of the quantities δa, δb, δe^2 any two may be regarded as independent, the third being determined from the result of differentiating $b = a\sqrt{1-e^2}$ logarithmically.

i. e.
$$\frac{\delta a}{a} - \frac{\delta b}{b} = \tfrac{1}{2}\frac{\delta e^2}{1-e^2} \quad \ldots\ldots\ldots \quad (19)$$

When $\delta a = \delta b = 1000$ metres $= 39370\cdot113$ inches $= 3280\cdot843$ feet

$$\tfrac{1}{2}\frac{\delta a}{a}\operatorname{cosec} 1'' = 16\cdot1718 \qquad \text{of which the logarithm is } 1\cdot2087596$$

$$\tfrac{1}{2}\frac{\delta b}{b}\operatorname{cosec} 1'' = 16\cdot2258 \qquad ,, \quad ,, \quad ,, \quad 1\cdot2102058$$

Also

$$\frac{\delta\nu}{\nu} = \frac{\delta a}{a} + \frac{\sin^2\lambda}{1-e^2\sin^2\lambda}\cdot\frac{\delta e^2}{2}$$

$$= \frac{\delta a}{a}\left(1 + \frac{(1-e^2)\sin^2\lambda}{1-e^2\sin^2\lambda}\right) - \frac{\delta b}{b}\cdot\frac{(1-e^2)\sin^2\lambda}{1-e^2\sin^2\lambda} \quad \ldots\ldots \quad \text{by (19)}$$

Hence equation (9) may be written, omitting the dashes

$$\left.\begin{aligned} R &= \sin 2\lambda\left\{16\cdot1718\left(1 + \frac{(1-e^2)\sin^2\lambda}{1-e^2\sin^2\lambda}\right)\delta a - 16\cdot2258\,\frac{(1-e^2)\sin^2\lambda}{1-e^2\sin^2\lambda}.\delta b\right\} \\ P &= u_0 - R \quad \ldots\ldots\ldots \\ Q &= w_0\,\beta\cos\lambda \quad \ldots\ldots\ldots \end{aligned}\right\} \quad (20)$$

where δa, δb are expressed in kilometres and u_0 and w_0 in seconds.

18. *Along a parallel of latitude*

Case I, when $\delta a = 1$, $\delta b = 0$, $u_0 = 0$, $w_0 = 0$

By (20)
$$\left.\begin{aligned} R &= 16\cdot1718\sin 2\lambda\left(1 + \frac{(1-e^2)\sin^2\lambda}{1-e^2\sin^2\lambda}\right) = -P \\ Q &= 0 \end{aligned}\right\} \quad \ldots\ldots \quad (21)$$

and (10) becomes

$$\left.\begin{aligned} u &= R\left(1 - \cos(\beta L)\right) \\ v - v_0 &= -R\left(\frac{L^\circ \cot\lambda}{57 \cdot 3} + \frac{1}{\beta}\tan\lambda\sin(\beta L)\right) \\ w &= -\frac{R}{\beta}\sec\lambda\sin(\beta L) \end{aligned}\right\} \quad . \quad . \quad (22)$$

where L is expressed in degrees.

Case II, when $\delta a = 0,\ \delta b = 1,\ u_0 = 0,\ w_0 = 0$

From (20)

$$R = -16 \cdot 2258 \sin 2\lambda. \frac{(1 - e^2)\sin^2\lambda}{1 - e^2\sin^2\lambda} \quad . \quad . \quad . \quad . \quad (23)$$

and equations (22) hold for this case also.

Case III, when $\delta a = 0,\ \delta b = 0,\ u_0 = 1,\ w_0 = 0$

In this case $\frac{\delta\nu}{\nu} = 0$ so that

$$\left.\begin{aligned} R &= 0 \\ P &= u_0 = 1 \\ Q &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (24)$$

and

$$\left.\begin{aligned} u &= \cos(\beta L) \\ v - v_0 &= \frac{1}{\beta}\tan\lambda\sin(\beta L) \\ w &= \frac{1}{\beta}\sec\lambda\sin(\beta L) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (25)$$

Case IV, when $\delta a = 0,\ \delta b = 0,\ u_0 = 0,\ w_0 = 1$

$$\left.\begin{aligned} R &= \mathrm{P} = 0 \\ Q &= -\beta\cos\lambda \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (26)$$

and

$$\left.\begin{aligned} u &= -\beta\cos\lambda\sin(\beta L) \\ v - v_0 &= -\sin\lambda\left(1 - \cos(\beta L)\right) \\ w &= \cos(\beta L) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (27)$$

19. From (10) and (19)

$$\frac{\delta\rho}{\rho} = \frac{\delta a}{a} - 2\left(\frac{\delta a}{a} - \frac{\delta b}{b}\right)\left(1 - \tfrac{3}{2}\frac{(1 - e^2)\sin^2\lambda}{1 - e^2\sin^2\lambda}\right)$$

$$= \frac{\delta a}{a} - 2\left(\frac{\delta a}{a} - \frac{\delta b}{b}\right)\left\{1 - \tfrac{3}{2}(1 - e^2)\sin^2\lambda(1 + e^2\sin^2\lambda + \quad . \quad . \quad . \quad . \quad)\right\}$$

$$= \frac{\delta a}{a} - 2\left(\frac{\delta a}{a} - \frac{\delta b}{b}\right)\left\{1 - \tfrac{3}{2}(1 - e^2)\left(\frac{1 - \cos 2\lambda}{2} + \frac{e^2}{8}(3 - 4\cos 2\lambda + \cos 4\lambda) \quad . \quad . \quad . \quad . \right)\right\}$$

Hence from (12)

$$u - u_0 = -\int_{\lambda_0}^{\lambda} \frac{\delta\rho}{\rho}\, d\lambda$$

$$= -\frac{\delta a}{a}(\lambda - \lambda_0) + 2\left(\frac{\delta a}{a} - \frac{\delta b}{b}\right)\left\{0 \cdot 2513(\lambda - \lambda_0) + 0 \cdot 3750(\sin 2\lambda - \sin 2\lambda_0) - 0 \cdot 0003(\sin 4\lambda - \sin 4\lambda_0)\right\}$$

Expressing in seconds and introducing δa, δb expressed in kilometres and λ, λ_0 in degrees this becomes

$$u - u_0 = \delta a \left\{ -0 \cdot 2807 \, (\lambda^\circ - \lambda_0^\circ) + 24 \cdot 26 \, (\sin 2\lambda + \sin 2\lambda_0) - 0 \cdot 019 \, (\sin 4\lambda - \sin 4\lambda_0) \right\}$$

$$+ \delta b \left\{ -0 \cdot 2847 (\lambda^\circ - \lambda_0^\circ) - 24 \cdot 34 \, (\sin 2\lambda - \sin 2\lambda_0) + 0 \cdot 019 (\sin 4\lambda - \sin 4\lambda_0) \right\} \quad (28)$$

20. *Along a meridian*

Case I, when $\delta a = 1$, $\delta b = 0$, $u_0 = 0$, $w_0 = 0$

By (28) and (17)

$$\left. \begin{aligned} u &= -0 \cdot 2807 (\lambda^\circ - \lambda_0^\circ) + 24 \cdot 26 \, (\sin 2\lambda - \sin 2\lambda_0) - 0 \cdot 02 \, (\sin 4\lambda - \sin 4\lambda_0) \\ v - v_0 &= 0 \\ w &= 0 \end{aligned} \right\} \quad (29)$$

Case II, when $\delta a = 0$, $\delta b = 1$, $u_0 = 0$, $w_0 = 0$

By (28) and (17)

$$\left. \begin{aligned} u &= -0 \cdot 2847 \, (\lambda^\circ - \lambda_0^\circ) - 24 \cdot 34 \, (\sin 2\lambda - \sin 2\lambda_0) + 0 \cdot 02 \, (\sin 4\lambda - \sin 4\lambda_0) \\ v - v_0 &= 0 \\ w &= 0 \end{aligned} \right\} \quad (30)$$

Case III, when $\delta a = 0$, $\delta b = 0$, $u_0 = 1$, $w_0 = 0$

Then
$$\left. \begin{aligned} u &= u_0 = 1 \\ v - v_0 &= 0 \\ w &= 0 \end{aligned} \right\} \quad \ldots \ldots \ldots \quad (31)$$

Case IV, when $\delta a = 0$, $\delta b = 0$, $u_0 = 0$, $w_0 = 1$

Then
$$\left. \begin{aligned} u - u_0 &= 0 \\ v - v_0 &= \frac{\nu_0 \cos \lambda_0}{a} . \sqrt{1 - e^2} \left[\tan \lambda - 0 \cdot 000{,}058{,}2 \lambda^c + 0 \cdot 000{,}004{,}2 \sin 2\lambda \right]_{\lambda_0}^{\lambda} \\ w &= \frac{\nu_0 \cos \lambda_0}{\nu \cos \lambda} \quad \ldots \ldots \end{aligned} \right\} \quad (32)$$

21. With the equations of §18, 20 changes can be computed at any point for any case. There are two possible routes. As an example consider Case I. We can first proceed along a parallel to the appropriate longitude. In proceeding thence along a meridian we have to apply Cases I. III and IV (because the values given by (22) now become initial values), and so find the values $u_y \; v_y \; w_y$: secondly we can proceed along a meridian and afterwards along a parallel and so find the second set of values $u_x \, v_x \, w_x$,

These computations have been made and the results for every degree square corner, so far as concerns the Indian triangulation, are exhibited in the following tables.

TABLE V.

Case I.—$\delta a =$ 1 km.

Values of u_x in seconds.

Lat. \ Long.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°
36°	Positive	2·044	1·979	1·920	1·867	1·820	1·779	1·743	1·713	1·689	1·670	1·658	1·651	1·650	1·655	1·666
35		2·037	1·974	1·917	1·865	1·819	1·779	1·744	1·715	1·692	1·674	1·662	1·655	1·654	1·659	1·669
34		2·003	1·942	1·880	1·836	1·701	1·752	1·718	1·690	1·667	1·649	1·637	1·631	1·630	1·635	1·645
33		1·942	1·883	1·828	1·779	1·736	1·698	1·665	1·637	1·615	1·598	1·586	1·580	1·579	1·584	1·594
32		1·855	1·797	1·744	1·696	1·654	1·616	1·584	1·557	1·536	1·519	1·508	1·502	1·501	1·505	1·515
31		1·741	1·685	1·633	1·586	1·545	1·509	1·477	1·451	1·430	1·414	1·403	1·397	1·396	1·401	1·410
30		1·602	1·546	1·496	1·451	1·410	1·375	1·345	1·319	1·299	1·283	1·272	1·266	1·265	1·270	1·279
29		1·437	1·383	1·334	1·290	1·251	1·216	1·187	1·162	1·142	1·120	1·116	1·110	1·109	1·114	1·123
28		1·248	1·195	1·148	1·105	1·066	1·033	1·004	0·979	0·960	0·945	0·935	0·929	0·928	0·933	0·941
27		1·035	0·988	0·937	0·895	0·857	0·824	0·796	0·773	0·754	0·739	0·729	0·724	0·723	0·727	0·735
26		0·798	0·748	0·703	0·662	0·625	0·593	0·565	0·542	0·523	0·509	0·500	0·495	0·494	0·498	0·506
25		0·538	0·490	0·445	0·405	0·369	0·338	0·311	0·288	0·270	0·256	0·247	0·242	0·241	0·245	0·253
24		0·257	0·209	0·166	0·127	0·092	0·061	0·035	0·013	0·005	0·019	0·028	0·033	0·034	0·030	0·022
23	Negative	0·046	0·093	0·135	0·174	0·208	0·238	0·264	0·286	0·303	0·316	0·325	0·330	0·331	0·327	0·319
22		0·370	0·416	0·457	0·495	0·528	0·558	0·583	0·604	0·622	0·635	0·643	0·648	0·649	0·645	0·638
21		0·714	0·759	0·800	0·837	0·869	0·898	0·923	0·944	0·961	0·974	0·982	0·987	0·988	0·984	0·977
20		1·077	1·121	1·161	1·198	1·230	1·258	1·282	1·303	1·319	1·332	1·340	1·345	1·346	1·342	1·335
19						1·609	1·687	1·661	1·681	1·697	1·709	1·718	1·722	1·723	1·719	1·712
18						2·006	2·033	2·057	2·077	2·093	2·105	2·113	2·117	2·118	2·115	2·108
17						2·420	2·447	2·470	2·490	2·505	2·517	2·525	2·529	2·530	2·519	2·520
16						2·850	2·877	2·900	2·919	2·934	2·946	2·954	2·958	2·959	2·956	2·949
15										3·379	3·391	3·399	3·403	3·404	3·401	3·394
14										3·839	3·850	3·858	3·862	3·863	3·860	3·853
13										4·312	4·323	4·331	4·335	4·336	4·333	4·326
12										4·798	4·809	4·817	4·821	4·822	4·819	4·812
11										5·296	5·307	5·315	5·319	5·320	5·317	5·310
10										5·805	5·816	5·824	5·828	5·826	5·825	5·819
9										6·324	6·335	6·343	6·347	6·347	6·344	6·338
8										6·852	6·864	6·871	6·875	6·875	6·872	6·866

Lat. \ Long.		81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	Positive	1·682	1·705												
35		1·685	1·707												
34		1·661	1·682												
33		1·609	1·629												
32		1·530	1·550	1·575	1·606	1·642	1·683	1·729	1·780	1·836	1·898				
31		1·425	1·444	1·469	1·499	1·533	1·578	1·618	1·668	1·723	1·783				
30		1·294	1·312	1·336	1·365	1·399	1·438	1·482	1·531	1·584	1·643				
29		1·136	1·155	1·178	1·207	1·240	1·278	1·320	1·368	1·420	1·477				
28		0·955	0·973	0·996	1·023	1·056	1·092	1·134	1·180	1·231	1·287	1·347	1·411	1·481	1·555
27		0·748	0·766	0·788	0·815	0·847	0·883	0·923	0·968	1·018	1·072	1·131	1·194	1·261	1·333
26		0·519	0·536	0·558	0·584	0·615	0·650	0·689	0·733	0·782	0·835	0·892	0·953	1·019	1·090
25		0·266	0·282	0·304	0·329	0·359	0·394	0·432	0·475	0·523	0·574	0·630	0·690	0·755	0·824
24		0·010	0·006	0·027	0·052	0·082	0·115	0·153	0·195	0·242	0·292	0·347	0·406	0·469	0·536
23	Negative	0·307	0·291	0·271	0·247	0·218	0·185	0·148	0·107	0·061	0·011	0·042	0·100	0·162	0·228
22		0·626	0·610	0·590	0·566	0·538	0·506	0·470	0·429	0·385	0·336	0·284	0·227	0·166	0·101
21		0·965	0·949	0·930	0·906	0·879	0·847	0·812	0·772	0·728	0·680	0·629	0·573	0·514	0·450
20		1·324	1·308	1·289	0·266	1·239	1·208	1·173	1·134	1·091	1·044	0·994	0·940	0·882	0·819
19		1·701	1·686	1·667	0·644	1·618	1·587								
18		2·097	2·082	2·063	2·041	2·015	1·985								
17		2·509	2·495	2·476	2·454	2·429	2·400								
16		2·938	2·924	2·906	2·884	2·859	2·830								
15		3·383	3·369												
14		3·843	3·829												
13		4·316	4·302												
12		4·802	4·788												
11		5·300	5·286												
10		5·809	5·795												
9		6·328	6·314												
8		6·856	6·842												

Values of $u_x - u_y$ in seconds.

Case I.—$\delta a = 1$ km.

TABLE VI.

Long. / Lat.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	Positive	0·102	0·086	0·070	0·057	0·044	0·034	0·024	0·016	0·010	0·005	0·002	0·000	0·000	0·001	0·004	0·008	0·014												
35		0·092	0·077	0·063	0·051	0·040	0·030	0·022	0·015	0·009	0·005	0·002	0·000	0·000	0·001	0·004	0·008	0·013												
34		0·082	0·068	0·056	0·045	0·035	0·027	0·019	0·013	0·008	0·004	0·002	0·000	0·000	0·001	0·003	0·007	0·011												
33		0·072	0·060	0·049	0·040	0·031	0·024	0·017	0·012	0·007	0·004	0·002	0·000	0·000	0·001	0·003	0·006	0·010												
32		0·063	0·052	0·043	0·035	0·027	0·020	0·015	0·010	0·006	0·003	0·001	0·000	0·000	0·001	0·003	0·005	0·009	0·013	0·019	0·025	0·032	0·040	0·049	0·059	0·070				
31		0·054	0·045	0·037	0·029	0·023	0·017	0·013	0·009	0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·004	0·007	0·011	0·016	0·021	0·027	0·034	0·042	0·051	0·060				
30		0·045	0·037	0·031	0·025	0·019	0·015	0·011	0·007	0·004	0·002	0·001	0·000	0·000	0·001	0·002	0·004	0·006	0·009	0·013	0·018	0·023	0·029	0·035	0·042	0·050				
29		0·036	0·030	0·025	0·020	0·015	0·012	0·009	0·006	0·004	0·002	0·001	0·000	0·000	0·000	0·001	0·003	0·005	0·008	0·011	0·014	0·019	0·023	0·028	0·034	0·040				
28		0·028	0·023	0·019	0·015	0·012	0·009	0·007	0·005	0·003	0·002	0·001	0·000	0·000	0·000	0·001	0·002	0·004	0·006	0·008	0·011	0·014	0·018	0·022	0·026	0·031	0·037	0·042	0·048	0·055
27		0·020	0·017	0·014	0·011	0·009	0·007	0·005	0·003	0·002	0·001	0·000	0·000	0·000	0·000	0·001	0·002	0·003	0·004	0·006	0·008	0·010	0·013	0·016	0·019	0·023	0·027	0·031	0·035	0·040
26		0·013	0·011	0·009	0·007	0·006	0·004	0·003	0·002	0·001	0·001	0·000	0·000	0·000	0·000	0·001	0·001	0·002	0·003	0·004	0·005	0·007	0·008	0·010	0·012	0·014	0·017	0·019	0·022	0·025
25		0·006	0·005	0·004	0·003	0·003	0·002	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·002	0·002	0·003	0·004	0·005	0·006	0·007	0·008	0·009	0·010	0·011
24	Negative	0·001	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·002
23		0·007	0·006	0·005	0·004	0·003	0·002	0·002	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·002	0·002	0·003	0·004	0·005	0·006	0·007	0·008	0·009	0·011	0·012	0·014
22		0·013	0·011	0·009	0·007	0·005	0·004	0·003	0·002	0·001	0·001	0·000	0·000	0·000	0·000	0·001	0·001	0·002	0·003	0·004	0·005	0·006	0·008	0·010	0·012	0·014	0·017	0·019	0·022	0·025
21		0·018	0·015	0·012	0·010	0·008	0·006	0·004	0·003	0·002	0·001	0·000	0·000	0·000	0·000	0·001	0·002	0·003	0·004	0·006	0·007	0·009	0·012	0·014	0·017	0·020	0·024	0·027	0·031	0·035
20		0·025	0·019	0·016	0·013	0·010	0·008	0·005	0·004	0·002	0·001	0·001	0·000	0·000	0·000	0·001	0·002	0·003	0·005	0·007	0·009	0·012	0·015	0·018	0·022	0·026	0·030	0·035	0·040	0·045
19						0·012	0·009	0·007	0·005	0·003	0·002	0·001	0·000	0·000	0·000	0·001	0·002	0·004	0·006	0·008	0·011	0·014								
18						0·014	0·011	0·008	0·005	0·003	0·002	0·001	0·000	0·000	0·001	0·001	0·003	0·005	0·007	0·010	0·013	0·016								
17						0·016	0·012	0·008	0·006	0·004	0·002	0·001	0·000	0·000	0·001	0·001	0·003	0·005	0·008	0·011	0·014	0·018								
16						0·017	0·013	0·009	0·006	0·004	0·002	0·001	0·000	0·000	0·001	0·002	0·003	0·005	0·008	0·012	0·016	0·020								
15										0·004	0·002	0·001	0·000	0·000	0·001	0·002	0·004	0·006												
14										0·004	0·002	0·001	0·000	0·000	0·001	0·002	0·004	0·006												
13										0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·004	0·007												
12										0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·004	0·007												
11										0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·004	0·007												
10										0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·005	0·008												
9										0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·005	0·008												
8										0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·005	0·008												

Case I.—δa= 1 km.

Values of v_x in seconds.

TABLE VII.

Positive

Lat. \ Long.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°
36°	8·579	7·845	7·112	6·377	5·642	4·906	4·169	3·433	2·696	1·958	1·221	0·483
35	8·480	7·755	7·029	6·303	5·576	4·849	4·121	3·393	2·665	1·936	1·207	0·477
34	8·386	7·669	6·951	6·233	5·514	4·795	4·075	3·355	2·635	1·914	1·193	0·472
33	8·297	7·587	6·877	6·166	5·455	4·744	4·032	3·319	2·606	1·893	1·180	0·467
32	8·212	7·509	6·806	6·103	5·399	4·695	3·990	3·285	2·579	1·874	1·168	0·462
31	8·131	7·435	6·739	6·043	5·346	4·648	3·950	3·252	2·554	1·855	1·156	0·458
30	8·054	7·365	6·675	5·985	5·295	4·603	3·912	3·221	2·529	1·837	1·145	0·453
29	7·980	7·297	6·614	5·930	5·246	4·561	3·876	3·191	2·506	1·820	1·135	0·449
28	7·909	7·232	6·555	5·877	5·199	4·521	3·842	3·163	2·484	1·804	1·125	0·445
27	7·841	7·170	6·499	5·827	5·155	4·482	3·809	3·136	2·462	1·789	1·115	0·441
26	7·777	7·111	6·445	5·779	5·112	4·445	3·777	3·110	2·442	1·774	1·106	0·438
25	7·715	7·055	6·394	5·733	5·071	4·409	3·747	3·085	2·422	1·759	1·097	0·434
24	7·655	7·000	6·344	5·688	5·031	4·375	3·718	3·061	2·403	1·746	1·088	0·431
23	7·598	6·947	6·296	5·645	4·993	4·341	3·689	3·037	2·385	1·733	1·080	0·427
22	7·542	6·896	6·250	5·604	4·957	4·310	3·662	3·015	2·367	1·720	1·072	0·424
21	7·489	6·847	6·206	5·564	4·921	4·279	3·636	2·993	2·350	1·707	1·064	0·421
20	7·437	6·800	6·162	5·525	4·887	4·249	3·611	2·972	2·334	1·695	1·057	0·418
19					4·854	4·220	3·586	2·952	2·318	1·684	1·050	0·415
18					4·821	4·192	3·562	2·932	2·303	1·673	1·043	0·413
17					4·790	4·165	3·539	2·913	2·288	1·662	1·036	0·410
16					4·759	4·138	3·516	2·894	2·273	1·651	1·029	0·407
15									2·258	1·641	1·023	0·405
14									2·244	1·630	1·016	0·402
13									2·231	1·620	1·010	0·400
12									2·217	1·610	1·004	0·397
11									2·203	1·601	0·998	0·395
10									2·190	1·591	0·992	0·392
9									2·177	1·582	0·986	0·390
8									2·164	1·572	0·980	0·388

Negative

Lat. \ Long.	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	0·254	0·992	1·729	2·467	3·205												
35	0·252	0·981	1·710	2·438	3·167												
34	0·249	0·970	1·691	2·411	3·132												
33	0·246	0·960	1·673	2·385	3·098												
32	0·244	0·950	1·655	2·361	3·066	3·771	4·476	5·181	5·885	6·589	7·292	7·994	8·696				
31	0·241	0·940	1·639	2·337	3·036	3·734	4·432	5·129	5·827	6·523	7·220	7·915	8·611				
30	0·239	0·931	1·623	2·315	3·007	3·698	4·389	5·080	5·771	6·461	7·151	7·840	8·529				
29	0·237	0·922	1·608	2·294	2·979	3·664	4·349	5·034	5·718	6·402	7·085	7·768	8·451				
28	0·235	0·914	1·594	2·273	2·952	3·631	4·310	4·989	5·667	6·345	7·023	7·700	8·376	9·052	9·728	10·403	11·077
27	0·233	0·906	1·580	2·254	2·927	3·600	4·273	4·946	5·619	6·291	6·963	7·634	8·305	8·975	9·645	10·314	10·983
26	0·231	0·899	1·567	2·235	2·903	3·570	4·238	4·905	5·572	6·239	6·905	7·571	8·236	8·901	9·566	10·230	10·893
25	0·229	0·892	1·554	2·217	2·879	3·542	4·204	4·866	5·527	6·189	6·850	7·510	8·171	8·830	9·490	10·149	10·807
24	0·227	0·885	1·542	2·199	2·857	3·514	4·171	4·828	5·484	6·141	6·797	7·452	8·108	8·762	9·417	10·071	10·724
23	0·225	0·878	1·530	2·183	2·835	3·488	4·140	4·792	5·443	6·095	6·746	7·396	8·047	8·697	9·346	9·996	10·644
22	0·224	0·871	1·519	2·167	2·814	3·462	4·109	4·756	5·403	6·050	6·697	7·342	7·988	8·633	9·278	9·923	10·567
21	0·222	0·865	1·508	2·151	2·794	3·437	4·080	4·722	5·364	6·007	6·649	7·290	7·931	8·572	9·213	9·853	10·493
20	0·220	0·859	1·498	2·136	2·775	3·413	4·051	4·689	5·327	5·965	6·602	7·240	7·876	8·513	9·149	9·785	10·421
19	0·219	0·853	1·488	2·122	2·756	3·390	4·024	4·657	5·291								
18	0·217	0·847	1·478	2·108	2·738	3·367	3·997	4·626	5·256								
17	0·216	0·842	1·468	2·094	2·720	3·345	3·971	4·596	5·221								
16	0·215	0·836	1·458	2·080	2·702	3·324	3·945	4·567	5·188								
15	0·213	0·831	1·449	2·067	2·685												
14	0·212	0·826	1·440	2·054	2·668												
13	0·211	0·821	1·431	2·042	2·652												
12	0·209	0·816	1·423	2·029	2·636												
11	0·208	0·811	1·414	2·017	2·620												
10	0·207	0·806	1·406	2·005	2·604												
9	0·206	0·801	1·397	1·993	2·588												
8	0·204	0·797	1·389	1·981	2·573												

TABLE VIII.

Case I.—$\delta a = 1$ km.

Values of $|v_x - v_y|$ in seconds.

Long. / Lat.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
	Positive												Negative																
36°	0·128	0·118	0·107	0·097	0·086	0·075	0·064	0·053	0·042	0·030	0·019	0·008	0·004	0·015	0·027	0·038	0·050												
35	0·104	0·095	0·087	0·078	0·070	0·061	0·052	0·043	0·034	0·025	0·015	0·003	0·003	0·012	0·022	0·031	0·040												
34	0·082	0·076	0·069	0·062	0·056	0·049	0·041	0·034	0·027	0·020	0·012	0·005	0·003	0·010	0·017	0·025	0·032												
33	0·064	0·059	0·054	0·049	0·043	0·038	0·032	0·027	0·021	0·015	0·010	0·004	0·002	0·008	0·014	0·019	0·025												
32	0·049	0·045	0·041	0·037	0·033	0·029	0·025	0·020	0·016	0·012	0·007	0·003	0·001	0·006	0·010	0·015	0·019	0·023	0·027	0·031	0·036	0·040	0·043	0·047	0·051				
31	0·035	0·033	0·030	0·027	0·024	0·021	0·018	0·015	0·012	0·009	0·005	0·002	0·001	0·004	0·008	0·011	0·014	0·017	0·020	0·023	0·026	0·029	0·032	0·035	0·037				
30	0·025	0·023	0·021	0·019	0·017	0·015	0·013	0·010	0·008	0·006	0·004	0·002	0·001	0·003	0·005	0·008	0·010	0·012	0·014	0·016	0·018	0·020	0·022	0·024	0·026				
29	0·016	0·015	0·014	0·013	0·011	0·010	0·008	0·007	0·005	0·004	0·002	0·001	0·001	0·002	0·004	0·005	0·006	0·008	0·009	0·011	0·012	0·013	0·014	0·016	0·017				
28	0·010	0·009	0·008	0·008	0·007	0·006	0·005	0·004	0·003	0·002	0·001	0·001	0·000	0·001	0·002	0·003	0·004	0·005	0·006	0·006	0·007	0·008	0·009	0·009	0·010	0·011	0·011	0·012	0·012
27	0·005	0·005	0·004	0·004	0·004	0·003	0·003	0·002	0·002	0·001	0·001	0·000	0·000	0·001	0·001	0·002	0·002	0·003	0·003	0·003	0·004	0·004	0·005	0·005	0·005	0·005	0·005	0·006	0·006
26	0·002	0·002	0·002	0·002	0·001	0·001	0·001	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·002	0·002	0·002	0·002	0·002	0·002	0·002
25	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
24	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
23	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·002	0·002	0·002	0·002
22	0·004	0·003	0·003	0·002	0·002	0·002	0·002	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·002	0·002	0·002	0·003	0·003	0·003	0·004	0·004	0·005	0·005	0·005
21	0·007	0·006	0·005	0·005	0·004	0·004	0·003	0·003	0·002	0·002	0·001	0·000	0·000	0·001	0·001	0·002	0·002	0·003	0·004	0·004	0·005	0·005	0·006	0·007	0·007	0·008	0·009	0·010	0·010
20	0·011	0·010	0·009	0·008	0·007	0·006	0·005	0·004	0·003	0·002	0·001	0·001	0·000	0·001	0·002	0·003	0·004	0·005	0·006	0·007	0·008	0·009	0·010	0·011	0·012	0·013	0·014	0·015	0·016
19					0·010	0·009	0·008	0·003	0·005	0·003	0·002	0·001	0·001	0·002	0·003	0·005	0·006	0·007	0·009	0·010	0·011								
18					0·014	0·012	0·010	0·009	0·007	0·005	0·003	0·001	0·001	0·002	0·004	0·006	0·008	0·010	0·012	0·014	0·015								
17					0·018	0·016	0·014	0·011	0·009	0·006	0·004	0·001	0·001	0·003	0·006	0·008	0·010	0·013	0·015	0·018	0·020								
16					0·023	0·020	0·017	0·014	0·011	0·008	0·005	0·002	0·001	0·004	0·007	0·010	0·013	0·016	0·019	0·022	0·025								
15									0·013	0·010	0·006	0·002	0·001	0·005	0·009	0·012	0·016												
14									0·016	0·012	0·007	0·003	0·001	0·006	0·010	0·015	0·019												
13									0·019	0·013	0·008	0·003	0·002	0·007	0·012	0·017	0·022												
12									0·021	0·015	0·010	0·004	0·002	0·008	0·014	0·019	0·025												
11									0·024	0·017	0·011	0·004	0·002	0·009	0·015	0·022	0·029												
10									0·027	0·020	0·012	0·005	0·002	0·010	0·017	0·025	0·032												
9									0·030	0·022	0·014	0·005	0·003	0·011	0·019	0·027	0·035												
8									0·033	0·024	0·015	0·006	0·003	0·012	0·021	0·030	0·039												

TABLE IX.

Values of w_x in seconds.

Case I.—$\delta a = 1$ km.

Positive

Lat. \ Long.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°
36	4·750	4·347	3·943	3·538	3·132	2·725	2·317	1·903	1·499	1·089	0·679	0·269
35	4·567	4·130	3·792	3·402	3·012	2·620	2·228	1·835	1·441	1·047	0·653	0·253
34	4·394	4·021	3·648	3·273	2·897	2·521	2·143	1·765	1·387	1·008	0·628	0·249
33	4·229	3·871	3·511	3·150	2·789	2·426	2·063	1·699	1·335	0·970	0·605	0·239
32	4·073	3·727	3·381	3·034	2·685	2·336	1·986	1·636	1·285	0·934	0·582	0·230
31	3·925	3·592	3·258	2·923	2·588	2·251	1·914	1·577	1·239	0·900	0·561	0·222
30	3·784	3·461	3·142	2·819	2·495	2·171	1·846	1·520	1·194	0·868	0·541	0·214
29	3·652	3·343	3·032	2·720	2·408	2·095	1·781	1·467	1·152	0·837	0·522	0·207
28	3·528	3·229	2·929	2·628	2·326	2·024	1·721	1·417	1·113	0·809	0·504	0·200
27	3·410	3·121	2·831	2·540	2·249	1·957	1·634	1·370	1·076	0·782	0·488	0·193
26	3·301	3·021	2·740	2·459	2·176	1·893	1·610	1·326	1·042	0·757	0·472	0·187
25	3·198	2·927	2·655	2·382	2·109	1·835	1·560	1·285	1·009	0·733	0·457	0·181
24	3·102	2·839	2·576	2·311	2·046	1·780	1·513	1·246	0·979	0·711	0·444	0·176
23	3·013	2·757	2·501	2·241	1·987	1·729	1·470	1·210	0·951	0·691	0·431	0·170
22	2·930	2·682	2·433	2·183	1·932	1·681	1·429	1·177	0·925	0·672	0·419	0·166
21	2·854	2·612	2·370	2·126	1·882	1·637	1·392	1·147	0·901	0·654	0·408	0·161
20	2·784	2·548	2·311	2·074	1·836	1·597	1·358	1·118	0·878	0·638	0·398	0·158
19					1·793	1·560	1·326	1·092	0·858	0·624	0·389	0·154
18					1·754	1·526	1·298	1·069	0·840	0·610	0·380	0·151
17					1·719	1·496	1·272	1·047	0·823	0·598	0·373	0·147
16					1·687	1·468	1·248	1·028	0·807	0·587	0·366	0·145
15									0·794	0·577	0·359	0·142
14									0·782	0·568	0·354	0·140
13									0·771	0·560	0·349	0·138
12									0·761	0·553	0·345	0·137
11									0·753	0·547	0·341	0·135
10									0·746	0·542	0·338	0·134
9									0·740	0·538	0·335	0·133
8									0·735	0·534	0·333	0·132

Negative

Lat. \ Long.	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36	0·142	0·552	0·962	1·372	1·781												
35	0·136	0·531	0·925	1·319	1·713												
34	0·131	0·511	0·890	1·269	1·648												
33	0·126	0·492	0·857	1·222	1·586												
32	0·121	0·473	0·825	1·176	1·527	1·878	2·228	2·577	2·926	3·274	3·620	3·966	4·310				
31	0·117	0·456	0·795	1·134	1·472	1·809	2·146	2·483	2·819	3·154	3·488	3·821	4·153				
30	0·113	0·440	0·767	1·093	1·419	1·745	2·070	2·395	2·719	3·042	3·364	3·685	4·005				
29	0·109	0·425	0·740	1·055	1·370	1·684	1·998	2·311	2·624	2·936	3·246	3·556	3·865				
28	0·105	0·410	0·715	1·019	1·323	1·627	1·930	2·232	2·534	2·835	3·136	3·435	3·733	4·030	4·326	4·621	4·914
27	0·102	0·397	0·691	0·985	1·279	1·573	1·866	2·158	2·450	2·741	3·032	3·321	3·609	3·896	4·182	4·467	4·751
26	0·099	0·384	0·669	0·954	1·238	1·522	1·806	2·089	2·371	2·653	2·934	3·214	3·493	3·771	4·047	4·323	4·598
25	0·095	0·372	0·648	0·924	1·200	1·475	1·750	2·024	2·298	2·571	2·843	3·114	3·385	3·654	3·922	4·189	4·455
24	0·093	0·361	0·628	0·896	1·164	1·431	1·697	1·963	2·229	2·494	2·758	3·021	3·283	3·544	3·805	4·064	4·322
23	0·090	0·350	0·610	0·870	1·130	1·390	1·649	1·907	2·165	2·422	2·678	2·934	3·189	3·442	3·695	3·946	4·197
22	0·087	0·341	0·594	0·847	1·099	1·351	1·603	1·855	2·106	2·356	2·605	2·854	3·101	3·348	3·593	3·838	4·082
21	0·085	0·332	0·578	0·824	1·070	1·316	1·561	1·806	2·050	2·294	2·537	2·779	3·021	3·261	3·500	3·738	3·976
20	0·083	0·324	0·564	0·804	1·044	1·284	1·523	1·762	2·000	2·238	2·475	2·711	2·946	3·181	3·414	3·646	3·878
19	0·081	0·316	0·551	0·786	1·020	1·254	1·488	1·721	1·954								
18	0·079	0·309	0·539	0·769	0·998	1·227	1·456	1·684	1·911								
17	0·078	0·303	0·528	0·753	0·978	1·202	1·426	1·650	1·873								
16	0·076	0·298	0·519	0·739	0·960	1·180	1·400	1·619	1·838								
15	0·075	0·292	0·509	0·726	0·943												
14	0·074	0·288	0·501	0·715	0·929												
13	0·073	0·284	0·494	0·705	0·916												
12	0·072	0·280	0·489	0·697	0·905												
11	0·071	0·277	0·483	0·689	0·895												
10	0·071	0·275	0·479	0·683	0·887												
9	0·070	0·272	0·475	0·677	0·880												
8	0·069	0·271	0·472	0·673	0·874												

Case I.—$\delta a = 1$ km.

Values of $w_x - w_y$ in seconds.

TABLE X.

Lat. \ Long.		65°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°
36°	Positive	1·239	1·134	1·039	0·923	0·817	0·711	0·605	0·498	0·391	0·284	0·177	0·070
35		1·100	1·007	0·913	0·830	0·736	0·631	0·537	0·443	0·347	0·252	0·157	0·063
34		0·968	0·838	0·804	0·721	0·638	0·555	0·472	0·339	0·305	0·222	0·138	0·055
33		0·842	0·771	0·699	0·627	0·555	0·483	0·411	0·338	0·266	0·193	0·120	0·048
32		0·723	0·663	0·601	0·539	0·477	0·415	0·353	0·290	0·228	0·166	0·103	0·041
31		0·611	0·559	0·507	0·455	0·403	0·350	0·298	0·245	0·193	0·140	0·087	0·035
30		0·504	0·461	0·419	0·376	0·332	0·280	0·243	0·202	0·159	0·116	0·072	0·029
29		0·404	0·370	0·335	0·301	0·266	0·232	0·197	0·163	0·128	0·093	0·058	0·023
28		0·310	0·284	0·257	0·231	0·204	0·173	0·151	0·125	0·098	0·071	0·044	0·018
27		0·222	0·203	0·184	0·165	0·146	0·127	0·108	0·089	0·070	0·051	0·032	0·013
26		0·140	0·128	0·116	0·104	0·092	0·080	0·068	0·053	0·044	0·032	0·020	0·008
25		0·063	0·057	0·052	0·047	0·041	0·036	0·031	0·025	0·020	0·015	0·009	0·004
24	Negative	0·008	0·008	0·007	0·006	0·006	0·005	0·004	0·003	0·003	0·002	0·001	0·001
23		0·074	0·068	0·062	0·055	0·049	0·043	0·037	0·030	0·023	0·017	0·011	0·004
22		0·135	0·123	0·112	0·100	0·089	0·077	0·066	0·054	0·043	0·031	0·019	0·008
21		0·190	0·174	0·158	0·142	0·126	0·109	0·093	0·076	0·030	0·014	0·027	0·011
20		0·241	0·220	0·200	0·179	0·159	0·138	0·117	0·097	0·076	0·055	0·035	0·014
19						0·189	0·164	0·140	0·115	0·091	0·066	0·041	0·016
18						0·216	0·188	0·160	0·132	0·104	0·075	0·047	0·019
17						0·241	0·209	0·178	0·147	0·115	0·084	0·052	0·021
16						0·262	0·228	0·194	0·160	0·123	0·092	0·057	0·023
15										0·135	0·098	0·061	0·024
14										0·143	0·104	0·065	0·026
13										0·150	0·109	0·068	0·027
12										0·156	0·113	0·071	0·028
11										0·161	0·117	0·073	0·029
10										0·165	0·120	0·075	0·029
9										0·168	0·122	0·076	0·030
8										0·171	0·124	0·077	0·031

Lat. \ Long.		78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	Negative	0·037	0·145	0·251	0·358	0·435												
35		0·033	0·128	0·223	0·318	0·413												
34		0·029	0·113	0·193	0·280	0·363												
33		0·025	0·098	0·171	0·243	0·316												
32		0·021	0·084	0·147	0·209	0·271	0·334	0·396	0·458	0·520	0·532	0·613	0·704	0·765				
31		0·018	0·071	0·124	0·173	0·229	0·281	0·334	0·383	0·439	0·491	0·543	0·595	0·616				
30		0·015	0·059	0·102	0·146	0·189	0·233	0·276	0·319	0·362	0·405	0·448	0·491	0·534				
29		0·012	0·047	0·082	0·117	0·152	0·186	0·221	0·256	0·290	0·325	0·359	0·393	0·428				
28		0·009	0·033	0·063	0·090	0·116	0·143	0·170	0·196	0·223	0·249	0·276	0·302	0·328	0·354	0·380	0·406	0·432
27		0·007	0·026	0·045	0·064	0·083	0·102	0·121	0·140	0·159	0·178	0·197	0·216	0·235	0·254	0·272	0·291	0·309
26		0·004	0·016	0·028	0·040	0·052	0·064	0·076	0·088	0·100	0·112	0·124	0·136	0·148	0·160	0·171	0·183	0·195
25		0·003	0·007	0·013	0·018	0·024	0·029	0·034	0·040	0·045	0·050	0·053	0·061	0·067	0·072	0·077	0·083	0·08[illegible]
24	Positive	0·000	0·001	0·002	0·002	0·003	0·004	0·005	0·005	0·006	0·007	0·007	0·008	0·009	0·009	0·010	0·011	0·012
23		0·002	0·009	0·015	0·022	0·028	0·034	0·041	0·047	0·054	0·030	0·066	0·072	0·078	0·085	0·091	0·097	0·103
22		0·004	0·016	0·027	0·039	0·051	0·062	0·074	0·085	0·097	0·108	0·120	0·131	0·142	0·154	0·165	0·176	0·187
21		0·005	0·022	0·038	0·055	0·071	0·088	0·104	0·120	0·137	0·153	0·169	0·185	0·201	0·217	0·233	0·249	0·265
20		0·007	0·028	0·049	0·070	0·090	0·111	0·132	0·152	0·173	0·193	0·214	0·234	0·255	0·275	0·295	0·315	0·335
19		0·009	0·033	0·058	0·083	0·108	0·132	0·157	0·182	0·208								
18		0·010	0·038	0·066	0·095	0·123	0·151	0·180	0·208	0·236								
17		0·011	0·042	0·074	0·106	0·137	0·169	0·200	0·231	0·262								
16		0·012	0·046	0·081	0·115	0·149	0·184	0·218	0·252	0·286								
15		0·013	0·050	0·087	0·123	0·160												
14		0·013	0·053	0·092	0·131	0·170												
13		0·014	0·055	0·096	0·137	0·178												
12		0·015	0·057	0·100	0·143	0·185												
11		0·015	0·059	0·103	0·147	0·191												
10		0·016	0·061	0·106	0·151	0·196												
9		0·016	0·062	0·108	0·154	0·200												
8		0·016	0·063	0·110	0·156	0·203												

TABLE XI.

Case II.—δb= 1 km. Values of u_x in seconds.

Long. / Lat.		66°	67°	68°	69	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	e	8·819	8·830	8·839	8·848	8·856	8·862	8·868	8·873	8·877	8·880	8·882	8·883	8·883	8·882	8·880	8·878	8·874												
35		7·762	7·772	7·781	7·788	7·796	7·802	7·807	7·811	7·815	7·818	7·820	7·821	7·821	7·820	7·818	7·816	7·813												
34	v	7·178	7·186	7·194	7·201	7·208	7·213	7·218	7·222	7·225	7·228	7·229	7·230	7·230	7·230	7·228	7·226	7·223												
33	i	6·566	6·574	6·581	6·587	6·593	6·598	6·602	6·605	6·608	6·610	6·612	6·613	6·613	6·612	6·611	6·609	6·607												
32		5·929	5·935	5·941	5·947	5·951	5·955	5·959	5·962	5·964	5·966	5·967	5·968	5·968	5·968	5·967	5·965	5·963	5·960	5·957	5·953	5·918	5·943	5·937	5·931	5·924				
31	t	5·266	5·271	5·275	5·280	5·283	5·287	5·290	5·292	5·294	5·205	5·296	5·297	5·297	5·297	5·296	5·294	5·293	5·290	5·288	5·284	5·281	5·277	5·272	5·267	5·262				
30	a	4·577	4·580	4·584	4·587	4·590	4·593	4·504	4·596	4·597	4·598	4·599	4·600	4·600	4·599	4·599	4·598	4·596	4·595	4·593	4·591	4·588	4·585	4·582	4·578	4·574				
29	g	3·863	3·866	3·867	3·869	3·871	3·872	3·873	3·874	3·875	3·876	3·876	3·877	3·877	3·877	3·876	3·876	3·875	3·874	3·873	3·871	3·870	3·868	3·866	3·864	3·862				
28		3·125	3·126	3·126	3·127	3·127	3·127	3·128	3·128	3·128	3·128	3·129	3·129	3·129	3·129	3·129	3·128	3·128	3·128	3·128	3·127	3·127	3·126	3·126	3·125	3·125	3·124	3·123	3·122	3·121
27	e	2·363	2·362	2·361	2·360	2·359	2·359	2·358	2·357	2·357	2·357	2·357	2·356	2·356	2·357	2·357	2·357	2·357	2·358	2·358	2·359	2·360	2·361	2·362	2·363	2·364	2·365	2·366	2·368	2·370
26		1·578	1·575	1·572	1·570	1·568	1·566	1·564	1·563	1·562	1·561	1·560	1·560	1·560	1·560	1·561	1·562	1·562	1·564	1·565	1·567	1·569	1·572	1·574	1·577	1·580	1·584	1·587	1·591	1·595
25	N	0·771	0·766	0·761	0·757	0·753	0·750	0·747	0·745	0·743	0·742	0·741	0·740	0·740	0·741	0·742	0·743	0·744	0·747	0·749	0·752	0·756	0·760	0·764	0·769	0·774	0·780	0·786	0·793	0·800
24		0·059	0·066	0·072	0·078	0·083	0·088	0·092	0·095	0·098	0·100	0·101	0·102	0·102	0·101	0·100	0·098	0·096	0·093	0·089	0·085	0·080	0·074	0·068	0·061	0·053	0·045	0·037	0·027	0·017
23		0·910	0·919	0·927	0·935	0·942	0·948	0·953	0·957	0·961	0·963	0·965	0·966	0·966	0·965	0·964	0·961	0·958	0·954	0·949	0·943	0·937	0·929	0·921	0·912	0·902	0·892	0·880	0·868	0·855
22		1·781	1·792	1·803	1·812	1·821	1·828	1·835	1·840	1·845	1·848	1·850	1·851	1·852	1·851	1·849	1·846	1·842	1·837	1·830	1·823	1·815	1·806	1·795	1·784	1·772	1·759	1·744	1·729	1·712
21	e	2·672	2·686	2·699	2·710	2·720	2·729	2·737	2·744	2·749	2·753	2·756	2·757	2·757	2·756	2·754	2·750	2·746	2·739	2·732	2·723	2·713	2·702	2·690	2·676	2·661	2·645	2·628	2·609	2·589
20	v	3·582	3·598	3·614	3·627	3·639	3·650	3·659	3·667	3·673	3·678	3·681	3·683	3·683	3·682	3·679	3·675	3·669	3·662	3·653	3·643	3·631	3·618	3·603	3·587	3·569	3·550	3·530	3·508	3·484
19	i					4·577	4·589	4·600	4·609	4·616	4·622	4·626	4·628	4·628	4·627	4·623	4·618	4·612	4·603	4·593	4·581	4·568								
18						5·533	5·547	5·559	5·569	5·578	5·584	5·588	5·591	5·591	5·589	5·585	5·580	5·572	5·563	5·551	5·537	5·522								
17	t					6·505	6·521	6·535	6·547	6·556	6·563	6·568	6·571	6·571	6·569	6·565	6·558	6·550	6·539	6·526	6·510	6·493								
16	i					7·494	7·512	7·527	7·540	7·551	7·559	7·564	7·567	7·567	7·565	7·561	7·554	7·544	7·532	7·517	7·505	7·480								
15										8·562	8·570	8·576	8·579	8·580	8·577	8·572	8·564	8·553												
14	s									9·587	9·596	9·603	9·606	9·607	9·604	9·598	9·590	9·578												
13	o									10·625	10·636	10·643	10·647	10·648	10·645	10·639	10·629	10·616												
12										11·677	11·689	11·697	11·701	11·702	11·698	11·692	11·681	11·667												
11	P									12·741	12·753	12·762	12·766	12·767	12·764	12·757	12·745	12·730												
10										13·815	13·829	13·838	13·843	13·844	13·840	13·833	13·820	13·804												
9										14·900	14·915	14·925	14·930	14·931	14·927	14·919	14·905	14·888												
8										15·994	16·010	16·020	16·026	16·027	16·023	16·014	16·000	15·980												

Values of $u_x - u_y$ in seconds.

Case II.—$\delta b = 1$ km. *TABLE XII.*

Long. / Lat.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	Positive	0·105	0·088	0·072	0·058	0·045	0·034	0·025	0·017	0·010	0·005	0·002	0·000	0·000	0·001	0·004	0·009	0·015												
35		0·100	0·084	0·069	0·055	0·043	0·033	0·024	0·016	0·010	0·005	0·002	0·000	0·000	0·001	0·004	0·008	0·014												
34		0·095	0·079	0·065	0·052	0·041	0·031	0·022	0·015	0·009	0·005	0·002	0·000	0·000	0·001	0·004	0·008	0·013												
33		0·088	0·074	0·060	0·049	0·038	0·029	0·021	0·014	0·009	0·005	0·002	0·000	0·000	0·001	0·004	0·007	0·012												
32		0·081	0·068	0·056	0·045	0·035	0·026	0·019	0·013	0·008	0·004	0·002	0·000	0·000	0·001	0·003	0·007	0·011	0·017	0·024	0·032	0·041	0·052	0·064	0·077	0·091				
31		0·073	0·061	0·050	0·041	0·032	0·024	0·017	0·012	0·007	0·004	0·001	0·000	0·000	0·001	0·003	0·006	0·010	0·015	0·022	0·029	0·037	0·047	0·058	0·069	0·082				
30		0·065	0·054	0·044	0·036	0·028	0·021	0·015	0·010	0·006	0·003	0·001	0·000	0·000	0·001	0·003	0·005	0·009	0·014	0·019	0·026	0·033	0·041	0·051	0·061	0·072				
29		0·055	0·046	0·038	0·031	0·024	0·018	0·013	0·009	0·005	0·003	0·001	0·000	0·000	0·001	0·002	0·005	0·008	0·012	0·016	0·022	0·028	0·036	0·044	0·052	0·062				
28		0·045	0·038	0·031	0·025	0·020	0·015	0·011	0·007	0·004	0·002	0·001	0·000	0·000	0·001	0·002	0·004	0·006	0·010	0·013	0·018	0·023	0·029	0·036	0·043	0·051	0·059	0·069	0·078	0·089
27		0·035	0·029	0·024	0·019	0·015	0·011	0·008	0·006	0·003	0·002	0·001	0·000	0·000	0·001	0·001	0·003	0·005	0·007	0·010	0·014	0·018	0·022	0·027	0·033	0·039	0·045	0·052	0·060	0·068
26		0·024	0·020	0·016	0·013	0·010	0·008	0·005	0·004	0·002	0·001	0·000	0·000	0·000	0·000	0·001	0·003	0·003	0·005	0·007	0·009	0·012	0·015	0·018	0·022	0·026	0·030	0·035	0·040	0·046
25		0·012	0·010	0·008	0·006	0·005	0·004	0·003	0·002	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·001	0·002	0·002	0·003	0·004	0·006	0·007	0·009	0·011	0·013	0·015	0·017	0·019	0·022
24	Negative	0·001	0·001	0·001	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·001	0·002	0·002	0·002	0·003	0·003
23		0·015	0·013	0·010	0·008	0·006	0·005	0·004	0·002	0·002	0·001	0·000	0·000	0·000	0·000	0·001	0·001	0·002	0·003	0·004	0·006	0·008	0·010	0·012	0·014	0·017	0·020	0·023	0·026	0·029
22		0·029	0·025	0·020	0·016	0·013	0·010	0·007	0·005	0·003	0·002	0·001	0·000	0·000	0·000	0·001	0·002	0·004	0·006	0·009	0·012	0·015	0·019	0·023	0·028	0·033	0·038	0·044	0·051	0·057
21		0·044	0·037	0·030	0·024	0·020	0·014	0·010	0·007	0·004	0·002	0·001	0·000	0·000	0·001	0·002	0·004	0·006	0·009	0·013	0·018	0·023	0·029	0·035	0·042	0·050	0·058	0·067	0·077	0·087
20		0·060	0·050	0·041	0·033	0·026	0·020	0·014	0·010	0·006	0·003	0·001	0·000	0·000	0·001	0·002	0·005	0·008	0·013	0·018	0·024	0·031	0·039	0·047	0·057	0·067	0·078	0·091	0·104	0·117
19						0·033	0·025	0·018	0·012	0·008	0·004	0·002	0·000	0·000	0·001	0·003	0·006	0·011	0·016	0·023	0·030	0·039								
18						0·040	0·030	0·022	0·015	0·009	0·005	0·002	0·000	0·000	0·001	0·004	0·008	0·013	0·020	0·028	0·037	0·048								
17						0·048	0·036	0·026	0·018	0·011	0·006	0·002	0·000	0·000	0·002	0·005	0·009	0·015	0·023	0·033	0·044	0·057								
16						0·056	0·042	0·030	0·021	0·013	0·007	0·003	0·000	0·000	0·002	0·005	0·011	0·018	0·027	0·038	0·051	0·066								
15										0·015	0·008	0·003	0·000	0·000	0·002	0·006	0·012	0·021												
14										0·016	0·009	0·003	0·001	0·000	0·002	0·007	0·014	0·023												
13										0·018	0·010	0·004	0·001	0·000	0·002	0·008	0·015	0·026												
12										0·020	0·011	0·004	0·001	0·000	0·003	0·008	0·017	0·029												
11										0·023	0·012	0·005	0·001	0·000	0·003	0·009	0·019	0·032												
10										0·025	0·013	0·005	0·001	0·000	0·003	0·010	0·021	0·035												
9										0·027	0·014	0·006	0·001	0·000	0·004	0·011	0·022	0·038												
8										0·029	0·015	0·006	0·001	0·000	0·004	0·012	0·024	0·041												

Case II.—δb= 1 km. **Values of v_s in seconds.** *TABLE XIII.*

Lat. \ Long.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°
	Negative											
36°	1·035	0·948	0·857	0·768	0·679	0·590	0·501	0·413	0·324	0·235	0·147	0·058
35	1·051	0·960	0·870	0·780	0·689	0·599	0·509	0·419	0·329	0·239	0·149	0·059
34	1·065	0·973	0·882	0·790	0·699	0·607	0·516	0·425	0·333	0·242	0·151	0·060
33	1·077	0·985	0·892	0·800	0·707	0·615	0·522	0·430	0·337	0·245	0·153	0·061
32	1·088	0·994	0·901	0·808	0·714	0·621	0·527	0·434	0·341	0·248	0·154	0·061
31	1·096	1·002	0·908	0·814	0·720	0·626	0·532	0·438	0·344	0·250	0·156	0·062
30	1·103	1·008	0·913	0·819	0·724	0·629	0·535	0·440	0·346	0·251	0·157	0·062
29	1·107	1·012	0·917	0·822	0·727	0·632	0·537	0·442	0·347	0·252	0·157	0·062
28	1·109	1·014	0·919	0·824	0·728	0·633	0·538	0·443	0·348	0·253	0·158	0·062
27	1·109	1·014	0·919	0·824	0·728	0·633	0·538	0·443	0·348	0·253	0·158	0·062
26	1·106	1·012	0·917	0·822	0·727	0·632	0·537	0·442	0·347	0·252	0·157	0·062
25	1·101	1·007	0·913	0·818	0·724	0·629	0·535	0·440	0·346	0·251	0·157	0·062
24	1·094	1·000	0·907	0·813	0·719	0·625	0·531	0·437	0·343	0·250	0·156	0·062
23	1·084	0·991	0·898	0·805	0·712	0·619	0·526	0·433	0·340	0·247	0·154	0·061
22	1·071	0·980	0·888	0·796	0·704	0·612	0·520	0·428	0·336	0·244	0·152	0·060
21	1·056	0·966	0·875	0·785	0·694	0·604	0·513	0·422	0·332	0·241	0·150	0·059
20	1·038	0·949	0·860	0·772	0·683	0·594	0·505	0·416	0·326	0·237	0·148	0·059
19					0·669	0·582	0·495	0·407	0·320	0·232	0·145	0·057
18					0·653	0·568	0·483	0·398	0·312	0·227	0·141	0·056
17					0·636	0·553	0·470	0·387	0·304	0·221	0·138	0·054
16					0·616	0·536	0·456	0·375	0·295	0·214	0·134	0·053
15									0·284	0·207	0·129	0·051
14									0·273	0·199	0·124	0·049
13									0·261	0·190	0·118	0·047
12									0·248	0·180	0·112	0·044
11									0·233	0·169	0·106	0·042
10									0·218	0·158	0·099	0·039
9									0·201	0·146	0·091	0·036
8									0·183	0·133	0·083	0·033

Lat. \ Long.	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
	Positive																
36°	0·031	0·119	0·208	0·296	0·385												
35	0·031	0·121	0·211	0·301	0·391												
34	0·032	0·123	0·214	0·305	0·396												
33	0·032	0·124	0·217	0·309	0·401												
32	0·032	0·125	0·219	0·312	0·405	0·498	0·592	0·685	0·778	0·872	0·965	1·059	1·152				
31	0·032	0·126	0·220	0·314	0·408	0·502	0·596	0·691	0·785	0·879	0·973	1·067	1·161				
30	0·033	0·127	0·222	0·316	0·411	0·505	0·600	0·695	0·789	0·884	0·979	1·074	1·168				
29	0·033	0·128	0·223	0·318	0·413	0·508	0·603	0·698	0·793	0·888	0·983	1·078	1·173				
28	0·033	0·128	0·223	0·318	0·414	0·509	0·604	0·699	0·794	0·889	0·984	1·080	1·175	1·270	1·365	1·460	1·555
27	0·033	0·128	0·223	0·318	0·414	0·509	0·604	0·699	0·794	0·889	0·984	1·079	1·175	1·270	1·365	1·460	1·555
26	0·033	0·128	0·223	0·318	0·413	0·509	0·603	0·698	0·792	0·887	0·982	1·077	1·172	1·267	1·361	1·456	1·551
25	0·033	0·127	0·222	0·316	0·411	0·506	0·600	0·695	0·789	0·883	0·978	1·072	1·167	1·261	1·355	1·449	1·543
24	0·032	0·126	0·220	0·314	0·408	0·503	0·596	0·690	0·784	0·878	0·971	1·065	1·159	1·252	1·345	1·439	1·532
23	0·032	0·125	0·218	0·312	0·405	0·498	0·591	0·684	0·777	0·870	0·962	1·055	1·148	1·240	1·333	1·425	1·518
22	0·032	0·124	0·216	0·308	0·400	0·492	0·584	0·676	0·768	0·860	0·951	1·043	1·134	1·226	1·317	1·408	1·500
21	0·031	0·122	0·213	0·304	0·394	0·485	0·576	0·666	0·757	0·847	0·938	1·028	1·118	1·208	1·298	1·388	1·478
20	0·031	0·120	0·209	0·299	0·388	0·477	0·566	0·655	0·744	0·833	0·922	1·010	1·009	1·187	1·270	1·364	1·453
19	0·030	0·118	0·205	0·293	0·380	0·468	0·555	0·642	0·729								
18	0·030	0·115	0·200	0·286	0·371	0·457	0·542	0·627	0·712								
17	0·029	0·112	0·195	0·278	0·361	0·444	0·527	0·610	0·693								
16	0·028	0·109	0·189	0·270	0·350	0·431	0·511	0·592	0·672								
15	0·027	0·105	0·183	0·260	0·338												
14	0·026	0·101	0·175	0·250	0·325												
13	0·025	0·096	0·167	0·239	0·310												
12	0·023	0·091	0·159	0·227	0·294												
11	0·022	0·086	0·150	0·213	0·277												
10	0·021	0·080	0·140	0·199	0·259												
9	0·019	0·074	0·129	0·184	0·239												
8	0·017	0·068	0·118	0·168	0·218												

Case II.—$\delta b = 1$ km.

Values of $v_x - v_y$ in seconds.

TABLE XIV.

Lat. \ Long.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
	Positive												Negative																
36°	0·178	0·159	0·144	0·130	0·115	0·100	0·085	0·070	0·055	0·040	0·025	0·010	0·005	0·021	0·036	0·051	0·066												
35	0·146	0·134	0·122	0·110	0·098	0·085	0·073	0·060	0·047	0·034	0·021	0·008	0·005	0·017	0·030	0·043	0·056												
34	0·121	0·111	0·102	0·092	0·081	0·071	0·061	0·050	0·039	0·029	0·018	0·007	0·004	0·014	0·025	0·036	0·047												
33	0·099	0·091	0·083	0·075	0·067	0·058	0·050	0·041	0·032	0·023	0·015	0·006	0·003	0·012	0·021	0·029	0·038												
32	0·079	0·072	0·066	0·060	0·053	0·046	0·040	0·033	0·026	0·019	0·012	0·005	0·002	0·009	0·016	0·023	0·030	0·037	0·044	0·051	0·058	0·064	0·070	0·077	0·083				
31	0·061	0·056	0·051	0·046	0·041	0·036	0·030	0·025	0·020	0·014	0·009	0·004	0·002	0·007	0·013	0·018	0·023	0·029	0·034	0·039	0·044	0·049	0·054	0·059	0·064				
30	0·045	0·041	0·038	0·034	0·030	0·026	0·023	0·019	0·015	0·011	0·007	0·003	0·001	0·005	0·009	0·013	0·017	0·021	0·025	0·029	0·033	0·037	0·040	0·044	0·047				
29	0·031	0·029	0·026	0·024	0·021	0·018	0·016	0·013	0·010	0·008	0·005	0·002	0·001	0·004	0·007	0·009	0·012	0·015	0·018	0·020	0·023	0·026	0·028	0·030	0·033				
28	0·020	0·018	0·017	0·015	0·014	0·012	0·010	0·008	0·007	0·005	0·003	0·001	0·001	0·002	0·004	0·006	0·008	0·010	0·011	0·013	0·015	0·016	0·018	0·019	0·021	0·022	0·023	0·024	0·025
27	0·011	0·010	0·009	0·009	0·008	0·007	0·006	0·005	0·004	0·003	0·002	0·001	0·000	0·001	0·002	0·003	0·004	0·005	0·006	0·007	0·008	0·009	0·010	0·011	0·012	0·012	0·013	0·014	0·014
26	0·005	0·005	0·004	0·004	0·003	0·003	0·003	0·002	0·002	0·001	0·001	0·000	0·000	0·001	0·001	0·002	0·002	0·003	0·003	0·003	0·004	0·004	0·005	0·005	0·005	0·005	0·006	0·006	0·006
25	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001
24	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
23	0·002	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·001	0·001	0·001	0·001	0·001	0·001	0·001	0·002	0·002	0·002	0·002	0·002	0·002
22	0·006	0·005	0·005	0·004	0·004	0·003	0·003	0·002	0·002	0·001	0·001	0·000	0·000	0·001	0·001	0·002	0·002	0·003	0·003	0·004	0·004	0·005	0·005	0·006	0·007	0·007	0·008	0·008	0·009
21	0·013	0·012	0·011	0·009	0·008	0·007	0·006	0·005	0·004	0·003	0·002	0·001	0·000	0·002	0·003	0·004	0·005	0·006	0·007	0·008	0·009	0·010	0·012	0·013	0·014	0·016	0·017	0·018	0·020
20	0·028	0·021	0·019	0·017	0·015	0·013	0·011	0·009	0·007	0·005	0·003	0·001	0·001	0·003	0·005	0·006	0·008	0·010	0·012	0·014	0·016	0·018	0·020	0·023	0·025	0·027	0·030	0·032	0·035
19					0·023	0·020	0·017	0·014	0·011	0·008	0·005	0·002	0·001	0·004	0·007	0·010	0·013	0·016	0·019	0·022	0·025								
18					0·033	0·029	0·025	0·020	0·016	0·012	0·007	0·003	0·002	0·006	0·010	0·015	0·019	0·023	0·028	0·032	0·037								
17					0·046	0·040	0·034	0·028	0·022	0·016	0·010	0·004	0·002	0·008	0·014	0·020	0·026	0·032	0·038	0·044	0·050								
16					0·060	0·052	0·044	0·037	0·029	0·021	0·013	0·005	0·003	0·011	0·018	0·026	0·034	0·042	0·050	0·058	0·066								
15									0·037	0·027	0·017	0·007	0·003	0·013	0·023	0·033	0·043												
14									0·045	0·033	0·021	0·008	0·004	0·017	0·029	0·042	0·054												
13									0·055	0·040	0·025	0·010	0·005	0·021	0·036	0·051	0·066												
12									0·066	0·048	0·030	0·012	0·006	0·025	0·043	0·061	0·079												
11									0·078	0·057	0·035	0·014	0·008	0·029	0·050	0·072	0·093												
10									0·091	0·066	0·041	0·016	0·009	0·034	0·059	0·084	0·109												
9									0·106	0·077	0·048	0·019	0·010	0·039	0·068	0·097	0·126												
8									0·121	0·088	0·055	0·022	0·011	0·045	0·078	0·111	0·144												

Case II.—δb = 1 km. Values of w_x in seconds. *TABLE XV.*

Long. / Lat.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°		78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	Positive	0·765	0·703	0·637	0·572	0·506	0·440	0·374	0·308	0·242	0·176	0·110	0·043	Negative	0·023	0·089	0·155	0·222	0·288												
35		0·697	0·633	0·579	0·519	0·460	0·400	0·340	0·280	0·220	0·160	0·100	0·039		0·021	0·081	0·141	0·202	0·262												
34		0·621	0·568	0·515	0·462	0·409	0·356	0·303	0·249	0·196	0·142	0·089	0·035		0·019	0·072	0·126	0·179	0·233												
33		0·539	0·493	0·447	0·401	0·355	0·309	0·263	0·216	0·170	0·123	0·077	0·030		0·016	0·063	0·109	0·156	0·202												
32		0·451	0·413	0·374	0·336	0·297	0·258	0·220	0·181	0·143	0·103	0·064	0·025		0·013	0·052	0·091	0·130	0·169	0·208	0·247	0·285	0·324	0·362	0·401	0·439	0·477				
31		0·357	0·327	0·296	0·266	0·235	0·205	0·174	0·143	0·113	0·082	0·051	0·020		0·011	0·042	0·072	0·103	0·134	0·165	0·196	0·226	0·257	0·287	0·318	0·348	0·378				
30		0·258	0·236	0·214	0·192	0·170	0·148	0·126	0·103	0·081	0·059	0·037	0·015		0·008	0·030	0·052	0·075	0·097	0·119	0·141	0·163	0·185	0·207	0·229	0·251	0·273				
29		0·152	0·139	0·126	0·113	0·100	0·087	0·074	0·061	0·048	0·035	0·022	0·009		0·005	0·018	0·031	0·044	0·057	0·070	0·083	0·096	0·109	0·122	0·135	0·148	0·161				
28		0·041	0·038	0·034	0·031	0·027	0·024	0·020	0·017	0·013	0·009	0·006	0·002		0·001	0·005	0·008	0·012	0·015	0·019	0·022	0·026	0·030	0·033	0·036	0·040	0·043	0·047	0·050	0·054	0·057
27	Negative	0·076	0·070	0·063	0·057	0·050	0·044	0·037	0·031	0·024	0·017	0·011	0·004	Positive	0·002	0·009	0·015	0·022	0·028	0·035	0·041	0·048	0·055	0·061	0·067	0·074	0·080	0·087	0·093	0·100	0·106
26		0·199	0·182	0·165	0·148	0·131	0·114	0·097	0·080	0·063	0·045	0·028	0·011		0·006	0·023	0·040	0·057	0·074	0·092	0·107	0·126	0·143	0·160	0·177	0·194	0·210	0·227	0·244	0·260	0·277
25		0·327	0·300	0·272	0·244	0·216	0·188	0·160	0·131	0·103	0·075	0·047	0·018		0·010	0·038	0·066	0·095	0·123	0·151	0·179	0·207	0·235	0·263	0·291	0·319	0·346	0·374	0·401	0·429	0·456
24		0·461	0·422	0·383	0·344	0·304	0·265	0·226	0·185	0·146	0·106	0·066	0·026		0·014	0·054	0·093	0·133	0·173	0·213	0·252	0·292	0·331	0·371	0·410	0·449	0·488	0·527	0·566	0·604	0·643
23		0·601	0·550	0·499	0·448	0·396	0·345	0·293	0·241	0·190	0·138	0·086	0·034		0·018	0·070	0·122	0·174	0·225	0·277	0·329	0·380	0·432	0·483	0·534	0·585	0·636	0·687	0·737	0·787	0·837
22		0·746	0·683	0·619	0·556	0·492	0·428	0·364	0·300	0·235	0·171	0·107	0·042		0·022	0·087	0·151	0·216	0·280	0·344	0·408	0·472	0·536	0·600	0·663	0·726	0·789	0·852	0·915	0·977	1·039
21		0·897	0·821	0·744	0·668	0·591	0·514	0·437	0·360	0·283	0·206	0·128	0·051		0·027	0·104	0·182	0·259	0·336	0·414	0·491	0·563	0·644	0·721	0·797	0·873	0·949	1·024	1·100	1·174	1·249
20		1·053	0·963	0·874	0·784	0·694	0·604	0·513	0·423	0·332	0·241	0·151	0·060		0·031	0·122	0·213	0·304	0·395	0·485	0·576	0·666	0·756	0·846	0·936	1·025	1·114	1·203	1·291	1·379	1·466
19						0·800	0·696	0·592	0·488	0·383	0·278	0·174	0·069		0·036	0·141	0·246	0·351	0·455	0·560	0·664	0·768	0·872								
18						0·910	0·792	0·673	0·554	0·435	0·316	0·197	0·078		0·041	0·160	0·280	0·399	0·518	0·636	0·755	0·873	0·991								
17						1·023	0·890	0·756	0·623	0·489	0·356	0·222	0·088		0·046	0·180	0·314	0·448	0·582	0·715	0·848	0·981	1·114								
16						1·139	0·991	0·842	0·694	0·545	0·396	0·247	0·098		0·052	0·201	0·350	0·499	0·648	0·796	0·945	1·093	1·241								
15										0·602	0·437	0·273	0·108		0·057	0·222	0·386	0·551	0·715												
14										0·660	0·480	0·299	0·118		0·062	0·243	0·424	0·604	0·785												
13										0·720	0·533	0·326	0·129		0·068	0·265	0·462	0·659	0·856												
12										0·781	0·567	0·354	0·140		0·074	0·288	0·501	0·715	0·928												
11										0·843	0·613	0·382	0·151		0·080	0·310	0·541	0·772	1·002												
10										0·907	0·659	0·411	0·163		0·086	0·334	0·582	0·830	1·078												
9										0·971	0·706	0·440	0·174		0·092	0·357	0·623	0·889	1·155												
8										1·037	0·754	0·470	0·186		0·098	0·382	0·666	0·949	1·233												

Case II.—δb = 1 km.

Values of $w_x - w_y$ in seconds.

TABLE XVI.

Long. / Lat.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°		78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	Positive	1·269	1·162	1·054	0·945	0·837	0·728	0·619	0·510	0·401	0·291	0·181	0·072	Negative	0·038	0·148	0·257	0·367	0·476												
35		1·193	1·092	0·990	0·888	0·786	0·684	0·582	0·479	0·376	0·273	0·170	0·067		0·036	0·139	0·242	0·345	0·447												
34		1·110	1·016	0·922	0·827	0·732	0·637	0·542	0·446	0·350	0·255	0·159	0·063		0·033	0·129	0·225	0·321	0·417												
33		1·023	0·936	0·849	0·762	0·674	0·587	0·499	0·411	0·323	0·234	0·146	0·058		0·031	0·119	0·207	0·295	0·384												
32		0·929	0·851	0·772	0·692	0·613	0·533	0·453	0·373	0·293	0·213	0·133	0·053		0·028	0·108	0·188	0·269	0·349	0·429	0·509	0·588	0·668	0·747	0·826	0·905	0·984				
31		0·831	0·760	0·690	0·619	0·548	0·477	0·405	0·334	0·262	0·190	0·119	0·047		0·025	0·097	0·169	0·240	0·312	0·383	0·455	0·526	0·597	0·668	0·738	0·809	0·879				
30		0·726	0·665	0·603	0·541	0·479	0·417	0·354	0·292	0·229	0·167	0·104	0·041		0·023	0·085	0·147	0·210	0·273	0·335	0·398	0·460	0·522	0·584	0·646	0·707	0·769				
29		0·616	0·564	0·512	0·459	0·406	0·354	0·301	0·248	0·195	0·141	0·088	0·035		0·018	0·072	0·125	0·178	0·231	0·284	0·337	0·390	0·443	0·495	0·548	0·600	0·652				
28		0·501	0·458	0·416	0·373	0·330	0·287	0·244	0·201	0·158	0·115	0·072	0·028		0·015	0·058	0·101	0·145	0·188	0·231	0·274	0·317	0·360	0·403	0·445	0·488	0·530	0·572	0·614	0·656	0·698
27		0·380	0·347	0·315	0·283	0·250	0·218	0·185	0·153	0·120	0·087	0·054	0·022		0·011	0·044	0·077	0·110	0·143	0·175	0·208	0·240	0·273	0·305	0·337	0·370	0·402	0·434	0·466	0·497	0·529
26		0·253	0·231	0·210	0·188	0·167	0·145	0·124	0·102	0·080	0·058	0·036	0·014		0·008	0·029	0·051	0·073	0·095	0·117	0·138	0·160	0·182	0·203	0·225	0·246	0·268	0·289	0·310	0·331	0·353
25		0·121	0·111	0·100	0·090	0·080	0·069	0·059	0·049	0·038	0·028	0·017	0·007		0·004	0·014	0·024	0·035	0·045	0·056	0·066	0·076	0·087	0·097	0·107	0·118	0·128	0·138	0·148	0·159	0·169
24		0·017	0·015	0·014	0·013	0·011	0·010	0·008	0·007	0·005	0·004	0·002	0·001		0·001	0·002	0·003	0·005	0·006	0·008	0·009	0·011	0·012	0·014	0·015	0·016	0·018	0·019	0·021	0·022	0·023
23	Negative	0·160	0·146	0·133	0·119	0·106	0·092	0·078	0·064	0·050	0·037	0·023	0·009	Positive	0·005	0·019	0·032	0·046	0·060	0·074	0·088	0·101	0·115	0·129	0·142	0·156	0·169	0·182	0·196	0·209	0·222
22		0·308	0·282	0·256	0·230	0·203	0·177	0·150	0·124	0·097	0·071	0·044	0·017		0·009	0·036	0·062	0·089	0·116	0·142	0·169	0·195	0·221	0·248	0·274	0·300	0·326	0·352	0·378	0·404	0·429
21		0·462	0·423	0·383	0·344	0·304	0·265	0·225	0·185	0·146	0·106	0·066	0·026		0·014	0·054	0·094	0·133	0·173	0·213	0·253	0·292	0·332	0·371	0·410	0·450	0·489	0·528	0·566	0·605	0·643
20		0·620	0·568	0·515	0·462	0·409	0·356	0·303	0·249	0·196	0·142	0·089	0·035		0·019	0·073	0·126	0·179	0·233	0·286	0·339	0·393	0·446	0·499	0·552	0·604	0·657	0·709	0·761	0·813	0·864
19						0·517	0·450	0·382	0·315	0·247	0·180	0·112	0·044		0·023	0·091	0·159	0·227	0·294	0·362	0·429	0·496	0·563								
18						0·628	0·547	0·465	0·383	0·301	0·219	0·136	0·054		0·028	0·111	0·193	0·275	0·357	0·439	0·521	0·603	0·684								
17						0·743	0·646	0·549	0·452	0·355	0·258	0·161	0·064		0·034	0·131	0·228	0·325	0·422	0·519	0·616	0·713	0·809								
16						0·860	0·748	0·636	0·524	0·412	0·299	0·186	0·074		0·039	0·152	0·264	0·377	0·489	0·601	0·713	0·825	0·937								
15										0·469	0·341	0·212	0·084		0·044	0·173	0·301	0·429	0·558												
14										0·526	0·384	0·239	0·095		0·050	0·194	0·339	0·483	0·628												
13										0·583	0·427	0·266	0·105		0·056	0·217	0·378	0·539	0·699												
12										0·650	0·473	0·294	0·116		0·061	0·239	0·417	0·595	0·773												
11										0·718	0·518	0·323	0·128		0·067	0·262	0·457	0·653	0·847												
10										0·777	0·564	0·352	0·139		0·073	0·286	0·498	0·711	0·923												
9										0·842	0·611	0·381	0·151		0·079	0·310	0·540	0·770	1·000												
8										0·908	0·659	0·411	0·163		0·086	0·334	0·582	0·831	1·079												

Case III.—$u_o = 1''$.

Values of u and v in seconds.

TABLE XVII.

Long. / Lat.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
	Values of u (Positive)																												
36°-8°	0·979	0·983	0·986	0·989	0·991	0·993	0·995	0·997	0·998	0·999	1·000	1·000	1·000	1·000	0·999	0·998	0·977	0·996	0·994	0·992	0·989	0·987	0·984	0·980	0·977	0·973	0·969	0·964	0·959
	Values of v																												
	Negative												Positive																
86°	0·147	0·134	0·122	0·109	0·097	0·084	0·072	0·059	0·046	0·034	0·021	0·008	0·004	0·017	0·030	0·043	0·055												
35	0·142	0·130	0·118	0·105	0·093	0·081	0·069	0·057	0·045	0·033	0·020	0·008	0·004	0·017	0·029	0·041	0·053												
34	0·136	0·125	0·113	0·102	0·090	0·078	0·066	0·055	0·043	0·031	0·020	0·008	0·004	0·016	0·028	0·039	0·051												
33	0·131	0·120	0·109	0·098	0·087	0·075	0·064	0·053	0·041	0·030	0·019	0·007	0·004	0·015	0·027	0·038	0·049												
32	0·126	0·116	0·105	0·094	0·083	0·072	0·062	0·051	0·040	0·029	0·018	0·007	0·004	0·015	0·026	0·037	0·047	0·058	0·069	0·080	0·091	0·102	0·112	0·123	0·134				
31	0·121	0·111	0·101	0·091	0·080	0·070	0·059	0·049	0·038	0·028	0·017	0·007	0·004	0·014	0·025	0·035	0·046	0·056	0·066	0·077	0·087	0·098	0·108	0·118	0·129				
30	0·117	0·107	0·097	0·087	0·077	0·067	0·057	0·047	0·037	0·027	0·017	0·007	0·004	0·014	0·024	0·034	0·044	0·054	0·064	0·074	0·084	0·094	0·104	0·114	0·124				
29	0·112	0·102	0·093	0·083	0·074	0·064	0·055	0·045	0·035	0·026	0·016	0·006	0·003	0·013	0·023	0·032	0·042	0·052	0·061	0·071	0·081	0·090	0·100	0·109	0·119				
28	0·107	0·098	0·089	0·080	0·071	0·062	0·052	0·043	0·034	0·025	0·015	0·006	0·003	0·013	0·022	0·031	0·040	0·050	0·059	0·068	0·077	0·086	0·096	0·105	0·114	0·123	0·132	0·141	0·150
27	0·103	0·094	0·085	0·077	0·068	0·059	0·050	0·041	0·033	0·024	0·015	0·006	0·003	0·012	0·021	0·030	0·039	0·048	0·056	0·065	0·074	0·083	0·092	0·100	0·109	0·118	0·126	0·135	0·144
26	0·099	0·090	0·082	0·073	0·065	0·057	0·048	0·040	0·031	0·023	0·014	0·006	0·003	0·012	0·020	0·029	0·037	0·045	0·054	0·062	0·071	0·079	0·088	0·096	0·104	0·113	0·121	0·129	0·137
25	0·094	0·086	0·078	0·070	0·062	0·054	0·046	0·038	0·030	0·022	0·014	0·005	0·003	0·011	0·019	0·027	0·035	0·043	0·052	0·060	0·068	0·076	0·084	0·092	0·100	0·108	0·116	0·124	0·131
24	0·090	0·082	0·075	0·067	0·059	0·052	0·044	0·036	0·028	0·021	0·013	0·005	0·003	0·011	0·018	0·026	0·034	0·042	0·049	0·057	0·065	0·072	0·080	0·088	0·095	0·103	0·110	0·118	0·125
23	0·086	0·078	0·071	0·064	0·057	0·049	0·042	0·035	0·027	0·020	0·012	0·005	0·003	0·010	0·017	0·025	0·032	0·040	0·047	0·054	0·062	0·069	0·076	0·084	0·091	0·098	0·105	0·112	0·119
22	0·082	0·075	0·068	0·061	0·054	0·047	0·040	0·033	0·026	0·019	0·012	0·005	0·002	0·010	0·017	0·024	0·031	0·038	0·045	0·052	0·059	0·066	0·073	0·080	0·087	0·093	0·100	0·107	0·114
21	0·078	0·071	0·064	0·058	0·051	0·044	0·038	0·031	0·025	0·018	0·011	0·004	0·002	0·009	0·016	0·023	0·029	0·036	0·042	0·049	0·056	0·062	0·069	0·076	0·082	0·089	0·095	0·102	0·108
20	0·074	0·067	0·061	0·055	0·049	0·042	0·036	0·030	0·023	0·017	0·011	0·004	0·002	0·009	0·015	0·021	0·028	0·034	0·040	0·047	0·053	0·059	0·065	0·072	0·078	0·084	0·090	0·096	0·102
19					0·046	0·040	0·034	0·028	0·022	0·016	0·010	0·004	0·002	0·008	0·014	0·020	0·026	0·032	0·038	0·044	0·050								
18					0·043	0·038	0·032	0·026	0·021	0·015	0·009	0·004	0·002	0·008	0·013	0·019	0·025	0·030	0·036	0·042	0·047								
17					0·041	0·036	0·030	0·025	0·020	0·014	0·009	0·004	0·002	0·007	0·013	0·018	0·023	0·029	0·034	0·039	0·044								
16					0·038	0·033	0·028	0·023	0·018	0·013	0·008	0·003	0·002	0·007	0·012	0·017	0·022	0·027	0·032	0·037	0·042								
15									0·017	0·013	0·008	0·003	0·002	0·006	0·011	0·016	0·020												
14									0·016	0·012	0·007	0·003	0·002	0·006	0·010	0·015	0·019												
13									0·015	0·011	0·007	0·003	0·001	0·005	0·009	0·013	0·017												
12									0·014	0·010	0·006	0·002	0·001	0·005	[illegible]	0·012	0·016												
11									[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]												
10									[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]												
[illegible]									[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]												

Case III.—$u_0 = 1''$

Values of w in seconds.

TABLE XVIII.

Lat. \ Long.	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
	Negative												Positive																
36°	0·250	0·229	0·207	0·186	0·165	0·143	0·122	0·100	0·079	0·057	0·036	0·014	0·007	0·029	0·051	0·072	0·094												
35	0·247	0·226	0·205	0·184	0·163	0·142	0·120	0·099	0·078	0·057	0·035	0·014	0·007	0·029	0·050	0·071	0·093												
34	0·244	0·223	0·202	0·182	0·161	0·140	0·119	0·098	0·077	0·056	0·035	0·014	0·007	0·028	0·049	0·070	0·091												
33	0·241	0·221	0·200	0·180	0·159	0·138	0·118	0·097	0·076	0·055	0·034	0·014	0·007	0·028	0·049	0·070	0·090												
32	0·238	0·218	0·198	0·178	0·157	0·137	0·116	0·096	0·075	0·055	0·034	0·014	0·007	0·028	0·048	0·069	0·089	0·110	0·130	0·151	0·171	0·191	0·212	0·232	0·252				
31	0·236	0·216	0·196	0·176	0·156	0·135	0·115	0·095	0·074	0·054	0·034	0·013	0·007	0·028	0·048	0·068	0·088	0·109	0·129	0·149	0·169	0·189	0·210	0·230	0·250				
30	0·233	0·214	0·194	0·174	0·154	0·134	0·114	0·094	0·074	0·054	0·033	0·013	0·007	0·027	0·047	0·067	0·088	0·108	0·128	0·148	0·168	0·188	0·207	0·227	0·247				
29	0·231	0·211	0·192	0·172	0·152	0·133	0·113	0·093	0·073	0·053	0·033	0·013	0·007	0·027	0·047	0·067	0·087	0·107	0·126	0·146	0·166	0·186	0·205	0·225	0·245				
28	0·229	0·209	0·190	0·170	0·151	0·131	0·112	0·092	0·072	0·053	0·033	0·013	0·007	0·027	0·046	0·066	0·086	0·106	0·125	0·145	0·164	0·184	0·203	0·223	0·242	0·261	0·281	0·300	0·319
27	0·227	0·208	0·188	0·169	0·150	0·130	0·111	0·091	0·072	0·052	0·032	0·013	0·007	0·026	0·046	0·066	0·085	0·105	0·124	0·144	0·163	0·183	0·202	0·221	0·240	0·259	0·278	0·297	0·316
26	0·225	0·206	0·187	0·168	0·148	0·129	0·110	0·090	0·071	0·052	0·032	0·013	0·007	0·026	0·046	0·065	0·084	0·104	0·123	0·142	0·162	0·181	0·200	0·219	0·238	0·257	0·276	0·294	0·313
25	0·223	0·204	0·185	0·166	0·147	0·128	0·109	0·090	0·070	0·051	0·032	0·013	0·007	0·026	0·045	0·064	0·084	0·103	0·122	0·141	0·160	0·179	0·198	0·217	0·236	0·255	0·273	0·292	0·311
24	0·221	0·202	0·184	0·165	0·146	0·127	0·108	0·089	0·070	0·051	0·032	0·013	0·007	0·026	0·045	0·064	0·083	0·102	0·121	0·140	0·159	0·178	0·197	0·215	0·234	0·253	0·271	0·290	0·308
23	0·220	0·201	0·182	0·164	0·145	0·126	0·107	0·088	0·069	0·050	0·031	0·012	0·007	0·026	0·045	0·063	0·082	0·101	0·120	0·139	0·158	0·176	0·195	0·214	0·232	0·251	0·269	0·287	0·306
22	0·218	0·199	0·181	0·162	0·144	0·125	0·106	0·088	0·069	0·050	0·031	0·012	0·007	0·025	0·044	0·063	0·082	0·101	0·119	0·138	0·157	0·175	0·194	0·212	0·231	0·249	0·267	0·285	0·304
21	0·217	0·198	0·180	0·161	0·143	0·124	0·106	0·087	0·068	0·050	0·031	0·012	0·006	0·025	0·044	0·063	0·081	0·100	0·118	0·137	0·156	0·174	0·192	0·211	0·229	0·247	0·265	0·284	0·302
20	0·215	0·197	0·179	0·160	0·142	0·123	0·105	0·086	0·068	0·049	0·031	0·012	0·006	0·025	0·044	0·062	0·081	0·099	0·118	0·136	0·154	0·173	0·191	0·209	0·228	0·246	0·264	0·282	0·300
19					0·141	0·123	0·104	0·086	0·067	0·049	0·031	0·012	0·006	0·025	0·043	0·062	0·080	0·099	0·117	0·135	0·154								
18					0·140	0·122	0·104	0·085	0·067	0·049	0·030	0·012	0·006	0·025	0·043	0·061	0·080	0·098	0·116	0·134	0·153								
17					0·139	0·121	0·103	0·085	0·067	0·048	0·030	0·012	0·006	0·025	0·043	0·061	0·079	0·097	0·116	0·134	0·152								
16					0·139	0·121	0·103	0·084	0·066	0·048	0·030	0·012	0·006	0·025	0·043	0·061	0·079	0·097	0·115	0·133	0·151								
15									0·066	0·048	0·030	0·012	0·006	0·024	0·042	0·060	0·078												
14									0·066	0·048	0·030	0·012	0·006	0·024	0·042	0·060	0·078												
13									0·065	0·048	0·030	0·012	0·006	0·024	0·043	0·060	0·078												
12									0·065	0·047	0·030	0·012	0·006	0·024	0·042	0·060	0·078												
11									0·065	0·047	0·029	0·012	0·006	0·024	0·042	0·060	0·077												
10									0·065	0·047	0·029	0·012	0·006	0·024	0·042	0·059	0·077												
9									0·065	0·047	0·029	0·012	0·006	0·024	0·041	0·059	0·077												
8									0·064	0·047	0·029	0·012	0·006	0·024	0·041	0·059	0·077												

TABLE XIX.

Case IV.—$w_o = 1''$.

Values of u and v in seconds.

Long. / Lat.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
		Values of u																												
		Positive												Negative																
36°-8°		0·185	0·170	0·154	0·138	0·122	0·106	0·090	0·074	0·059	0·043	0·027	0·011	0·005	0·022	0·038	0·054	0·070	0·086	0·101	0·117	0·133	0·149	0·165	0·181	0·196	0·212	0·227	0·243	0·258
		Values of v																												
36°	Positive	0·239	0·242	0·244	0·245	0·247	0·249	0·250	0·251	0·252	0·252	0·253	0·253	0·253	0·253	0·253	0·252	0·251												
35		0·216	0·218	0·220	0·222	0·224	0·225	0·226	0·227	0·228	0·229	0·229	0·229	0·229	0·229	0·229	0·228	0·227												
34		0·193	0·195	0·197	0·199	0·200	0·202	0·203	0·204	0·205	0·205	0·206	0·206	0·206	0·206	0·205	0·205	0·204												
33		0·171	0·173	0·175	0·176	0·178	0·179	0·180	0·181	0·182	0·183	0·183	0·183	0·183	0·183	0·183	0·182	0·181												
32		0·149	0·151	0·153	0·154	0·156	0·157	0·158	0·159	0·160	0·160	0·161	0·161	0·161	0·161	0·160	0·160	0·159	0·158	0·157	0·156	0·155	0·153	0·151	0·150	0·148				
31		0·128	0·130	0·131	0·133	0·134	0·135	0·136	0·137	0·138	0·139	0·139	0·139	0·139	0·139	0·139	0·138	0·137	0·137	0·136	0·135	0·133	0·132	0·130	0·128	0·126				
30		0·107	0·109	0·110	0·112	0·113	0·114	0·115	0·116	0·117	0·117	0·117	0·118	0·118	0·118	0·117	0·117	0·116	0·115	0·114	0·113	0·112	0·111	0·109	0·107	0·105				
29		0·086	0·088	0·090	0·091	0·093	0·093	0·094	0·095	0·096	0·096	0·097	0·097	0·097	0·097	0·096	0·096	0·095	0·095	0·094	0·093	0·091	0·090	0·088	0·087	0·085				
28		0·066	0·068	0·069	0·071	0·072	0·073	0·074	0·075	0·075	0·076	0·076	0·076	0·076	0·076	0·076	0·075	0·075	0·074	0·073	0·072	0·071	0·070	0·068	0·067	0·065	0·063	0·061	0·059	0·057
27		0·046	0·048	0·049	0·051	0·052	0·053	0·054	0·055	0·055	0·056	0·056	0·056	0·056	0·056	0·056	0·055	0·055	0·054	0·053	0·052	0·051	0·050	0·048	0·047	0·045	0·043	0·041	0·039	0·037
26		0·027	0·029	0·030	0·031	0·032	0·033	0·034	0·035	0·035	0·036	0·036	0·036	0·036	0·036	0·036	0·036	0·035	0·034	0·034	0·033	0·031	0·030	0·029	0·028	0·026	0·024	0·022	0·020	0·018
25		0·008	0·010	0·011	0·012	0·013	0·014	0·015	0·015	0·016	0·016	0·017	0·017	0·017	0·017	0·017	0·016	0·016	0·015	0·014	0·013	0·012	0·011	0·010	0·009	0·007	0·005	0·004	0·002	0·000
24	Negative	0·011	0·009	0·008	0·007	0·006	0·005	0·004	0·004	0·003	0·003	0·002	0·002	0·002	0·002	0·003	0·003	0·004	0·004	0·005	0·006	0·007	0·008	0·009	0·010	0·012	0·013	0·015	0·017	0·019
23		0·029	0·028	0·027	0·026	0·025	0·024	0·023	0·022	0·022	0·021	0·021	0·021	0·021	0·021	0·022	0·022	0·023	0·023	0·024	0·024	0·025	0·026	0·027	0·028	0·030	0·032	0·033	0·035	0·037
22		0·047	0·046	0·045	0·044	0·043	0·042	0·041	0·041	0·040	0·040	0·040	0·040	0·040	0·040	0·040	0·040	0·041	0·041	0·042	0·043	0·044	0·045	0·046	0·047	0·048	0·050	0·051	0·053	0·055
21		0·065	0·064	0·063	0·062	0·061	0·060	0·060	0·059	0·059	0·058	0·058	0·058	0·058	0·058	0·058	0·059	0·059	0·060	0·060	0·061	0·062	0·063	0·064	0·065	0·066	0·068	0·069	0·071	0·072
20		0·083	0·082	0·081	0·080	0·079	0·078	0·078	0·077	0·077	0·076	0·076	0·076	0·076	0·076	0·076	0·077	0·077	0·077	0·078	0·079	0·080	0·080	0·081	0·082	0·084	0·085	0·086	0·088	0·090
19						0·097	0·096	0·095	0·095	0·094	0·094	0·094	0·094	0·094	0·094	0·094	0·094	0·095	0·095	0·096	0·097	0·097								
18						0·114	0·113	0·113	0·112	0·112	0·112	0·112	0·112	0·112	0·112	0·112	0·113	0·112	0·113	0·113	0·114	0·115								
17						0·131	0·131	0·130	0·130	0·129	0·129	0·129	0·129	0·129	0·129	0·129	0·129	0·130	0·130	0·131	0·131	0·132								
16						0·149	0·148	0·147	0·147	0·147	0·146	0·146	0·146	0·146	0·146	0·146	0·147	0·147	0·147	0·148	0·148	0·149								
15										0·164	0·163	0·163	0·163	0·163	0·163	0·163	0·164	0·164												
14										0·181	0·180	0·180	0·180	0·180	0·180	0·180	0·180	0·181												
13										0·197	0·197	0·197	0·197	0·197	0·197	0·197	0·197	0·197												
12										0·214	0·214	0·214	0·213	0·213	0·214	0·214	0·214	0·214												
11										0·230	0·230	0·230	0·230	0·230	0·230	0·230	0·230	0·230												
10										0·247	0·247	0·246	0·246	0·246	0·246	0·246	0·247	0·247												
9										0·263	0·263	0·263	0·263	0·263	0·263	0·263	0·263	0·263												
8										0·279	0·279	0·279	0·279	0·279	0·279	0·279	0·279	0·279												

Case IV.—$w_o = 1''$.

Values of w in seconds.

TABLE XX.

Long. / Lat.		66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°
36°		1·104	1·108	1·112	1·115	1·117	1·120	1·122	1·124	1·125	1·126	1·127	1·128	1·128	1·127	1·127
35		1·091	1·095	1·098	1·101	1·104	1·106	1·108	1·110	1·111	1·112	1·113	1·114	1·114	1·114	1·113
34		1·077	1·081	1·085	1·088	1·091	1·093	1·095	1·097	1·098	1·099	1·100	1·100	1·100	1·100	1·099
33		1·065	1·069	1·072	1·075	1·078	1·080	1·082	1·084	1·086	1·087	1·087	1·088	1·088	1·088	1·087
32		1·054	1·057	1·060	1·063	1·066	1·068	1·070	1·072	1·074	1·075	1·075	1·076	1·076	1·076	1·075
31		1·042	1·046	1·049	1·052	1·055	1·057	1·059	1·061	1·063	1·063	1·064	1·064	1·064	1·064	1·063
30		1·032	1·035	1·038	1·041	1·044	1·046	1·048	1·050	1·051	1·052	1·053	1·054	1·054	1·053	1·053
29		1·022	1·025	1·028	1·031	1·034	1·036	1·038	1·040	1·041	1·042	1·043	1·043	1·043	1·043	1·042
28		1·012	1·016	1·019	1·022	1·024	1·026	1·028	1·030	1·031	1·032	1·033	1·034	1·034	1·033	1·033
27	e	1·003	1·007	1·010	1·013	1·015	1·017	1·019	1·021	1·022	1·023	1·024	1·024	1·024	1·024	1·024
26		0·994	0·998	1·001	1·004	1·006	1·009	1·011	1·012	1·013	1·014	1·015	1·015	1·015	1·015	1·015
25	v	0·986	0·990	0·993	0·996	0·998	1·000	1·002	1·004	1·005	1·006	1·007	1·007	1·007	1·007	1·006
24	i	0·978	0·982	0·985	0·988	0·990	0·992	0·994	0·996	0·997	0·998	0·999	0·999	0·999	0·999	0·998
23	t	0·971	0·974	0·977	0·980	0·983	0·985	0·987	0·988	0·989	0·990	0·991	0·992	0·992	0·991	0·991
22		0·964	0·968	0·970	0·973	0·976	0·978	0·980	0·981	0·982	0·983	0·984	0·984	0·984	0·984	0·984
21	i	0·958	0·961	0·964	0·967	0·969	0·971	0·973	0·975	0·976	0·977	0·977	0·978	0·978	0·978	0·977
20	s	0·951	0·955	0·958	0·960	0·963	0·965	0·967	0·968	0·969	0·970	0·971	0·971	0·971	0·971	0·971
19						0·957	0·959	0·961	0·962	0·964	0·964	0·965	0·965	0·965	0·965	0·965
18	o					0·951	0·953	0·955	0·957	0·958	0·959	0·960	0·960	0·960	0·960	0·959
17						0·946	0·948	0·950	0·952	0·953	0·954	0·954	0·955	0·955	0·954	0·954
16	P					0·941	0·943	0·945	0·947	0·948	0·949	0·950	0·950	0·950	0·950	0·949
15						0·937	0·939	0·941	0·943	0·944	0·944	0·945	0·945	0·945	0·945	0·945
14						0·933	0·935	0·937	0·938	0·939	0·940	0·941	0·941	0·941	0·941	0·940
13						0·929	0·931	0·933	0·934	0·935	0·936	0·937	0·937	0·937	0·937	0·936
12						0·925	0·927	0·929	0·930	0·931	0·932	0·933	0·934	0·934	0·933	0·933
11										0·928	0·929	0·930	0·930	0·930	0·930	0·930
10										0·925	0·926	0·927	0·927	0·927	0·927	0·927
9										0·923	0·924	0·924	0·925	0·925	0·924	0·924
8										0·920	0·921	0·922	0·922	0·922	0·922	0·921

Long. / Lat.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°
36°	1·126	1·124												
35	1·112	1·111												
34	1·098	1·097												
33	1·086	1·085												
32	1·074	1·073	1·071	1·069	1·067	1·064	1·061	1·058	1·055	1·051	1·047	1·042	1·037	1·032
31	1·062	1·061	1·060	1·058	1·056	1·053	1·050	1·047	1·044	1·040	1·036	1·031	1·026	1·021
30	1·052	1·051	1·049	1·047	1·045	1·042	1·039	1·036	1·033	1·029	1·025	1·021	1·016	1·011
29	1·041	1·040	1·039	1·037	1·035	1·032	1·029	1·026	1·023	1·019	1·015	1·011	1·006	1·001
28	1·032	1·031	1·029	1·027	1·025	1·023	1·020	1·017	1·013	1·009	1·005	1·001	0·997	0·992
27	1·023	1·021	1·020	1·018	1·016	1·013	1·011	1·008	1·004	1·000	0·997	0·992	0·988	0·983
26	1·014	1·013	1·011	1·009	1·007	1·005	1·002	0·999	0·996	0·992	0·988	0·984	0·979	0·974
25	1·005	1·004	1·003	1·001	0·999	0·996	0·994	0·991	0·987	0·984	0·980	0·975	0·971	0·966
24	0·997	0·996	0·995	0·993	0·991	0·989	0·986	0·983	0·980	0·976	0·972	0·968	0·963	0·958
23	0·990	0·989	0·987	0·985	0·983	0·981	0·978	0·975	0·972	0·969	0·965	0·961	0·956	0·951
22	0·983	0·982	0·980	0·978	0·976	0·974	0·971	0·968	0·965	0·962	0·958	0·954	0·949	0·944
21	0·976	0·975	0·974	0·972	0·970	0·967	0·965	0·962	0·959	0·955	0·951	0·947	0·943	0·938
20	0·970	0·969	0·967	0·965	0·963	0·961	0·959	0·956	0·952	0·949	0·945	0·941	0·937	0·932
19	0·964	0·963	0·961	0·960	0·958	0·955	0·952	0·950	0·947	0·943	0·939	0·935	0·931	0·926
18	0·958	0·957	0·956	0·954	0·952	0·950	0·947	0·944	0·941	0·938	0·934	0·930	0·926	0·921
17	0·953	0·952	0·951	0·949	0·947	0·945	0·942	0·939	0·936	0·933	0·929	0·925	0·921	0·916
16	0·948	0·947	0·946	0·944	0·942	0·940	0·937	0·934	0·931	0·928	0·924	0·920	0·916	0·911
15	0·944	0·942												
14	0·939	0·938												
13	0·935	0·934												
12	0·932	0·931												
11	0·929	0·928												
10	0·926	0·925												
9	0·923	0·922												
8	0·920	0·919												

The closing errors.

22. The tables just given exhibit the "closing errors" or differences between $u_x\ v_x\ w^x$ and $u_y\ v_y\ w_y$ respectively. The formulæ for these differences $u_x - u_y$ etc. for the four cases will now be considered separately and approximate expressions found for them.

Case I, when $\delta a = 1,\ \delta b = 0,\ u_0 = 0,\ w_0 = 0$

Along the parallel OM the changes at M are given by equations (21) and (22) in which suffix zero may be added to β, R, λ to indicate that it applies to latitude λ_0. Apply the changes at M to the case of a meridian: it is necessary to consider cases I, III, IV of § 20 and the following equations are deduced:

$$\left.\begin{aligned}
u_y &= \Big[-0\cdot2807\lambda+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}+R_0\Big(1-(\cos\beta_0L)\Big)\\
v_y - v_0 &= -R_0\left(\frac{L^\circ\cot\lambda_0}{57\cdot3}+\frac{1}{\beta_0}\tan\lambda_0\sin(\beta_0L)\right)\\
&\quad -\frac{R_0\nu_0}{\beta_0\,a}\sqrt{1-e^2}\sin(\beta_0L)\Big[\tan\lambda-0\cdot000{,}058,\,2\lambda^\circ+0\cdot000{,}004{,}2\sin2\lambda\Big]_{\lambda_0}^{\lambda}\\
w_y &= -\frac{R_0\,\nu_0}{\beta_0\,\nu}\sec\lambda\sin(\beta_0L)\ \ldots\ldots\ldots\ldots\\
R_0 &= 16\cdot1718\sin2\lambda_0\left(1+\frac{\sin^2\lambda_0\,(1-e^2)}{1-e^2\sin^2\lambda_0}\right)\ \ldots\ldots\ldots
\end{aligned}\right\}\ \ldots (33)$$

Next proceeding along O N, the changes at N are given by (29), and applying these initial values to the parallel NP the following equations are formed by consideration of cases I and III of § 18

$$\left.\begin{aligned}
u_x &= R\Big(1-\cos(\beta L)\Big)+\cos(\beta L)\Big[-0\cdot2807\lambda^\circ+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}\\
v_x - v_0 &= -R\left(\frac{L^\circ\cot\lambda}{57\cdot3}+\frac{1}{\beta}\tan\lambda\sin(\beta L)\right)\\
&\quad +\frac{1}{\beta}\tan\lambda\sin(\beta L)\Big[-0\cdot2807\lambda^\circ+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}\\
w_x &= -\frac{R}{\beta}\sec\lambda\sin(\beta L)+\frac{1}{\beta}\sec\lambda\sin(\beta L)\Big[-0\cdot2807\lambda^\circ+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}
\end{aligned}\right\}(34)$$

From (33) and (34) it follows that

$$\left.\begin{aligned}
u_x-u_y &= \Big[R\Big(1-\cos(\beta L)\Big)\Big]_{\lambda_0}^{\lambda}+\Big(1-\cos(\beta L)\Big)\Big[0\cdot2807\lambda^\circ-24\cdot26\sin2\lambda+0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}\\
v_x-v_y &= -\left[R\left(\frac{L^\circ\cot\lambda}{57\cdot3}+\frac{1}{\beta}\tan\lambda\sin(\beta L)\right)\right]_{\lambda_0}^{\lambda}\\
&\quad+\frac{1}{\beta}\tan\lambda\sin(\beta L)\Big[-0\cdot280\,\lambda L^\circ+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}\\
&\quad+\frac{R_0\nu_0}{\beta_0a}\sqrt{1-e^2}\sin(\beta_0L)\Big[\tan\lambda-0\cdot000{,}058,\,2\lambda^\circ+0\cdot000{,}004{,}2\sin2\lambda\Big]_{\lambda_0}^{\lambda}\\
w_x-w_y &= -\frac{R}{\beta}\sec\lambda\sin(\beta L)+\frac{1}{\beta}\sec\lambda\sin(\beta L)\Big[-0\cdot2807\lambda^\circ+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\Big]_{\lambda_0}^{\lambda}\\
&\quad+\frac{R_0\nu_0}{\beta_0\,\nu}\sec\lambda\sin(\beta_0L)
\end{aligned}\right\}(35)$$

23. Equations (35) may be simplified and written in approximate form if terms depending on e^2 are neglected. The closing errors will still be expressed with sufficient accuracy. Then β becomes unity and "a" may be substituted for ν. Denote $\lambda-\lambda_0$ by θ. In what follows u_s etc. are expressed in seconds and λ, L, θ are expressed in radians except when the degree mark is added—thus λ°, and $\lambda^\circ/\lambda=57\cdot3$. The successive terms of (35) are taken one by one and reduced. $\doteq$ denotes approximate equality.

Case I, when $\delta a=1, \delta b=0, u_0=0, w_0=0$

Here $\dfrac{R}{16\cdot17}\doteq\sin2\lambda\,(1+\sin^2\lambda)=\frac{3}{2}\sin2\lambda-\frac{1}{4}\sin4\lambda$

$$u_s-u_y\doteq(1-\cos L)\left[16\cdot17\left(\tfrac{3}{2}\sin2\lambda-\tfrac{1}{4}\sin4\lambda\right)+16\cdot08\lambda-24\cdot26\sin2\lambda+0\cdot02\sin4\lambda\right]_{\lambda_0}^{\lambda}$$

$$=(1-\cos L)\left[16\cdot08\lambda-4\cdot02\sin4\lambda\right]_{\lambda_0}^{\lambda}$$

$$=16\cdot1\,(1-\cos L)\left\{\theta-\tfrac{1}{2}\sin2\theta\cos2(\lambda+\lambda_0)\right\}$$

$$=\cdot281\,\theta^\circ(1-\cos L)\left\{1-\frac{\sin2\theta}{2\theta}\cos2(\lambda+\lambda_0)\right\}\quad\ldots\ldots\quad(36)$$

$$\frac{R}{16\cdot17}\cot\lambda=2\cos^2\lambda\,(1+\sin^2\lambda)=\tfrac{5}{4}+\cos2\lambda-\tfrac{1}{4}\cos4\lambda$$

$$\frac{R}{16\cdot17}\tan\lambda=2\sin^2\lambda\,(1+\sin^2\lambda)=\tfrac{7}{4}-2\cos2\lambda+\tfrac{1}{4}\cos4\lambda$$

$$\left[-R\left(\frac{L^\circ\cot\lambda}{57\cdot3}+\tan\lambda\sin L\right)\right]_{\lambda_0}^{\lambda}=-16\cdot17\frac{L^\circ}{57\cdot3}\left[\tfrac{5}{4}+\cos2\lambda-\tfrac{1}{4}\cos4\lambda+\frac{\sin L}{L}\left(\tfrac{7}{4}-2\cos2\lambda+\tfrac{1}{4}\cos4\lambda\right)\right]_{\lambda_0}^{\lambda}$$

$$=+\cdot2823\,L^\circ\left\{2\left(1-2\frac{\sin L}{L}\right)\sin\theta\sin(\lambda+\lambda_0)-\tfrac{1}{2}\left(1-\frac{\sin L}{L}\right)\sin2\theta\sin2(\lambda+\lambda_0)\right\}$$

$$=-0\cdot0098\,L^\circ\theta^\circ\left\{\left(2\frac{\sin L}{L}-1\right)\frac{\sin\theta}{\theta}\sin(\lambda+\lambda_0)+\tfrac{1}{2}\left(1-\frac{\sin L}{L}\right)\frac{\sin2\theta}{2\theta}\sin2(\lambda+\lambda_0)\right\}$$

$$\frac{1}{\beta}\tan\lambda\sin(\beta L)\left[-0\cdot2807\,\lambda^\circ+24\cdot26\sin2\lambda-0\cdot02\sin4\lambda\right]_{\lambda_0}^{\lambda}$$

$$\doteq\theta^\circ\tan\lambda\sin L\left\{-0\cdot2807+\cdot8469\frac{\sin\theta}{\theta}\cos(\lambda+\lambda_0)-\cdot0014\frac{\sin2\theta}{2\theta}\cos2(\lambda+\lambda_0)\right\}$$

$$\doteq-0\cdot0049\,L^\circ\theta^\circ\tan\lambda\frac{\sin L}{L}\left\{1-3\frac{\sin\theta}{\theta}\cos(\lambda+\lambda_0)\right\}$$

$$\frac{R_0\,\nu_0}{\beta_0\,a}\sqrt{1-e^2}\sin(\beta_0L)\left[\tan\lambda-0\cdot000{,}058,\,2\lambda^\circ+0\cdot000{,}004,\,2\sin2\lambda\right]_{\lambda_0}^{\lambda}$$

$$\doteq16\cdot17\sin2\lambda_0(1+\sin^2\lambda_0)\sin L\left\{\frac{\sin\theta}{\cos\lambda\cos\lambda_0}-0\cdot000{,}058{,}2\theta^\circ+0\cdot000{,}008{,}4\cos(\lambda+\lambda_0)\sin\theta\right\}$$

$$\doteq+0\cdot0049\,L^\circ\theta^\circ\frac{\sin L}{L}\sin2\lambda_0\,(1+\sin^2\lambda_0)\left\{\sec\lambda\sec\lambda_0\frac{\sin\theta}{\theta}-0\cdot00834\right\}$$

Hence

$$v_x - v_y = +0{\cdot}0049\,L^\circ\,\theta^\circ \frac{\sin L}{L}\Big\{ 2\left(\frac{L}{\sin L}-2\right)\sin(\lambda+\lambda_0)\frac{\sin\theta}{\theta}+\left(1-\frac{L}{\sin L}\right)\sin 2(\lambda+\lambda_0)\frac{\sin 2\theta}{\theta}$$
$$+\left(-1+3\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\right)\tan\lambda$$
$$+\sin 2\lambda_0\,(1+\sin^2\lambda_0)\left(\sec\lambda\sec\lambda_0\frac{\sin\theta}{\theta}-0{\cdot}00334\right)\Big\}$$

$$=0{\cdot}0049\,L^\circ\,\theta^\circ\frac{\sin L}{L}\Big[-\tan\lambda-0{\cdot}00334\sin 2\lambda_0\,(1+\sin^2\lambda_0)$$
$$+\frac{\sin\theta}{\theta}\Big\{2\left(\frac{L}{\sin L}-2\right)\sin(\lambda+\lambda_0)+3\cos(\lambda+\lambda_0)\tan\lambda+2\sin\lambda_0(1+\sin^2\lambda_0)\sec\lambda\Big\}$$
$$+\left(1-\frac{L}{\sin L}\right)\sin 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta}\Big]$$

Now $\frac{\sin\theta}{\theta}\doteq 1$ and $\frac{L}{\sin L}\doteq 1$ the error being about $\cdot01$ when θ or $L=15^\circ$: hence $\frac{\sin\theta}{\theta}$ or $\frac{\sin 2\theta}{2\theta}$ may be treated as unity when multiplied by $\frac{L}{\sin L}-1$. It follows that

$$\left.\begin{array}{l} v_x-v_y \doteq +0{\cdot}0049L^\circ\theta^\circ\frac{\sin L}{L}\Big\{\tan\lambda\Big(\cos(\lambda+\lambda_0)-1\Big)-0{\cdot}00334\sin2\lambda_0\,(1+\sin^2\lambda_0) \\ \qquad +2\sec\lambda\sin^3\lambda_0+\left(\frac{L}{\sin L}-1\right)\Big(2\sin(\lambda+\lambda_0)-\sin2(\lambda+\lambda_0)\Big)\Big\}\end{array}\right\} \quad (37)$$

$$-\frac{R}{\beta}\sec\lambda\sin(\beta L)+\frac{R_0\nu_0}{\beta_0\nu}\sec\lambda\sin(\beta_0 L)\doteq-\sec\lambda\sin L\,(R-R_0)$$
$$=-0{\cdot}2823\,L^\circ\frac{\sin L}{L}\sec\lambda\Big[\tfrac{3}{2}\sin2\lambda-\tfrac{1}{4}\sin4\lambda\Big]_{\lambda_0}^{\lambda}$$
$$=-0{\cdot}0049\,L^\circ\theta^\circ\frac{\sin L}{L}\sec\lambda\Big\{3\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}-\cos2(\lambda+\lambda_0)\frac{\sin2\theta}{2\theta}\Big\}$$

$$\frac{1}{\beta}\sec\lambda\sin(\beta L)\Big[-0{\cdot}2807\,\lambda^\circ+24{\cdot}26\sin2\lambda-0{\cdot}02\sin4\lambda\Big]_{\lambda_0}^{\lambda}$$
$$\doteq\sec\lambda\sin L\Big\{-0{\cdot}2807\,\theta^\circ+48{\cdot}52\cos(\lambda+\lambda_0)\sin\theta\Big\}$$
$$=-0{\cdot}0049\,L^\circ\theta^\circ\sec\lambda\frac{\sin L}{L}\Big\{1-3\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\Big\}$$

Hence

$$w_x-w_y\doteq-\cdot0049\,L^\circ\theta^\circ\frac{\sin L}{L}\sec\lambda\Big\{1-\cos2(\lambda+\lambda_0)\frac{\sin2\theta}{2\theta}\Big\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

24. Case II, when $\delta a=0$, $\delta b=1$, $u_0=0$, $w_0=0$. Here

$$R=-16{\cdot}2258\,\frac{(1-e^2)\sin^2\lambda}{1-e^2\sin^2\lambda}\sin2\lambda$$
$$\doteq-8{\cdot}11\,(\sin2\lambda-\tfrac{1}{2}\sin4\lambda)$$

Equations (35) hold for this case if we use the above value of R and change the quantity $-0{\cdot}2807\lambda+24{\cdot}26\sin2\lambda-0{\cdot}02\sin4\lambda$ into $-0{\cdot}2847\lambda-24{\cdot}34\sin2\lambda+0{\cdot}02\sin4\lambda$.

Then

$$u_x - u_y \doteq (1-\cos L)\Big[-8\cdot 11\,(\sin 2\lambda - \tfrac{1}{2}\sin 4\lambda) + 16\cdot 3\,\lambda + 24\cdot 26\sin 2\lambda - 0\cdot 02\sin 4\lambda\Big]_{\lambda_0}^{\lambda}$$

$$= (1-\cos L)\Big[16\cdot 3\lambda + 16\cdot 1\sin 2\lambda + 4\cdot 04\sin 4\lambda\Big]_{\lambda_0}^{\lambda}$$

$$= \cdot 283\,\theta^\circ(1-\cos L)\left\{1 + 2\frac{\sin\theta}{\theta}\cos(\lambda+\lambda_0) + \frac{\sin 2\theta}{2\theta}\cos 2(\lambda+\lambda_0)\right\} \quad . \; . \; . \; . \; . \quad (39)$$

$$\frac{R_y}{16\cdot 23}\cot\lambda = -2\sin^3\lambda\cos\lambda\cot\lambda = -2\sin^2\lambda\cos^2\lambda = -\tfrac{1}{4}(1-\cos 4\lambda)$$

$$\frac{R_y}{16\cdot 23}\tan\lambda = -2\sin^3\lambda\cos\lambda\tan\lambda = -2\sin^4\lambda = -\tfrac{1}{4}(3-4\cos 2\lambda + \cos 4\lambda)$$

$$\therefore \quad \left[-R\left(\frac{L^\circ\cot\lambda}{57\cdot 3} + \tan\lambda\sin L\right)\right]_{\lambda_0}^{\lambda} = \frac{16\cdot 23}{4}\cdot\frac{L^\circ}{57\cdot 3}\left[1-\cos 4\lambda + \frac{\sin L}{L}(3-4\cos 2\lambda+\cos 4\lambda)\right]$$

$$= 0\cdot 0049\,L^\circ\theta^\circ\left\{\left(1-\frac{\sin L}{L}\right)\sin 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta} + 2\frac{\sin L}{L}\sin(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\right\}$$

$$\frac{1}{\beta}\tan\lambda\sin\beta L\Big[-0\cdot 2847\lambda^\circ - 24\cdot 34\sin 2\lambda + 0\cdot 02\sin 4\lambda\Big]_{\lambda_0}^{\lambda}$$

$$= \tan\lambda\sin L\theta^\circ\left\{-0\cdot 2847 - 0\cdot 8497\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta} + 0\cdot 0014\cos 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta}\right\}$$

$$\doteq -L^\circ\theta^\circ\tan\lambda\frac{\sin L}{L}\left\{+\cdot 00497 + 0\cdot 0148\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta} - 0\cdot 00002\cos 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta}\right\}$$

$$\doteq -0\cdot 0050\,L^\circ\theta^\circ\tan\lambda\frac{\sin L}{L}\left\{1+3\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\right\}$$

$$\frac{R_0 v_0}{\beta_0 a}\sqrt{1-e^2}\sin(\beta_0 L)\Big[\tan\lambda - 0\cdot 000{,}058,\,2\lambda^\circ + 0\cdot 000{,}004,\,2\sin 2\lambda\Big]_{\lambda_0}^{\lambda}$$

$$\doteq -16\cdot 23\sin 2\lambda_0\sin^2\lambda_0\sin L\left\{\frac{\sin\theta}{\cos\lambda\cos\lambda_0} - 0\cdot 000058\,2\theta^\circ + 0\cdot 0000084\cos(\lambda+\lambda_0)\sin\theta\right\}$$

$$= -0\cdot 0050\,L^\circ\theta^\circ\frac{\sin L}{L}\sin 2\lambda_0\sin^2\lambda_0\left\{\sec\lambda\sec\lambda_0\frac{\sin\theta}{\theta} - 0\cdot 00834\right\}$$

Hence

$$v_x - v_y \doteq 0\cdot 0050\,L^\circ\theta^\circ\frac{\sin L}{L}\left\{\left(\frac{L}{\sin L}-1\right)\sin 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta} + 2\sin(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\right.$$

$$- \left(1+3\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\right)\tan\lambda$$

$$\left. -\sin 2\lambda_0\sin^2\lambda_0\left(-0\cdot 00834 + \sec\lambda\sec\lambda_0\frac{\sin\theta}{\theta}\right)\right\}$$

$$\doteq 0\cdot 0050\,L^\circ\theta^\circ\frac{\sin L}{L}\left\{\frac{\sin\theta}{\theta}\left(\sin 2\lambda_0\cos\lambda_0\sec\lambda - \cos(\lambda+\lambda_0)\tan\lambda\right) - \tan\lambda\right.$$

$$\left. + \left(\frac{L}{\sin L}-1\right)\sin 2(\lambda+\lambda_0) + 0\cdot 00834\sin 2\lambda_0\sin^2\lambda_0\right\}. \quad (40)$$

$$-\frac{R}{\beta}\sec\lambda\sin(\beta L)+\frac{R_0\nu_0}{\beta_0\nu}\sec\lambda\sin(\beta_0 L) = -\sec\lambda\sin L\,(R-R_0)$$

$$= +16\cdot 23\sec\lambda\sin L\Big[\tfrac{1}{2}\sin 2\lambda-\tfrac{1}{4}\sin 4\lambda\Big]_{\lambda_0}^{\lambda}$$

$$=0\cdot 0050\,L^\circ\theta^\circ\frac{\sin L}{L}\sec\lambda\left\{\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}-\cos 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta}\right\}$$

$$\frac{1}{\beta}\sec\lambda\sin(\beta L)\Big[-0\cdot 2847\lambda-24\cdot 34\sin 2\lambda+0\cdot 02\sin 4\lambda\Big]_{\lambda_0}^{\lambda}$$

$$\doteq\sec\lambda\sin L\left\{-0\cdot 2847\theta^\circ-48\cdot 68\cos(\lambda+\lambda_0)\sin\theta\right\}$$

$$\doteq -0\cdot 0050\,L^\circ\theta^\circ\sec\lambda\frac{\sin L}{L}\left\{1+3\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}\right\}$$

Hence

$$w_x-w_y=-0\cdot 0050\,L^\circ\theta^\circ\frac{\sin L}{L}\sec\lambda\left\{1+2\cos(\lambda+\lambda_0)\frac{\sin\theta}{\theta}+\cos 2(\lambda+\lambda_0)\frac{\sin 2\theta}{2\theta}\right\} \quad . \quad . \quad . \quad . \quad (41)$$

25. The closing errors for cases III and IV have also been considered and are practically zero. This is at once evident also from the computed values of $u_x u_y$ etc. which agree to at least 0·001 of a second. It is otherwise clear that there would be no closing error on a sphere caused by moving the origin: and accordingly the effect on a spheroid must vanish with e^2 and accordingly have e^2 as a factor. In considering closing errors then it is only necessary to take cases I and II into account, and this may be done by means of equations (36) to (41). The form of these equations explains how the closing errors found in tables II, III, IV approximately satisfied the empirical relations (18). The relations would not have been equally satisfactory for case I and case II considered independently.

In the case of Indian triangulation θ° only exceeds 8° for values of L° between -7° and -1° and is greater than -8° for values of L° between -5° and $+3^\circ$: so that we can always consider one of the quantities, θ° or L°, numerically less than 8°. Closing errors for the elementary area $dL\,d\lambda$ are now deduced from the equations (36) to (41). In what follows L is treated as identical with $\sin L$.

Putting U_1 for u_x-u_y in case I, U_2 for u_x-u_y in case II etc. we have, omitting small terms

$$dU_1\doteq 16\cdot 1\sin L\,(1-\cos 4\lambda)\,dL\,d\lambda \quad . \quad . \quad . \quad . \quad (42)$$

$$dU_2\doteq 16\cdot 1\sin L\,(1+2\cos 2\lambda+\cos 4\lambda)\,dL\,d\lambda \quad . \quad . \quad . \quad . \quad (43)$$

$$dV_1\doteq -16\cdot 17\cos L\left\{-2\sin 2\lambda+\sin 4\lambda+4\sin 2\lambda-\sin 4\lambda\right\}dL\,d\lambda$$

$$+\cos L\left\{\sec^2\lambda\,(-16\cdot 08\lambda+24\cdot 26\sin 2\lambda)+\tan\lambda(-16\cdot 08+48\cdot 52\cos 2\lambda)\right\}dL\,d\lambda$$

$$+16\cdot 17\sin 2\lambda_0\,(1+\sin^2\lambda_0)\cos L\left\{\sec^2\lambda-0\cdot 00334\right\}dL\,d\lambda$$

$$\doteq 16\cdot 1\cos L\left\{\sec^2\lambda\left(-\lambda+\tfrac{3}{2}\sin 2\lambda+\sin 2\lambda_0\,(1+\sin^2\lambda_0)\right)+\tan\lambda(3\cos 2\lambda-1)-2\sin 2\lambda\right\}dL\,d\lambda$$

$$\doteq 16\cdot 1\cos L\left\{\sec^2\lambda\,(0\cdot 871-\lambda+\tfrac{3}{2}\sin 2\lambda)+\sin 2\lambda-4\tan\lambda\right\}dL\,d\lambda.$$

$$\doteq 16\cdot 1\cos L\left\{\sec^2\lambda\,(0\cdot 871-\lambda)+\sin 2\lambda-\tan\lambda\right\}dL\,d\lambda \quad . \quad . \quad . \quad . \quad (44)$$

$$dV_2 \doteq 16\cdot23\cos L\left\{\sin4\lambda+2\sin2\lambda-\sin4\lambda\right\}dL\,d\lambda$$

$$+\cos L\left\{\sec^2\lambda\,(-16\cdot3\lambda-24\cdot34\sin2\lambda)+\tan\lambda\,(-16\cdot3-48\cdot68\cos2\lambda)\right\}dL\,d\lambda$$

$$-16\cdot23\sin2\lambda_0\sin^2\lambda_0\cos L\sec^2\lambda\,dL\,d\lambda$$

$$\doteq 16\cdot2\cos L\left\{2\sin2\lambda+\sec^2\lambda\,(-\lambda-\tfrac{3}{2}\sin2\lambda-\sin2\lambda_0\sin^2\lambda_0)+\tan\lambda\,(-1-3\cos2\lambda)\right\}dL\,d\lambda$$

$$\doteq -16\cdot2\cos L\left\{\sec^2\lambda\,(0\cdot125+\lambda+\tfrac{3}{2}\sin2\lambda)+2\tan\lambda-\sin2\lambda\right\}dL\,d\lambda \quad \ldots\ldots \quad (45)$$

$$\doteq -16\cdot2\cos L\left\{\sec^2\lambda\,(0\cdot125+\lambda)+5\tan\lambda-\sin2\lambda\right\}dL\,d\lambda$$

Putting $\lambda_0=24^\circ\,7'\,11''\cdot26$

$$dW_1 \doteq -32\cdot34\cos L\cos\lambda\,(1+3\sin^2\lambda)\,dL\,d\lambda$$

$$+\cos L\left\{\sin\lambda\sec^2\lambda\,(-16\cdot1\lambda+24\cdot26\sin2\lambda)+\sec\lambda\,(-16\cdot1+48\cdot5\cos2\lambda)\right\}dL\,d\lambda$$

$$\doteq -16\cdot1\cos L\left\{2\cos\lambda\,(1+3\sin^2\lambda)+\sin\lambda\sec^2\lambda\,(\lambda-\tfrac{3}{2}\sin2\lambda)+\sec\lambda\,(1-3\cos2\lambda)\right\}dL\,d\lambda$$

$$\doteq -16\cdot1\cos L\left\{\lambda\tan\lambda+1+5\cos^2\lambda-6\cos^4\lambda\right\}\sec\lambda\,dL\,d\lambda \quad \ldots\ldots \quad (46)$$

$$dW_2 \doteq 16\cdot2\cos L\times6\cos\lambda\sin^2\lambda\,dL\,d\lambda$$

$$+\cos L\left\{\sin\lambda\sec^2\lambda(-16\cdot3\lambda-24\cdot34\sin2\lambda)+\sec\lambda(-16\cdot3-48\cdot68\cos2\lambda)\right\}dL\,d\lambda.$$

$$\doteq -16\cdot2\cos L\left\{-6\cos\lambda\sin^2\lambda+\sec\lambda\,(\lambda\tan\lambda+\tfrac{3}{2}\tan\lambda\sin2\lambda+1+3\cos2\lambda)\right\}dL\,d\lambda$$

$$\doteq -16\cdot2\cos L\sec\lambda\left\{\lambda\tan\lambda+1-3\cos^2\lambda+6\cos4\lambda\right\}dL\,d\lambda \quad \ldots\ldots \quad (47)$$

By means of equations (42) to (47) it is possible to find the changes u, v, w at P as computed by any route. For

$$u=u_y+\int dU$$

the integration being taken over the area between the desired route (upper limit) and the central parallel and the meridian through P: and u_y being the quantity to be found by properly combining the four cases. This obviously does not get rid of the multiple values obtainable for u, v, w according to the route followed, but if any route has special advantages, results of following it become available. One such route is the geodesic, or the shortest path between any point and the origin. There is something to be said in favour of following this route, and the subject of the geodesic is accordingly considered in some detail in the following chapter, where a direct method of finding the quantities u, v, w along a geodesic is also made use of.

In concluding this chapter it may be pointed out that the equations (42) to (47) enable the differences of the values of u, v, w to be rapidly estimated. This makes it clear at once how far it is a matter of importance to strictly adhere to any route that may be selected; for the difference in values that will be found by any two routes is the closing error.

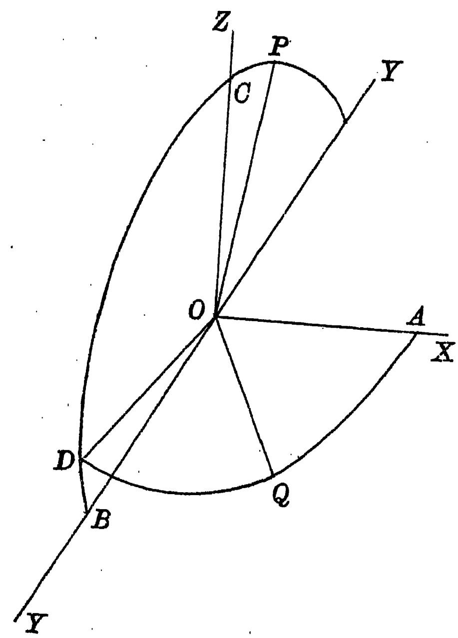
CHAPTER II.

Geodesics on a Spheroid.

1. It is now necessary to develop some properties and relations of a geodesic in order that the changes of coordinates due to change of axes may be computed along geodesics. A fundamental relation of a geodesic on a conicoid is*

$$pD = \text{constant} \quad \ldots \ldots \quad (1)$$

where p is the perpendicular from the centre on the tangent plane at a point and D is the semidiameter of the quadric parallel to the tangent to the curve at the same point. In the case of a spheroid there is symmetry about the polar axis. In the figure ZOX is the equatorial plane and YCY' any meridian. Let P be any point on a geodesic: then the plane through O parallel to the tangent plane at P is the plane DOX, where OD is the diameter conjugate to OP. If ϕ is the eccentric angle of P so that the coordinates of P are O, $a\cos\phi$, $b\sin\phi$ then



$$Od^2 = a^2\sin^2\phi + b^2\cos^2\phi$$

For a geodesic proceeding from P in azimuth A, the semidiameter parallel to it is OQ where $DOQ = 180° - A$ and so

$$D = \left(\frac{\sin^2 A}{a^2} + \frac{\cos^2 A}{a^2\sin^2\phi + b^2\cos^2\phi}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{a^2\,(a^2\sin^2\phi + b^2\cos^2\phi)}{a^2 + \sin^2 A\cos^2\phi\,(b^2 - a^2)}\right)^{\frac{1}{2}}$$

Also

$$p^2 = \frac{1}{\dfrac{\cos^2\phi}{a^2} + \dfrac{\sin^2\phi}{b^2}}$$

hence

$$p^2 D^2 = \frac{a^4\,b^2}{a^2 + \sin^2 A\cos^2\phi\,(b^2 - a^2)}$$

which is constant along a geodesic. It follows that

$$\sin A\cos\phi = \text{constant} = k = \sin A_0 \quad \ldots \ldots \quad (2)$$

along any geodesic, A_0 being the azimuth of the geodesic on crossing the equator.

* *Geometry of Three Dimensions* by George Salmon, 3rd edition, § 397.

2. Take two consecutive points on a spheroid and let A be the azimuth of the elementary line joining them. Then if the latitudes and longitudes are $\lambda, L;\ \lambda + d\lambda,\ L+dL$ it follows that

$$\tan A = \frac{\nu \cos\lambda\, dL}{\rho\, d\lambda} \quad \ldots \quad (3)$$

where ρ is the radius of curvature of the meridian and ν is the normal terminated by the minor axis.

The relation between the latitude λ and the eccentric angle or "reduced latitude" ϕ is

$$\tan\phi = \frac{b}{a}\tan\lambda = \sqrt{1-e^2}\tan\lambda = (1-\epsilon)\tan\lambda \quad \ldots \quad (4)$$

Differentiating (2) logarithmically

$$\frac{dA}{d\phi} = \tan A \tan\phi \quad \ldots \quad (5)$$

and by (4)

$$\frac{d\phi}{d\lambda} = \frac{b}{a}\,\frac{\cos^2\phi}{\cos^2\lambda} \quad \ldots \quad (6)$$

Multiplying (5) and (6)

$$\frac{dA}{d\lambda} = \frac{b}{a}\tan A\tan\phi\frac{\cos^2\phi}{\cos^2\lambda} = \frac{b^2}{a^2}\cdot\frac{\tan A\tan\lambda}{\cos^2\lambda\left(1+\frac{b^2}{a^2}\tan^2\lambda\right)}$$

$$= \frac{b^2}{a^2}\cdot\frac{\tan A\tan\lambda}{1-e^2\sin^2\lambda} = \frac{\rho}{\nu}\tan A\tan\lambda \quad \ldots \quad (7)$$

Equation (3) may be written

$$L = \int\frac{\rho}{\nu\cos\lambda}\cdot\tan A\, d\lambda \quad \ldots \quad (8)$$

and the integration of this will now be performed.

$$\cot^2 A = \operatorname{cosec}^2 A - 1 = \frac{\cos^2\phi}{k^2} - 1 \quad \ldots \text{ by } (2)$$

$$= \frac{1}{k^2\left(1+\frac{b^2}{a^2}\tan^2\lambda\right)} - 1 = \frac{\cos^2\lambda}{k^2(1-e^2\sin^2\lambda)} - 1$$

$$\therefore\quad \tan A = \pm\frac{k\sqrt{1-e^2\sin^2\lambda}}{\sqrt{1-k^2-(1-k^2e^2)\sin^2\lambda}} = \pm\frac{k}{\sqrt{1-k^2}}\cdot\frac{\sqrt{1-e^2\sin^2\lambda}}{\sqrt{1-a^2\sin^2\lambda}}$$

where $a^2 = \dfrac{1-k^2e^2}{1-k^2}$ and the $+$ sign is taken for 1st and 4th quadrants and the minus sign for the 2nd and 3rd quadrants.

Hence by (8)

$$L = \pm\frac{k(1-e^2)}{\sqrt{1-k^2}}\int\frac{1}{\sqrt{1-e^2\sin^2\lambda}\sqrt{1-a^2\sin^2\lambda}}\,\frac{d\lambda}{\cos\lambda}$$

$$\text{since } \frac{\rho}{\nu} = \frac{1-e^2}{1-e^2\sin^2\lambda}$$

Put x for $\sin\lambda$: then

$$L = \pm\frac{k(1-e^2)}{\sqrt{1-k^2}}\int\frac{dx}{\sqrt{1-e^2x^2}\sqrt{1-a^2x^2}\,(1-x^2)} \quad \ldots \quad (9)$$

This is an elliptic integral which cannot be integrated exactly: but it may be developed in a series of integrable terms as follows.

Put $1-x^2=y^2$, then

$$\frac{1}{\sqrt{1-e^2x^2}}\cdot\frac{1}{1-x^2}=\frac{1}{y^2\sqrt{1-e^2+e^2y^2}}=\frac{1}{\sqrt{1-e^2}}\cdot\frac{1}{y^2\sqrt{1+\beta^2y^2}}$$

$$=\frac{1}{\sqrt{1-e^2}}\left\{\frac{1}{y^2}-\tfrac{1}{2}\beta^2+\frac{1.3}{2^2\lfloor 2}\beta^4y^2\quad\ldots\ldots\right\}\quad\ldots\quad(10)$$

where $\beta^2=e^2/(1-e^2)$: hence

$$\int\frac{dx}{\sqrt{1-e^2x^2}\sqrt{1-a^2x^2}(1-x^2)}=\frac{1}{\sqrt{1-e^2}}\int\frac{dx}{\sqrt{1-a^2x^2}}\left\{\frac{1}{1-x^2}-\tfrac{1}{2}\beta^2+\tfrac{3}{8}\beta^4(1-x^2)\ .\right\}\ .\ (11)$$

Finally put

$$\sin\theta=ax=a\sin\lambda\quad\ldots\ldots\ldots\ldots\quad(12)$$

Then

$$\int\frac{dx}{\sqrt{1-a^2x^2}}=\frac{1}{a}\int\frac{\cos\theta\,d\theta}{\cos\theta}=\frac{\theta}{a}\quad\ldots\ldots\ldots\ldots\quad(13)$$

$$\int\frac{dx}{(1-x^2)\sqrt{1-a^2x^2}}=\frac{1}{a}\int\frac{d\theta}{1-\dfrac{\sin^2\theta}{a^2}}=a\int\frac{d\tan\theta}{a^2+\overline{a^2-1}\,|\tan^2\theta}$$

$$=\frac{1}{\sqrt{a^2-1}}\tan^{-1}\left(\frac{\sqrt{a^2-1}}{a}\tan\theta\right)\quad\ldots\ldots\ldots\quad(14)$$

The remaining terms of (11) may be dealt with by the formula of reduction (17) now deduced.

$$u_n=\int\frac{x^n\,dx}{\sqrt{1-a^2x^2}}=\frac{1}{a^2}\int\frac{x^{n-2}\,(1-\overline{1-a^2x^2})\,dx}{\sqrt{1-a^2x^2}}$$

$$=\frac{1}{a^2}\cdot u_{n-2}-\frac{1}{a^2}\int x^{n-2}\sqrt{1-a^2x^2}\,dx\quad\ldots\ldots\ldots\quad(15)$$

Integrating by parts

$$u_n=-\frac{1}{a^2}\int x^{n-1}d\ \sqrt{1-a^2x^2}$$

$$=-\frac{1}{a^2}x^{n-1}\sqrt{1-a^2x^2}+\frac{n-1}{a^2}\int x^{n-2}\sqrt{1-a^2x^2}\,dx\quad\ldots\quad(16)$$

Multiplying (15) by $(n-1)$ and adding to (16)

$$nu_n=\frac{n-1}{a^2}\cdot u_{n-2}-\frac{1}{a^2}\cdot x^{n-1}\sqrt{1-a^2x^2}\quad\ldots\ldots\ldots\quad(17)$$

Hence

$$\int\frac{1-x^2}{\sqrt{1-a^2x^2}}dx=\frac{\theta}{a}\left(1-\frac{1}{2a^2}\right)+\frac{1}{2a^2}x\sqrt{1-a^2x^2}\quad\ldots\ldots\ldots\quad(18)$$

and from (9), (11), (13), (14) and (18)

$$L-L'=\pm\frac{k\sqrt{1-e^2}}{\sqrt{1-k^2}}\left[\frac{1}{\sqrt{a^2-1}}\tan^{-1}\left(\frac{\sqrt{a^2-1}}{a}\tan\theta\right)-\tfrac{1}{2}\beta^2.\ \frac{\theta}{a}\right.$$

$$\left.+\tfrac{3}{8}\beta^4\left\{\frac{\theta}{a}\left(1-\frac{1}{2a^2}\right)+\frac{1}{2a^2}\sin\lambda\cos\theta\right\}\quad\ldots\right]\ .\ (19)$$

in which

$$\left.\begin{aligned} \theta &= \sin^{-1}(\alpha \sin\lambda) \\ \alpha^2 &= \frac{1-k^2e^2}{1-k^2} \\ \beta^2 &= \frac{e^2}{1-e^2} \end{aligned}\right\} \quad \ldots\ldots \quad (20)$$

Hence

$$\sin\theta = \sqrt{\frac{1-k^2e^2}{1-k^2}}\sin\lambda$$

$$\tan\theta = \frac{\sqrt{1-k^2e^2}\sin\lambda}{\sqrt{1-k^2-(1-k^2e^2)\sin^2\lambda}} = \frac{\sqrt{1-k^2e^2}\tan\lambda}{\sqrt{1-k^2-k^2\tan^2\lambda(1-e^2)}}$$

$$= \sqrt{\frac{1-k^2e^2}{1-e^2}} \quad \frac{\tan\phi}{\sqrt{1-k^2\sec^2\phi}} \qquad \text{by} \quad (4)$$

$$= \pm\sqrt{\frac{1-k^2e^2}{1-e^2}}\tan\phi\sec A \quad \ldots\ldots \quad (21)$$

$$\therefore \quad \frac{\sqrt{\alpha^2-1}}{\alpha}\tan\theta = \pm\, k\tan\phi\sec A = \pm\tan A\sin\phi \quad \ldots\ldots \quad (22)$$

Since θ is given by (20) we may always arrange that it shall be in 1st quadrant and the sign in (22) must be taken accordingly.

Put

$$\tan\psi = \pm\tan A\sin\phi \quad \ldots\ldots \quad (23)$$

ψ being always in the first quadrant.

Then (19) may be written

$$L-L' = \pm\left[\pm\psi - \frac{e^2}{2}(1+\overline{1+k^2}|\frac{e^2}{8}+\ldots)k\theta + \frac{3e^4}{16}(1+\ldots)k\sqrt{1-k^2}\sin\lambda\cos\theta + \ldots\right] \quad \ldots \quad (24)$$

Now by (2) it follows that

$$\tan A\sin\phi = \frac{\sin A_0\sec\phi}{\sqrt{1-\sin^2 A_0\sec^2\phi}}\sin\phi = \frac{\sin A_0\tan\phi}{\sqrt{\cos^2 A_0-\sin^2 A_0\tan^2\phi}} = \frac{\tan A_0\tan\phi}{\sqrt{1-\tan^2 A_0\tan^2\phi}}$$

Hence

$$\psi = \pm\tan^{-1}(\tan A\sin\phi) = \pm\sin^{-1}(\tan A_0\tan\phi) \quad \ldots \quad (25)$$

Neglecting terms involving e^4 (23) may be written

$$\left[L\right] = \pm\left[\pm\psi - \frac{ke^2}{2}\theta\right] \quad \ldots\ldots \quad (26)$$

where ψ and θ are both in first quadrant and are defined by (25) and (20) respectively. This result is correct to nearest second for the terrestrial spheroid.

The rules for the double sign outside bracket are	+ 1st and 4th quadrant of azimuth − 2nd and 3rd quadrant ,,
and for double signs before ψ	+ 1st and 2nd quadrant ,, − 3rd and 4th quadrant ,,

The quantity θ may also be found from (21) which may be written

$$\tan\theta = \frac{1}{k}\sqrt{\frac{1-k^2e^2}{1-e^2}}\cdot\tan\psi \quad . \; . \; . \; . \; . \; . \; . \quad (27)$$

3. To solve (26) for A_0 take the first approximation to A_0, namely A_1 such that

$$\Big[\sin^{-1}(\tan A_1 \tan\phi)\Big]_{\phi'}^{\phi} = L - L' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (28)$$

and for brevity put $\tan A_1 \tan\phi = x$, $\tan A_1 \tan\phi' = y$ and $L - L' = \theta$

Then (26) becomes

$$\sin^{-1}x - \sin^{-1}y = \theta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (29)$$

$$\therefore \quad x\sqrt{1-y^2} - y\sqrt{1-x^2} = \sin\theta$$

Squaring and transposing

$$x^2 + y^2 - 2x^2y^2 - \sin^2\theta = 2xy\sqrt{(1-x^2)(1-y^2)}$$

Squaring again

$$\therefore \quad \sin^4\theta - 2\sin^2\theta\,(x^2+y^2-2x^2y^2) + (x^2-y^2)^2 = 0$$

$$\text{or} \quad (x^2-y^2)^2 + 4x^2y^2\sin^2\theta - 2\,(x^2+y^2)\sin^2\theta + \sin^4\theta = 0$$

and putting this into factors

$$(x^2+y^2+2xy\cos\theta-\sin^2\theta)\,(x^2+y^2-2xy\cos\theta-\sin^2\theta) = 0$$

Substituting for x and y it follows that

$$\tan^2 A_1\Big(\tan^2\phi + \tan^2\phi' \pm 2\tan\phi\tan\phi'\cos(L-L')\Big) = \sin^2(L-L') \quad . \; . \; . \quad (30)$$

The double signs have been introduced by the process of squaring and it is necessary to return to (28) to decide which signs give the required solution.

First supposing $\tan\phi$ and $\tan\phi'$ of the same sign and $\phi > \phi'$: then from (28) changing the sign of $\tan\phi'$ diminishes the value of $\tan A_1$: hence by (30) we see that the lower sign must be taken. The same is true if $\tan\phi$ and $\tan\phi'$ are of opposite sign, and as ϕ and ϕ' are interchangeable this shows that the lower sign in (29) must always be taken.

Again if $\phi > \phi'$ the sign of $\tan A$ is the same as that of $L-L'$, and if $\phi < \phi'$ the sign of $\tan A$ is opposite to that of $L-L'$.

Hence we may write (30)

$$\tan A_1 = \pm\frac{\sin(L-L')}{\sqrt{\tan^2\phi+\tan^2\phi'-2\tan\phi\tan\phi'\cos\overline{L-L'}}} \quad . \; . \; . \; . \; . \; . \; . \quad (31)$$

the upper or lower sign being taken according as $\phi >$ or $< \phi'$.

Also $\quad \tan^2\phi + \tan^2\phi' - 2\tan\phi\tan\phi'\cos\overline{L-L'}$

$$= (1-e^2)\Big\{\tan^2\lambda + \tan^2\lambda' - 2\tan\lambda\tan\lambda'\cos\overline{L-L'}\Big\} \quad \text{by (4)}$$

$$= (1-e^2)\Big(\tan\lambda - \tan\lambda'\Big)^2\Big\{1 + \frac{4\tan\lambda\tan\lambda'}{(\tan\lambda-\tan\lambda')^2}\sin^2\frac{L-L'}{2}\Big\}$$

$$= (1-e^2)\Big(\frac{\sin(\lambda-\lambda')}{\cos\lambda\cos\lambda'}\Big)^2\sec^2\omega$$

where $$\tan^2 \omega = \sin^2 \frac{L-L'}{2} \cdot \frac{\sin 2\lambda \sin 2\lambda'}{\sin^2 (\lambda-\lambda')}$$

so that finally

$$\left.\begin{aligned} \tan A_1 &= + \frac{\sin (L-L') \cos\omega \cos\lambda \cos\lambda'}{\sqrt{1-e^2} \sin (\lambda-\lambda')} \\ \tan \omega &= \frac{\sin \frac{1}{2} (L-L')}{\sin (\lambda-\lambda')} \sin 2\lambda \sin 2\lambda' \end{aligned}\right\} \quad \ldots \ldots \quad (32)$$

where

and $\cos\omega$ is taken positive.

4. Denote by ${}_1A$ the approximate value of A which corresponds to A_1 which is an approximate value of A_0.

Then

$$\left.\begin{aligned} \sin {}_1A \cos\phi &= \sin A_1 \\ {}_1A + \delta_1 A &= A \end{aligned}\right\} \quad \ldots \ldots \quad (33)$$

and

Hence

$$\tan A_1 = \frac{\sin {}_1A \cos\phi}{\sqrt{1-\sin^2 {}_1A \cos^2\phi}} = \frac{1}{\sqrt{\sec^2\phi \operatorname{cosec}^2 {}_1A - 1}}$$

$$= \frac{1}{\sqrt{\sec^2\phi \cot^2 {}_1A + \tan^2\phi}}$$

By (30)

$$\sin^2 (L-L') (\tan^2\phi + \sec^2\phi \cot^2 {}_1A) = \tan^2\phi + \tan^2\phi' - 2 \tan\phi \tan\phi' \cos (L-L')$$

$$\cot^2 {}_1A \sec^2\phi \sin^2 (L-L') = \left\{ \tan\phi \cos (L-L') - \tan\phi' \right\}^2$$

$$\therefore \quad \tan {}_1A = + \frac{\sec\phi \sin (L-L')}{\tan\phi \cos (L-L') - \tan\phi'} \quad \ldots \ldots \quad (34)$$

Similarly $$\tan {}_1A' = - \frac{\sec\phi' \sin (L-L')}{\tan\phi' \cos (L-L') - \tan\phi} \quad \ldots \ldots \quad (35)$$

By differentiating (33) logarithmically with regard to A_1 and ${}_1A$ we get the relation between δA_1 and $\delta_1 A$ as follows:

$$\cot {}_1A \, \delta_1 A = \cot A_1 \, \delta A_1 \quad \ldots \ldots \quad (36)$$

Equations (34) and (35) correspond to the ordinary equations of spherical trigonometry to which they reduce if the eccentric angles ϕ, ϕ' are replaced by latitudes λ, λ'.

5. Suppose next that

$$A_0 \doteq A_1 + \delta A_1 \quad \ldots \ldots \quad (37)$$

where δA_1 gives a second approximation to A_0. Then by (26) neglecting terms in e^4, it follows that

$$\frac{e^2 k}{2} (\theta - \theta') \doteq \pm \delta \Big[\psi \Big] \doteq \delta \Big[\sin^{-1} (\tan A_1 . \tan\phi) \Big]_{\phi'}^{\phi}$$

$$= \delta A_1 \left[\frac{\sec^2 A_1 \tan\phi}{\sqrt{1-\tan^2 A_1 \tan^2\phi}} \right]_{\phi'}^{\phi} \quad \ldots \ldots \quad (38)$$

With notation of (20)

$$\sin\theta = a\sin\lambda \doteq \sqrt{\frac{1-e^2\sin^2 A_1}{1-\sin^2 A_1}}\sin\lambda = \sec A_1 \sin\lambda\sqrt{1-e^2\sin^2 A_1}$$

since $\qquad k = \sin A_0 \doteq \sin A_1$

$$\therefore \qquad \cos^2\theta = 1-\sin^2\lambda\sec^2 A_1\,(1-e^2\sin^2 A_1)$$

$$= \cos^2\lambda - (1-e^2)\tan^2 A_1\sin^2\lambda$$

$$\therefore \quad \frac{\sec^2 A_1\tan\phi}{\sqrt{1-\tan^2 A_1\tan^2\phi}} = \frac{\sqrt{1-e^2}\sec^2 A_1\tan\lambda}{\sqrt{1-(1-e^2)\tan^2 A_1\tan^2\lambda}} = \sqrt{1-e^2}\sec^2 A_1\sin\lambda\sec\theta$$

$\therefore$ by (38)

$$\delta A_1 = \frac{e^2\sin A_1\cos^2 A_1}{2\sqrt{1-e^2}}\;\frac{(\theta-\theta')}{\Big[\sin\lambda\sec\theta\Big]_{\lambda'}^{\lambda}}$$

Now

$$\sin\lambda\sec\theta = \frac{1}{a}\tan\theta$$

$$\left.\begin{aligned} \therefore \qquad \delta A_1 &= \frac{e^2\sin 2A_1}{4}\sqrt{\frac{1-e^2\sin^2 A_1}{1-e^2}}\cdot\frac{\theta-\theta'}{\tan\theta-\tan\theta'} \\ \text{in which} \qquad \theta &= \sin^{-1}\left(\sin\lambda\sec A_1\sqrt{1-e^2\sin^2 A_1}\right) \end{aligned}\right\} \quad \cdots\cdots \quad (39)$$

For computation

$$\sin\theta \doteq \sin\phi\sec A_1\left\{1+\frac{e^2}{2}(\cos^2\phi-\sin^2 A_1)\right\} \qquad \text{since } \sin\lambda = \sin\phi\sqrt{1-e^2\cos^2\phi}$$

$$\doteq \sin\phi\sec A_1\left(1+\frac{e^2}{2}\cos^2\phi\cos^2{}_1A\right) \qquad \text{since } \sin A_1 = \sin{}_1A\cos\phi$$

Let $\quad \sin\theta_1 = \sin\phi\sec A_1 \quad$ and $\theta = \theta_1 + \delta\theta$

$$\tan\theta_1 = \frac{\sin\phi}{\sqrt{\cos^2 A_1-\sin^2\phi}} = \frac{\sin\phi}{\sqrt{\cos^2\phi-\sin^2{}_1A\cos^2\phi}} = \tan\phi\sec{}_1A$$

and

$$\delta\theta = \frac{e^2}{2}\cos^2\phi\cos^2{}_1A\tan\theta_1 = \frac{e^2}{4}\sin 2\phi\cos{}_1A.$$

With a given value of the larger quantity θ, θ' say θ it is clear that $\dfrac{\theta-\theta'}{\tan\theta-\tan\theta'}$ is greater the smaller the value of θ'; its maximum value accordingly is $\dfrac{\theta}{\tan\theta}$ and the maximum value of this quantity occurs when $\theta=0$, when it becomes unity. It is quite clear then that δA_1 cannot exceed $\frac{1}{4}e^2\sin 2A$, *i. e.* $6'.\sin 2A_1$ in the case of the terrestrial spheroid where $e^2 \doteq \dfrac{1}{150}$.

6. If ds is the length of an elementary line

$$\rho\, d\lambda = -ds\cos A$$

$$\therefore \qquad s = -\int\rho\sec A\, d\lambda$$

On a geodesic

$$\cos A = \sqrt{1-\sin^2 A_0 \sec^2\phi} = \cos A_0 \sqrt{1-\tan^2 A_0 \tan^2\phi}$$

$$\rho = \frac{a\,(1-e^2)}{(1-e^2\sin^2\lambda)^{\frac{3}{2}}} \quad ; \quad \tan\lambda = \frac{\tan\phi}{\sqrt{1-e^2}}$$

$$\sin\lambda = \frac{\sin\phi}{\sqrt{1-e^2\cos^2\phi}}; \quad 1-e^2\sin^2\lambda = \frac{1-e^2}{1-e^2\cos^2\phi}\ ; \ \cos\lambda = \cos\phi\sqrt{\frac{1-e^2}{1-e^2\cos^2\phi}}$$

whence

$$\rho = \frac{a}{\sqrt{1-e^2}}\,(1-e^2\cos^2\phi)^{\frac{3}{2}}$$

Also $\dfrac{d\lambda}{\sin\lambda\cos\lambda} = \dfrac{d\phi}{\sin\phi\cos\phi}$

$$d\lambda = d\phi\frac{\sqrt{1-e^2}}{1-e^2\cos^2\phi}$$

$$\therefore \quad s = -\int\frac{a\,(1-e^2\cos^2\phi)^{\frac{3}{2}}}{\sqrt{1-e^2}}\cdot\frac{\sec A_0}{\sqrt{1-\tan^2 A_0\tan^2\phi}}\cdot\frac{\sqrt{1-e^2}}{1-e^2\cos^2\phi}\cdot d\phi$$

$$= -a\sec A_0\int\sqrt{\frac{1-e^2\cos^2\phi}{1-\tan^2 A_0\tan^2\phi}}\cdot d\phi$$

Put $\sin\phi = x \quad d\phi = \dfrac{dx}{\sqrt{1-x^2}}$

$$s = -a\sec A_0\int\sqrt{\frac{1-e^2(1-x^2)}{1-\tan^2 A_0\, x^2/(1-x^2)}}\cdot\frac{dx}{\sqrt{1-x^2}}$$

$$= -a\int\sqrt{\frac{1-e^2(1-x^2)}{\cos^2 A_0 - x^2}}\cdot dx = -a\sqrt{1-e^2}\int\frac{\sqrt{1+\beta^2x^2}}{\sqrt{\cos^2 A_0-x^2}}\cdot dx$$

where $\beta^2 = \dfrac{e^2}{1-e^2}$

Put $x = \cos A_0 \sin\chi = \sin\phi$

then

$$s = -a\sqrt{1-e^2}\int_{\chi'}^{\chi}\frac{\sqrt{1+\beta^2\cos^2 A_0\sin^2\chi}}{\cos A_0\cos\chi}\cdot\cos A_0\cos\chi d\chi$$

$$= -a\sqrt{1-e^2}\int_{\chi'}^{\chi}\sqrt{1+h^2\sin^2\chi}\,d\chi \qquad \text{where } h^2 = \beta^2\cos^2 A_0 = \frac{e^2\cos^2 A_0}{1-e^2}$$

$$= -a\sqrt{1-e^2}\Big[\chi + \tfrac{1}{2}h^2\int\sin^2\chi\,d\chi - \tfrac{1}{8}h^4\int\sin^4\chi\,d\chi \ \ \ldots\ldots\Big]_{\chi'}^{\chi}$$

Now $\displaystyle\int\sin^n\chi\,d\chi = -\frac{1}{n}\cos\chi\sin^{n-1}\chi + \frac{(n-1)}{n}\int\sin^{n-2}\chi\,d\chi$

$$\therefore \quad \int\sin^2\chi\,d\chi = -\tfrac{1}{2}\cos\chi\sin\chi + \frac{\chi}{2}$$

$$\int \sin^4 \chi \, d\chi = -\tfrac{1}{4} \cos\chi \sin^3\chi - \tfrac{3}{4} \int \sin^2 \chi \, d\chi$$

$$= -\tfrac{1}{4} \cos\chi \sin^3\chi - \tfrac{3}{8} \cos\chi \sin\chi + \tfrac{3}{8}\chi$$

etc.

$$\therefore \quad s = -a\sqrt{1-e^2}\left[\chi\left(1+\tfrac{1}{4}h^2 - \frac{3}{64}h^4 \ldots\right) - \frac{h^2}{4}\cos\chi\sin\chi\left(1 - \frac{3}{16}h^2 \ldots\right) + \frac{h^4}{32}\cos\chi\sin^3\chi\,(1 \quad \ldots \quad) - \quad \ldots \quad \right]_{\chi'}^{\chi} \quad \ldots \quad (40)$$

where

$$\left.\begin{aligned} \sin\chi &= \frac{\sin\phi}{\cos A_0} \\ h^2 &= \frac{e^2 \cos^2 A_0}{1-e^2} \end{aligned}\right\} \quad \ldots \ldots \quad (41)$$

Otherwise since $\sin A \cos\phi = \sin A_0$

$$\sin\chi = \frac{\sin\phi \sin A_0}{\sin A \cos\phi \cos A_0} = \tan\phi \tan A_0 \operatorname{cosec} A$$

Also

$$\tan^2 A_0 = \frac{\sin^2 A \cos^2\phi}{1-\sin^2 A \cos^2\phi} = \frac{\sin^2 A}{\tan^2\phi + \cos^2 A}$$

and

$$\sin\chi = \frac{\tan\phi}{\pm\sqrt{\tan^2\phi + \cos^2 A}}$$

$$\tan\chi = \pm \tan\phi \sec A \quad \ldots \ldots \quad (42)$$

7. To facilitate reductions, tables are now given enabling the conversion from λ to ϕ and *vice versa* to be easily performed. They have been computed from formula $\lambda - \phi = \tfrac{1}{2}.\dfrac{\epsilon \sin^2\lambda}{1-\epsilon\sin^2\lambda}$ which is readily deducible from (4).

Table XXI.

λ	$\phi-\lambda$	λ	$\phi-\lambda$	λ	$\phi-\lambda$	λ	$\phi-\lambda$
°	′ ″	°	′ ″	°	′ ″	°	′ ″
0	−0 0·0	10	−1 57·3	20	−3 40·5	30	−4 57·1
1	−0 12·0	11	−2 8·4	21	−3 49·5	31	−5 3·0
2	−0 23·8	12	−2 19·4	22	−3 58·2	32	−5 8·5
3	−0 35·8	13	−2 30·3	23	−4 6·7	33	−5 13·5
4	−0 47·7	14	−2 41·0	24	−4 14·9	34	−5 18·2
5	−0 59·5	15	−2 51·4	25	−4 22·8	35	−5 22·5
6	−1 11·3	16	−3 1·8	26	−4 30·4	36	−5 26·4
7	−1 23·0	17	−3 11·9	27	−4 37·6	37	−5 29·9
8	−1 34·5	18	−3 21·6	28	−4 44·5	38	−5 33·1
9	−1 45·9	19	−3 31·2	29	−4 50·9	39	−5 35·8
10	−1 57·3	20	−3 40·5	30	−4 57·1	40	−5 38·2

Table XXII.

ϕ	$\lambda-\phi$	ϕ	$\lambda-\phi$	ϕ	$\lambda-\phi$	ϕ	$\lambda-\phi$
°	′ ″	°	′ ″	°	′ ″	°	′ ″
0	+0 0·0	10	+1 57·7	20	+3 41·1	30	+4 57·6
1	+0 12·0	11	+2 8·8	21	+3 50·1	31	+5 3·5
2	+0 23·9	12	+2 19·8	22	+3 58·8	32	+5 8·9
3	+0 35·9	13	+2 30·7	23	+4 7·3	33	+5 13·9
4	+0 47·9	14	+2 41·5	24	+4 15·5	34	+5 18·6
5	+0 59·7	15	+2 51·9	25	+4 23·4	35	+5 22·8
6	+1 11·5	16	+3 2·3	26	+4 30·9	36	+5 26·7
7	+1 23·3	17	+3 12·4	27	+4 38·1	37	+5 30·2
8	+1 34·8	18	+3 22·1	28	+4 45·0	38	+5 33·3
9	+1 46·2	19	+3 31·7	29	+4 51·4	39	+5 36·0
10	+1 57·7	20	+3 41·1	30	+4 57·6	40	+5 38·4

Values of A_0, k, s together with certain of the quantities by means of which they are computed are now given in tabular form for geodesics passing through the origin and points L, ϕ, for values of $L'-L$ differing by 4° from 0 to 24° and for values of ϕ from 10° to 38°. It is clear that for longitudes east of the origin the value of A is 360− (its value in table): and that for s there is no change.

TABLE XXIII.

ϕ	$L'-L$	0°	4°	8°	12°	16°	20°	24°
38°	$180°-A_1$	0° 0′ 0″	11° 40′ 32″·2	21° 58′ 5″·6	30° 12′ 43″·8	36° 27′ 5″·0	41° 2′ 45″·3	44° 23′ 53″·7
	$-\delta A_1$	0 0	1 37·2	2 41·6	3 6·7	3 2·2	2 48·9	2 27·1
	$180°-A_0$	0 0 0	11 42 9	22 0 47	30 15 51	36 30 7	41 5 34	44 26 21
	log k	$-\infty$	$\bar{1}$·3071363	$\bar{1}$·5738213	$\bar{1}$·7024180	$\bar{1}$·7744081	$\bar{1}$·8177511	$\bar{1}$·8451916
	h^2	0·006682	0·006407	0·005743	0·004985	0·004318	0·003795	0·003407
	s/b	0·243714	0·250921	0·271391	0·302418	0·341099	0·385090	0·432724
34°	$180°-A_1$	0 0 0	16 46 26·0	30 1 52·5	39 8 30·0	45 6 17·2	48 59 53·3	51 34 18·2
	$-\delta A_1$	0 0	2 14·0	3 21·5	3 20·0	2 53·0	2 21·7	1 52·7
	$180°-A_0$	0 0 0	16 48 40	30 5 14	39 11 50	45 9 10	49 2 15	51 36 6
	log k	$-\infty$	$\bar{1}$·4612245	$\bar{1}$·7001131	$\bar{1}$·8007114	$\bar{1}$·8506405	$\bar{1}$·8780268	$\bar{1}$·8941560
	h^2	0·006682	0·006123	0·005003	0·004013	0·003323	0·002872	0·002578
	s/b	0·173820	0·184246	0·212464	0·252555	0·299755	0·351166	0·405139
30°	$180°-A_1$	0 0 0	27 11 14·3	43 2 45·7	50 56 59·8	55 2 12·4	57 17 10·9	58 34 48·1
	$-\delta A_1$	0 0	3 24·5	3 27·8	2 36·8	1 51·6	1 18·3	0 54·9
	$180°-A_0$	0 0 0	27 14 39	43 6 14	50 59 37	55 4 4	57 18 29	58 35 48
	log k	$-\infty$	$\bar{1}$·6606593	$\bar{1}$·8346251	$\bar{1}$·8904627	$\bar{1}$·9137238	$\bar{1}$·9250992	$\bar{1}$·9312075
	h^2	0·006682	0·005282	0·003562	0·002647	0·002191	0·001949	0·001814
	s/b	0·103942	0·121206	0·162319	0·213923	0·270030	0·328293	0·387702
26°	$180°-A_1$	0 0 0	52 54 33·6	60 59 16·4	62 59 19·7	63 41 3·3	63 56 27·7	
	$-\delta A_1$	0 0	2 46·9	1 7·7	0 32·1	0 16·4	0 6·5	
	$180°-A_0$	0 0 0	52 57 21	61 0 24	62 59 52	63 41 20	63 56 34	
	log k	$-\infty$	$\bar{1}$·9020954	$\bar{1}$·9418474	$\bar{1}$·9498721	$\bar{1}$·9525018	$\bar{1}$·9534486	
	h^2	0·006682	0·002425	0·001570	0·001378	0·001318	0·001289	
	s/b	0·034077	0·072033	0·131409	0·198359	0·255990	0·318858	
22°	$180°-A_1$	0 0 0	53 31 3·3	62 24 14·3	64 41 45·0	65 31 8·4	65 50 39·9	65 56 48·9
	$-\delta A_1$	0 0	3 5·1	1 19·0	0 38·7	0 20·4	0 9·4	0 2·1
	$180°-A_0$	0 0 0	53 34 8	62 25 33	64 42 24	65 31 29	65 50 49	65 56 51
	log k	$-\infty$	$\bar{1}$·9055652	$\bar{1}$·9476361	$\bar{1}$·9562317	$\bar{1}$·9591081	$\bar{1}$·9602121	$\bar{1}$·9605528
	h^2	0·006682	0·002357	0·001432	0·001220	0·001147	0·001119	0·001110
	s/b	0·035775	0·073722	0·133777	0·196608	0·260182	0·324032	0·387973
18°	$180°-A_1$	0 0 0	29 19 11·0	46 25 6·2	55 5 13·3	59 41 41·7	62 18 45·2	63 52 30·2
	$-\delta A_1$	0 0	4 2·5	4 7·9	3 11·5	2 21·1	1 43·1	1 15·0
	$180°-A_0$	0 0 0	29 23 14	46 29 14	55 8 25	59 44 3	62 20 28	63 53 45
	log k	$-\infty$	$\bar{1}$·6908226	$\bar{1}$·8604705	$\bar{1}$·9141068	$\bar{1}$·9363608	$\bar{1}$·9473002	$\bar{1}$·9532744
	h^2	0·006682	0·005073	0·003168	0·002183	0·001697	0·001440	0·001294
	s/b	0·105616	0·124192	0·168012	0·222619	0·281789	0·343184	0·405645
14°	$180°-A_1$	0 0 0	19 22 52·8	34 31 14·1	44 51 50·8	51 41 49·6	56 15 27·3	59 21 59·3
	$-\delta A_1$	0 0	3 8·3	4 27·9	4 27·1	3 54·8	3 17·9	2 43·8
	$180°-A_0$	0 0 0	19 26 1	34 35 42	44 56 18	51 45 44	56 18 45	59 24 43
	log k	$-\infty$	$\bar{1}$·5220722	$\bar{1}$·7541789	$\bar{1}$·8490169	$\bar{1}$·8951186	$\bar{1}$·9201629	$\bar{1}$·9349266
	h^2	0·006682	0·005943	0·004528	0·003348	0·002560	0·002056	0·001730
	s/b	0·157964	0·187488	0·219676	0·264754	0·317253	0·374042	0·433406
10°	$180°-A_1$	0 0 0	14 27 13·6	27 2 2·7	36 58 9·7	44 26 58·7	50 0 9·0	54 8 7·8
	$-\delta A_1$	0 0	2 29·4	4 5·5	4 41·4	4 38·9	4 18·7	3 51·3
	$180°-A_0$	0 0 0	14 29 43	27 6 8	37 2 51	44 31 38	50 4 28	54 11 59
	log k	$-\infty$	$\bar{1}$·3984612	$\bar{1}$·6585649	$\bar{1}$·7799407	$\bar{1}$·8458708	$\bar{1}$·8847263	$\bar{1}$·9090537
	h^2	0·006682	0·006264	0·005295	0·004257	0·003396	0·002752	0·002287
	s/b	0·245270	0·254192	0·279254	0·316635	0·362531	0·414102	0·469464

TABLE XXIV.

ϕ	$L'-L$	4°	8°	12°	16°	20°	24°	$L'-L$	4°	8°	12°	16°	20°	24°
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″		° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
38°	ψ	9 18 48·0	18 24 47·6	27 7 19·7	35 19 17·2	42 57 7·6	50 0 29·7	ψ'	5 18 13·3	10 23 38·1	15 5 35·6	19 16 57·9	22 54 14·4	25 57 2·0
	θ	39 2 46	41 41 34	45 32 6	50 2 52	54 49 40	59 36 57	θ'	24 39 44	26 8 25	28 12 42	30 30 56	32 46 56	34 50 54
	$-\chi$	38 57 23·4	41 36 40·7	45 27 50·1	49 59 15·0	54 46 36·4	59 34 23·7	$-\chi'$	24 35 33·5	26 4 32·5	28 9 8·0	30 27 41·7	32 43 58·1	34 48 9·4
	$180°-A$	14 54 57	28 24 7	39 45 34	49 0 53	56 31 21	62 41 9	$180°-A'$	12 49 59	24 13 58	33 29 48	40 38 48	46 2 4	50 3 31
34°	ψ	11 45 31·0	23 0 15·0	33 22 16·3	42 41 47·9	50 58 58·9	58 19 40·8	ψ'	7 44 54·5	14 59 2·1	21 20 26·9	26 39 22·0	30 55 57·1	34 16 3·1
	θ	35 49 35	40 19 53	46 14 27	52 30 24	58 34 31	64 13 30	θ'	25 15 46	28 9 24	31 46 34	35 20 40	38 28 29	41 2 20
	$-\chi$	35 44 36·7	40 15 39·3	46 11 1·0	52 27 38·7	58 32 19·7	64 11 46·3	$-\chi'$	25 11 43·3	28 5 50·3	31 43 29·4	35 17 59·0	38 26 4·7	41 0 8·1
	$180°-A$	20 25 3	37 12 26	49 40 9	58 46 54	65 37 29	70 57 58	$180°-A'$	18 27 50	33 17 47	43 47 37	50 55 50	55 47 0	59 6 56
30°	ψ	17 17 39·5	32 42 25·8	45 27 48·4	55 45 11·5	64 6 17·8	71 1 40·0	ψ'	13 17 1·7	24 41 9·8	33 25 54·1	39 42 39·5	44 3 9·4	46 57 53·6
	θ	34 17 29	43 16 20	52 38 4	60 51 33	67 47 47	73 39 40	θ'	27 20 35	33 58 30	40 23 14	45 24 11	49 0 29	51 28 28
	$-\chi$	34 13 16·3	43 13 17·5	52 35 54·0	60 49 57·2	67 46 36·8	73 38 49·5	$-\chi'$	27 16 53·2	33 55 39·9	40 20 59·8	45 22 18·5	48 58 49·2	51 26 56·8
	$180°-A$	31 54 40	52 5 42	63 48 14	71 12 6	76 21 23	80 14 48	$180°-A'$	30 5 9	48 26 27	58 18 49	63 51 53	67 9 30	69 10 9
26°	ψ	40 15 21·3	61 39 31·3	73 9 48·7	80 31 41·5	85 55 33·0		ψ'	36 14 41·9	53 38 12·2	61 7 51·0	64 29 4·6	65 52 17·5	
	θ	46 43 33	64 45 38	74 55 13	81 30 7	86 20 26		θ'	42 36 4	57 14 20	63 51 15	66 51 1	68 5 36	
	$-\chi$	46 41 28·7	64 44 35·1	74 54 37·8	81 29 39·5	86 20 19·5		$-\chi'$	42 33 59·6	57 13 6·2	63 50 18·6	66 50 9·7	68 4 50·3	
	$180°-A'$	62 37 43	76 41 49	82 26 36	85 49 2	88 12 42		$180°-A'$	60 55 52	73 17 55	77 20 15	78 59 35	79 39 20	
22°	ψ	33 11 16·7	50 41 8·8	58 45 26·4	62 34 8·8	64 17 12·1	64 51 13·0	ψ'	37 11 57·8	58 42 30·0	70 47 27·9	78 36 50·4	84 20 35·0	88 55 15·0
	θ	39 8 33	54 2 38	61 16 15	64 43 37	66 17 29	66 48 31	θ'	43 21 54	61 42 9	72 31 30	79 37 11	84 50 18	89 0 54
	χ	39 6 34·5	54 1 28·4	61 15 21·6	64 42 51·2	66 16 46·1	66 47 49·1	χ'	43 19 53·1	61 41 7·8	72 30 54·2	79 36 49·8	84 50 7·7	89 0 52·3
	A	60 11 58	72 56 44	77 11 49	78 59 49	79 46 27	80 1 36	A'	61 46 9	76 5 18	81 55 9	85 18 31	87 41 19	89 33 39
18°	ψ	10 32 37·5	20 0 48·5	27 48 16·9	33 50 3·8	38 18 48·4	41 32 18·6	ψ'	14 33 19·3	28 2 12·2	39 50 22·2	40 52 51·1	58 22 16·3	65 36 27·9
	θ	20 49 13	26 42 15	32 45 22	37 50 18	41 45 18	44 37 51	θ'	27 56 41	36 19 59	45 30 30	53 58 35	61 24 15	67 51 24
	χ	20 46 19·6	26 40 3·8	32 43 39·5	37 48 53·7	41 44 7·0	44 36 44·2	χ'	27 53 5·0	36 17 23·8	45 28 38·0	53 57 11·3	61 23 12·4	67 50 37·4
	A	31 3 42	49 41 20	59 37 49	65 15 0	68 38 23	70 46 15	A'	32 30 12	52 34 32	63 58 4	71 2 53	75 54 38	79 31 50
14°	ψ	5 2 47·9	9 54 8·1	14 24 21·7	18 26 45·7	21 57 51·5	24 56 51·9	ψ'	9 3 30·7	17 55 33·6	26 26 30·3	34 29 36·7	42 1 25·3	49 1 7·3
	θ	14 54 25	17 7 37	20 0 55	23 2 6	25 52 56	28 24 27	θ'	25 40 9	29 43 42	35 11 35	41 12 57	47 18 42	53 14 3
	χ	14 51 52·8	17 5 26·5	19 59 3·3	23 0 32·1	25 51 33·0	28 23 11·6	χ'	25 36 10·8	29 40 21·6	35 8 52·9	41 10 46·4	47 16 56·3	53 12 35·0
	A	20 3 13	35 48 50	46 42 59	54 2 48	59 2 33	62 31 26	A'	21 22 1	38 26 36	50 40 4	59 19 43	55 40 1	70 30 19
10°	ψ	2 36 46·0	5 10 38·7	7 38 55·8	9 59 16·4	12 9 46·7	14 9 4·2	ψ'	6 37 29·5	31 12 5·9	19 41 6·8	26 2 11·0	32 13 24·1	38 13 24·9
	θ	10 21 50	11 16 40	12 35 31	14 7 14	15 43 7	17 17 14	θ'	24 57 38	27 18 20	30 45 21	34 54 30	39 27 20	44 11 29
	χ	10 19 56·4	11 14 55·3	12 33 58·2	14 5 51·2	15 41 52·9	17 16 7·5	χ'	24 53 31·7	27 14 38·1	30 42 9·1	34 51 45·9	39 25 1·4	44 9 31·7
	A	14 43 26	27 33 20	37 43 4	45 24 11	51 8 33	55 26 39	A'	15 54 28	29 55 34	41 16 49	50 9 57	57 6 57	62 38 32

For the case $L'-L=0$ it is clear that

$$\psi=\psi'=0; \quad \theta=\lambda; \quad \theta'=\lambda'; \quad \chi=\pm\phi; \quad \chi'=\pm\phi'$$

8. It may be of interest to find the expression for the azimuthal angle of a vertical plane at the origin which passes through any given point on the earth, so that the difference of this and the geodesic may be studied.

The spheroid may be expressed

$$\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1 \qquad (43)$$

and P and Q two points on surface

$P \qquad a\cos\phi, 0, b\sin\phi$

$Q \qquad a\cos\phi'\cos L, a\cos\phi'\sin L, b\sin\phi'$

Tangent plane at P is

$$\frac{x\cos\phi}{a}+\frac{z\sin\phi}{b}=1 \qquad (44)$$

The vertical plane at P which passes through Q also is

$$lx+my+nz=1$$

subject to the conditions

$$la\cos\phi + nb\sin\phi = 1 \qquad \text{since } P \text{ is in it}$$

$$la\cos\phi'\cos L + ma\cos\phi'\sin L + nb\sin\phi' = 1 \qquad \text{since } Q \text{ is in it}$$

$$\frac{l\cos\phi}{a} + \frac{n\sin\phi}{b} = 0 \qquad \text{since it is perpendicular to (44)}$$

$$\therefore \quad \frac{la\cos\phi}{a^2} = \frac{nb\sin\phi}{-b^2} = \frac{la\cos\phi + nb\sin\phi}{a^2-b^2} = \frac{1}{a^2-b^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (45)$$

Also

$$\frac{1}{a^2-b^2} = \frac{la}{a^2\sec\phi} = \frac{nb}{-b^2\operatorname{cosec}\phi} = \frac{la\cos\phi'\cos L + nb\sin\phi' - 1}{a^2\sec\phi\cos\phi'\cos L - b^2\operatorname{cosec}\phi\sin\phi' - a^2 + b^2}$$

$$= \frac{m\,a\cos\phi'\sin L}{a^2\left(1 - \dfrac{\cos\phi'\cos L}{\cos\phi}\right) - b^2\left(1 - \dfrac{\sin\phi'}{\sin\phi}\right)} \quad . \; . \; . \; . \; . \; . \; . \quad (46)$$

The azimuthal angle of Q from P as determined by the vertical plane through P is the angle this plane makes with ZOX or otherwise it is the angle between the normal to this plane, whose direction cosines are proportional to $l\,m\,n$, and the axis OY.

The angle accordingly is $\cos^{-1}\dfrac{m}{\sqrt{l^2+m^2+n^2}} = \cot^{-1}\dfrac{m}{\sqrt{l^2+n^2}} = \psi$ say

$$\therefore \quad \tan\psi = \frac{\sqrt{l^2+n^2}}{m} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (47)$$

Now

$$\frac{l}{a\sec\phi} = \frac{n}{-b\operatorname{cosec}\phi} = \frac{1}{a^2-b^2} = \frac{\sqrt{l^2+n^2}}{\sqrt{a^2\sec^2\phi + b^2\operatorname{cosec}^2\phi}}$$

$$\therefore \quad \sqrt{l^2+n^2} = \frac{a}{\sin\phi\cos\phi}\sqrt{\sin^2\phi + (1-e^2)\cos^2\phi} \cdot \frac{1}{a^2e}$$

$$= \frac{\sqrt{1-e^2\cos^2\phi}}{ae^2\sin\phi\cos\phi} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (48)$$

and by (46)

$$m = \frac{1 - \dfrac{\cos'\phi\cos L}{\cos\phi} - (1-e^2)\left(1 - \dfrac{\sin\phi'}{\sin\phi}\right)}{ae^2\cos\phi'\sin L}$$

$$= \left\{\frac{\sin\phi'}{\sin\phi} - \frac{\cos\phi'\cos L}{\cos\phi} + e^2\left(1 - \frac{\sin\phi'}{\sin\phi}\right)\right\} \Big/ ae^2\cos\phi'\sin L$$

$$\therefore \quad \tan\psi = \frac{\sqrt{1-e^2\cos^2\phi}}{ae^2\sin\phi\cos\phi} \cdot \frac{ae^2\cos\phi'\sin L}{\dfrac{\sin\phi'}{\sin\phi} - \dfrac{\cos\phi'\cos L}{\cos\phi} + e^2\left(1 - \dfrac{\sin\phi'}{\sin\phi}\right)}$$

$$= \frac{2\cos\phi'\sin L\sqrt{1-e^2\cos^2\phi}}{\sin 2\phi\left\{\dfrac{\sin\phi'}{\sin\phi} - \dfrac{\cos\phi'\cos L}{\cos\phi} + e^2\left(1 - \dfrac{\sin\phi'}{\sin\phi}\right)\right\}} \quad . \; . \quad (49)$$

Substitute for ϕ in terms of λ by means of (4)

$$\tan\phi = \frac{1}{\sqrt{1-e^2}}\tan\lambda; \quad \frac{\sin\phi}{\sqrt{1-e^2}\sin\lambda} = \frac{\cos\phi}{\cos\lambda} = \frac{1}{\sqrt{1-e^2\sin^2\lambda}}$$

Hence the value of $\tan\psi$ is

$$\frac{\dfrac{\cos\lambda'\sin L}{\sqrt{1-e^2\sin^2\lambda}}\sqrt{\dfrac{1-e^2}{1-e^2\sin^2\lambda}}}{\dfrac{(\sin\lambda'\cos\lambda-\cos\lambda'\sin\lambda\cos L)\sqrt{1-e^2}}{\sqrt{(1-e^2\sin^2\lambda)(1-e^2\sin^2\lambda')}}+\dfrac{e^2\cos\lambda\sqrt{1-e^2}}{\sqrt{1-e^2\sin^2\lambda}}\left(\dfrac{\sin\lambda}{\sqrt{1-e^2\sin^2\lambda}}-\dfrac{\sin\lambda'}{\sqrt{1-e^2\sin^2\lambda'}}\right)}$$

from which it follows that

$$\tan\psi = \cos\lambda'\sin L\Big/\left\{\sin\lambda'\cos\lambda-\cos\lambda'\sin\lambda\cos L+e^2\cos\lambda\left(\sin\lambda\sqrt{\frac{1-e^2\sin^2\lambda'}{1-e^2\sin^2\lambda}}-\sin\lambda'\right)\right\}. \quad (50)$$

ψ being the azimuthal angle of ϕ from P.

The case of a sphere is found by putting $e=o$, when (50) becomes

$$\tan\psi_0=\frac{\cos\lambda'\sin L}{\sin\lambda'\cos\lambda-\cos\lambda'\sin\lambda\cos L} \quad \dots\dots\dots\dots \quad (51)$$

which is the ordinary formula.

Let $\psi=\psi_0+\delta\psi$: then $\cot\overline{\psi_0+\delta\psi}-\cot\psi_0=-\dfrac{\sin\delta\psi}{\sin\psi\sin\psi_0}$

$$-\delta\psi \doteq e^2\frac{\sin^2\psi_0\cos\lambda}{\sin L\cos\lambda'}\left\{\sin\lambda\sqrt{\frac{1-e^2\sin^2\lambda'}{1-e^2\sin^2\lambda}}-\sin\lambda'\right\} \quad \dots\dots\dots \quad (52)$$

This formula gives the correction to be applied to the azimuthal angle, found from the formula for a sphere, to obtain the spheroidal azimuth.

CHAPTER III.

Changes of coordinates of triangulated points, due to changes in axes of the terrestrial spheroid, calculated along geodesics.

1. Consider a geodesic on any surface, and let A B C be three consecutive points on it. Then A B C is the osculating plane at B and from the fundamental property of the geodesic it contains the normal to the surface at B. This shows that measured from B the azimuths of A and C differ by two right angles. It is possible then to describe a geodesic on a surface of unknown form by fulfilling this condition: and, to take a practical case, a traverse along a geodesic can be observed on the earth without knowing its figure if access to a level surface is possible. It follows that if there is a geodesic on one surface which has been selected as representing the earth and it is desired to change to another surface, the geodesic on the first surface will, on transfer to the second surface, remain a geodesic. This property makes it possible to differentiate along a geodesic with respect to the constants of the first surface and so to find relations between changes in these constants and the quantities defining the position of points.

This fact will now be made use of in connection with the equations of Chapter II for the case of a spheroid. Now it has been shown in Chapter I that the effect of slightly changing the latitude and azimuth at the origin may be computed, and that the result is practically independent of the route followed—values of u_x and u_y etc. being identical to nearer than 0·001 of a second: and the resulting changes of latitude, longitude and azimuth for unit changes at the origin are given in tables XVII–XX. These values then are equally applicable to a geodesic and so it is only necessary to consider the effects of changes in a and b.

It is however found convenient not to alter the constant k of the geodesic; and since this is equal to $\sin A \cos \phi$, this is given effect to by not changing the values of A and ϕ at the origin. It is moreover more convenient in dealing with geodesics on a spheroid to make use of the eccentric angle, or reduced latitude ϕ, in place of the latitude λ. Now the relation between λ and ϕ involves e^2 and so if ϕ is unchanged at the origin while e^2 undergoes a change it follows that λ must also change at the origin. In the solution which follows the effect of this origin change of λ occurs: but as its amount can be found from tables XVII, XVIII there is no difficulty in removing it.

2. For convenience of reference equations (26), (25), (27), (2), (40), (42), (41) of Chapter II are repeated.

$$\left[L\right] = \pm\left[\pm\psi - \frac{ke^2}{2}\theta\right] \quad \ldots \quad (1)$$

$$\psi = \pm\tan^{-1}(\tan A\sin\phi) = \pm\sin^{-1}(\tan A_0\tan\phi) \quad \ldots \quad (2)$$

$$\tan\theta = \frac{1}{k}\sqrt{\frac{1-k^2e^2}{1-e^2}}\tan\psi \quad \ldots \quad (3)$$

$$k = \sin A\cos\phi = \sin A_0 \quad \ldots \quad (4)$$

$$s = -a\sqrt{1-e^2}\left[\chi\left(1+\tfrac{1}{4}h^2\ .\ \right) - \frac{h^2}{8}\sin 2\chi\left(1-\ .\ \right) +\ .\ \right]_{\chi'}^{\chi} \quad (5)$$

$$\tan\chi = \pm\tan\phi\sec A = \pm\frac{1}{k}\tan\psi \quad \ldots \quad (6)$$

$$h^2 = \frac{e^2\cos^2 A_0}{1-e^2} \quad \ldots \quad (7)$$

$$\sin\chi = \sin\phi\sec A_0 \quad \ldots \quad (8)$$

The signs occurring in (1) and (2) are to be determined as explained in Chapter II. The sign of χ is determined by (8). It is best to consider a and e^2 as independent variables and b as dependent on these. At the end there is no difficulty in passing to the case of a and b considered as independent and e^2 as the dependent variable.

3. Suppose then that a and e^2 are changed, while the azimuth and reduced latitude ϕ of the origin remain unchanged. Values at the origin will be denoted by dashes.

Differentiating (4) and keeping k constant

$$\delta k = 0 = \cos A\cos\phi\,\delta A - \sin A\sin\phi\,\delta\phi$$

Changes of *reduced latitude*, longitude and azimuth, in keeping with the notation of Chapter I, will be denoted by u_1, v, w: and the above equation may be written

$$w = \tan A\tan\phi.\,u_1 \quad \ldots \quad (9)$$

Differentiating (1) and remembering that $v'=0$

$$\pm v = \left[\delta\psi\right] - \frac{ke^2}{2}\left[\delta\theta\right] - \frac{k\delta e^2}{2}\left[\theta\right] \quad \ldots \quad (10)$$

By differentiating logarithmically $\tan\psi = \pm\tan A\sin\phi$ which is the same as (2): and by (3) and (6)

$$\frac{\delta\psi}{\sin\psi\cos\psi} = \frac{w}{\sin A\cos A} + u_1\cot\phi = \frac{\delta\chi}{\sin\chi\cos\chi} \doteq \frac{\delta\theta}{\sin\theta\cos\theta} - (1-k^2)\frac{\delta e^2}{2} \equiv x \quad . \quad . \quad (11)$$

any of these expressions being denoted by x. This quantity x vanishes at the origin.

Finally differentiate (5) keeping s constant, replacing $\delta\chi$ by means of (11) and $a\sqrt{1-e^2}$ by b

$$\frac{s}{b}\left(-\frac{\delta a}{a} + \frac{\delta e^2}{2(1-e^2)}\right) = -\frac{x}{2}\sin 2\chi\left\{\left(1+\frac{h^2}{4}+..\right) - \frac{h^2}{4}\cos 2\chi(1..)..\right\} - \frac{\delta h^2}{8}\left[2\chi(1...) - \sin 2\chi(...)\right]$$

$$\doteq -\frac{x}{2}\sin 2\chi\left\{1+\frac{h^2}{4}(1-\cos 2\chi)\right\} - \frac{\delta h^2}{8}\left[2\chi - \sin 2\chi\right]. \quad \ldots \quad (12)$$

where

$$\delta h^2 = \frac{\cos^2 A_0}{(1-e^2)^2}.\,\delta e^2 \quad \ldots \quad (13)$$

Equations (12) and (13) serve to determine x in terms of $\frac{\delta a}{a}$ and δe^2. For the other quantities from (9) and (11) it follows that

$$w = \tan A \tan\phi . u_1 = \frac{x \tan A}{\sec^2 A + \cot^2\phi} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and from (10) and (11)

$$\pm v = \frac{x}{2}\sin 2\psi - \frac{ke^2}{4}\left\{\left[\sin 2\theta\right](1-k^2)\frac{\delta e^2}{2} + x\sin 2\theta\right\} - \frac{k\delta e^2}{2}\left[\theta\right] \quad . \quad . \quad . \quad (15)$$

In all the above equations square brackets indicate that the quantity enclosed has to be taken between limits.

4. It remains to give the relation between u and u_1

From (4) of Chapter II

$$\tan\phi = \sqrt{1-e^2}\tan\lambda$$

Differentiating this logarithmically

$$\frac{u_1}{\sin\phi\cos\phi} = \frac{u}{\sin\lambda\cos\lambda} - \frac{1}{2}\frac{\delta e^2}{1-e^2}$$

or

$$u = \frac{\sin 2\lambda}{\sin 2\phi} . u_1 + \frac{1}{4}\frac{\delta e^2}{1-e^2}\sin 2\lambda$$

Now

$$\frac{\sin 2\lambda}{\sin 2\phi} = \frac{\sqrt{1-e^2}}{1-e^2\cos^2\phi} \doteq 1 + \frac{e^2}{2}(2\cos^2\phi - 1)$$

$$\doteq 1 + \frac{e^2}{2}\cos 2\phi$$

so that

$$u \doteq \left(1 + \frac{e^2}{2}\cos 2\phi\right)u_1 + \frac{1}{4}\frac{\delta e^2}{1-e^2}\sin 2\phi \quad . \quad . \quad . \quad . \quad (16)$$

At the origin $u_1' = 0$: hence

$$u' = \frac{1}{4}\frac{\delta e^2}{1-e^2}\sin 2\phi' \quad . \quad . \quad . \quad . \quad . \quad (17)$$

5. It may be noticed that by this method the changes u, v, w appear to be found without any integration, whereas in Chapter I simultaneous differential equations occurred which had to be solved. The case under consideration is a particular case of the general equations (2) of Chapter I. The decision to follow a geodesic introduces a relation by which these equations can be reduced to total differential equations: and the integration of these equations would lead to the same results as may be obtained from equations (12) to (17). The same results will be seen to be obtainable by application to values of u_x, v_x, w_x of the appropriate closing errors. For the case now under consideration the equations formed in Chapter II give the results of integration: and so no further integration is necessary.

6. In making use of the equations (12) to (17) two cases are considered in which

(i) $\delta a = 1$ km. and $\delta e^2 = 0$

(ii) $\delta a = 0$ and $\delta e^2 = \cdot 0001$

The first of these corresponds to a combination of cases I and II of Chapter I, while the second corresponds to a combination of cases II and III. This arrangement simplifies computation and there is no difficulty in deriving cases I and II when the computations are complete.

As no azimuthal change is being made at the origin it is clear that there is symmetry about a central meridian. In Chapter II values of $\psi, \theta, \chi, A, \frac{s}{b}, h^2, k$ (*vide* tables XXIII, XXIV) have already been given for every 4° of ϕ from 10° to 38° and for longitude differences of 4° from 4° to 24°. With the help of these the values of u_1, u, v, w exhibited in the following two tables have been found. A double sign is prefixed to v and w and of these the upper or lower is to be taken according as the point is west or east of the origin. The results are given to three places of decimals as found by the computations: but the last figure is liable to error, which is not sufficiently large to be practically important for the present purpose.

TABLE XXV.

$\delta a = 1$ km., $(\delta e^2 = 0)$ $\delta b = \frac{b}{a} = \cdot 9967$ km.

ϕ	$L - L'$	4	8	12	16	20	24
38	u_1	− 7·834	− 7·710	− 7·510	− 7·227	− 6·863	− 6·415
	u	− 7·840	− 7·716	− 7·516	− 7·233	− 6·868	− 6·420
	$\pm v$	+ 2·644	+ 5·280	+ 7·914	+10·532	+13·143	+15·733
	$\pm w$	+ 1·630	+ 3·258	+ 4·882	+ 6·499	+ 8·106	+ 9·703
34	u_1	− 5·577	− 5·468	− 5·281	− 5·019	− 4·683	− 4·270
	u	− 5·584	− 5·475	− 5·288	− 5·025	− 4·689	− 4·275
	$\pm v$	+ 2·498	+ 4·996	+ 7·489	+ 9·967	+12·437	+14·894
	$\pm w$	+ 1·400	+ 2·799	+ 4·195	+ 5·586	+ 6·971	+ 8·348
30	u_1	− 3·325	− 3·222	− 3·052	− 2·812	− 2·502	− 2·123
	u	− 3·331	− 3·227	− 3·057	− 2·817	− 2·506	− 2·127
	$\pm v$	+ 2·384	+ 4·768	+ 7·143	+ 9·516	+11·871	+14·228
	$\pm w$	+ 1·195	+ 2·390	+ 3·581	+ 4·769	+ 5·951	+ 7·128
26	u_1	− 1·071	− 0·977	− 0·820	− 0·603	− 0·322	+ 0·024
	u	− 1·073	− 0·979	− 0·822	− 0·604	− 0·323	+ 0·024
	$\pm v$	+ 2·294	+ 4·586	+ 6·873	+ 9·152	+11·435	+13·718
	$\pm w$	+ 1·009	+ 2·016	+ 3·019	+ 4·024	+ 5·025	+ 6·024
22	u_1	+ 1·185	+ 1·268	+ 1·409	+ 1·606	+ 1·860	+ 2·172
	u	+ 1·188	+ 1·271	+ 1·412	+ 1·610	+ 1·864	+ 2·177
	$\pm v$	+ 2·225	+ 4·447	+ 6·666	+ 8·881	+11·088	+13·291
	$\pm w$	+ 0·836	+ 1·671	+ 2·504	+ 3·338	+ 4·164	+ 4·991
18	u_1	+ 3·439	+ 3·513	+ 3·639	+ 3·814	+ 4·044	+ 4·319
	u	+ 3·448	+ 3·522	+ 3·649	+ 3·824	+ 4·055	+ 4·331
	$\pm v$	+ 2·172	+ 4·343	+ 6·512	+ 8·674	+10·834	+12·981
	$\pm w$	+ 0·673	+ 1·346	+ 2·018	+ 2·688	+ 3·360	+ 4·024
14	u_1	+ 5·695	+ 5·760	+ 5·869	+ 6·024	+ 6·223	+ 6·467
	u	+ 5·712	+ 5·777	+ 5·886	+ 6·042	+ 6·241	+ 6·486
	$\pm v$	+ 2·135	+ 4·271	+ 6·401	+ 8·536	+10·660	+12·777
	$\pm w$	+ 0·519	+ 1·037	+ 1·554	+ 2·070	+ 2·587	+ 3·100
10	u_1	+ 7·952	+ 8·008	+ 8·102	+ 8·232	+ 8·404	+ 8·616
	u	+ 7·977	+ 8·033	+ 8·127	+ 8·258	+ 8·430	+ 8·643
	$\pm v$	+ 2·114	+ 4·229	+ 6·343	+ 8·451	+10·559	+12·662
	$\pm w$	+ 0·369	+ 0·737	+ 1·105	+ 1·472	+ 1·840	+ 2·206

TABLE XXVI.

$$\delta a = 0\,;\ (\delta e^2 = 0\cdot0001)\,;\ \delta b = -\frac{a^2\delta e^2}{2b} = -0\cdot3200 \text{ km.};\ u_0 = 3''\cdot872.$$

ϕ	L − L′	4°	8°	12°	16°	20°	24°
38°	u_1	+ 1·837	+ 1·807	+ 1·757	+ 1·689	+ 1·600	+ 1·493
	u	+ 6·881	+ 6·850	+ 6·800	+ 6·733	+ 6·643	+ 6·536
	$\pm v$	− 0·094	− 0·187	− 0·279	− 0·370	− 0·455	− 0·540
	$\pm w$	− 0·382	− 0·763	− 1·142	− 1·519	− 1·890	− 2·258
34°	u_1	+ 1·364	+ 1·336	+ 1·289	+ 1·223	+ 1·139	+ 1·037
	u	+ 6·185	+ 6·157	+ 6·111	+ 6·045	+ 5·960	+ 5·857
	$\pm v$	− 0·060	− 0·121	− 0·182	− 0·240	− 0·295	− 0·346
	$\pm w$	− 0·342	− 0·684	− 1·024	− 1·362	− 1·696	− 2·027
30°	u_1	+ 0·845	+ 0·818	+ 0·775	+ 0·713	+ 0·633	+ 0·536
	u	+ 5·350	+ 5·324	+ 5·280	+ 5·218	+ 5·138	+ 5·041
	$\pm v$	− 0·033	− 0·067	− 0·100	− 0·131	− 0·160	− 0·186
	$\pm w$	− 0·304	− 0·607	− 0·909	− 1·208	− 1·506	− 1·800
26°	u_1	+ 0·282	+ 0·257	+ 0·216	+ 0·158	+ 0·084	− 0·007
	u	+ 4·381	+ 4·357	+ 4·315	+ 4·258	+ 4·184	+ 4·096
	$\pm v$	− 0·011	− 0·023	− 0·033	− 0·041	− 0·049	− 0·056
	$\pm w$	− 0·265	− 0·530	− 0·794	− 1·057	− 1·317	− 1·579
22°	u_1	− 0·321	− 0·344	− 0·382	− 0·435	− 0·504	− 0·587
	u	+ 3·294	+ 3·271	+ 3·233	+ 3·180	+ 3·111	+ 3·027
	$\pm v$	+ 0·008	+ 0·015	+ 0·023	+ 0·032	+ 0·043	+ 0·055
	$\pm w$	− 0·227	− 0·454	− 0·679	− 0·904	− 1·127	− 1·349
18°	u_1	− 0·960	− 0·980	− 1·015	− 1·063	− 1·126	− 1·201
	u	+ 2·098	+ 2·078	+ 2·043	+ 1·995	+ 1·932	+ 1·856
	$\pm v$	+ 0·024	+ 0·046	+ 0·068	+ 0·092	+ 0·116	+ 0·143
	$\pm w$	− 0·188	− 0·375	− 0·563	− 0·749	− 0·935	− 1·119
14°	u_1	− 1·628	− 1·646	− 1·677	− 1·720	− 1·775	− 1·844
	u	+ 0·812	+ 0·794	+ 0·762	+ 0·719	+ 0·664	+ 0·596
	$\pm v$	+ 0·036	+ 0·069	+ 0·102	+ 0·134	+ 0·169	+ 0·205
	$\pm w$	− 0·148	− 0·296	− 0·444	− 0·591	− 0·738	− 0·884
10°	u_1	− 2·322	− 2·337	− 2·364	− 2·401	− 2·450	− 2·508
	u	− 0·547	− 0·562	− 0·590	− 0·627	− 0·676	− 0·734
	$\pm v$	+ 0·042	+ 0·083	+ 0·122	+ 0·162	+ 0·202	+ 0·244
	$\pm w$	− 0·108	− 0·215	− 0·322	− 0·429	− 0·536	− 0·642

By the help of tables XVII and XVIII of Chapter I and their extensions (*vide* tables XXXV, XXXVI below) the effect of u_0 is eliminated: the figures in these tables give by interpolation values of u for each point in table XXVI, and these multiplied by $-3\cdot872$ are applied to the corresponding figures in XXVI, leaving residuals due to case $\delta b = -\cdot3200$. The residuals are multiplied by $-\dfrac{1}{\cdot32}$ and the result is the case II,— $\delta b = 1, \delta a = 0$. This case is then multiplied by $-\cdot9967$ and applied to table XXV, leaving as a residual case I,— $\delta a = 1, \delta b = 0$. From these tables values corresponding to even degrees of λ are interpolated and the results are exhibited in tables XXVII, XXVIII.

Case I.—$\delta a = 1$ km.

Values of u_g in seconds.

TABLE XXIX.

Lat. \ Long.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°
40°													1·392	1·370	1·351	1·335	1·321					
39													1·514	1·491	1·472	1·456	1·443					
38													1·606	1·582	1·563	1·547	1·535					
37													1·669	1·644	1·625	1·609	1·598					
36									1·846	1·803	1·764	1·731	1·701	1·677	1·658	1·642	1·631	1·624	1·623	1·627	1·638	1·656
35									1·849	1·806	1·768	1·735	1·706	1·682	1·663	1·647	1·636	1·630	1·629	1·633	1·644	1·661
34									1·824	1·782	1·744	1·711	1·683	1·659	1·640	1·624	1·614	1·609	1·608	1·612	1·622	1·638
33									1·775	1·733	1·695	1·662	1·634	1·610	1·591	1·575	1·565	1·561	1·560	1·564	1·574	1·589
32	2·197	2·119	2·046	1·977	1·912	1·852	1·796	1·746	1·700	1·658	1·620	1·587	1·559	1·535	1·516	1·500	1·490	1·485	1·484	1·488	1·498	1·514
31	2·089	2·012	1·939	1·871	1·807	1·747	1·692	1·642	1·596	1·555	1·518	1·485	1·457	1·433	1·414	1·398	1·387	1·382	1·381	1·385	1·395	1·411
30	1·954	1·877	1·805	1·737	1·674	1·616	1·562	1·512	1·467	1·426	1·300	1·357	1·329	1·305	1·286	1·270	1·259	1·253	1·253	1·256	1·207	1·282
29	1·795	1·720	1·647	1·580	1·517	1·459	1·405	1·356	1·312	1·271	1·235	1·203	1·175	1·151	1·132	1·116	1·105	1·100	1·099	1·103	1·112	1·127
28	1·608	1·533	1·463	1·397	1·334	1·277	1·223	1·175	1·132	1·090	1·054	1·022	0·904	0·971	0·953	0·937	0·926	0·921	0·920	0·924	0·933	0·948
27	1·398	1·324	1·255	1·189	1·127	1·070	1·017	0·969	0·924	0·885	0·849	0·816	0·787	0·765	0·748	0·733	0·723	0·718	0·717	0·721	0·730	0·744
26	1·164	1·091	1·032	0·956	0·896	0·841	0·787	0·739	0·695	0·656	0·620	0·589	0·562	0·538	0·520	0·506	0·496	0·491	0·490	0·494	0·502	0·516
25	0·910	0·838	0·769	0·704	0·644	0·587	0·534	0·487	0·444	0·405	0·369	0·339	0·312	0·289	0·270	0·256	0·246	0·241	0·240	0·244	0·253	0·266
24								0·213	0·171	0·132	0·096	0·066	0·039	0·018	0·002	0·016	0·026	0·033	0·034	0·028	0·010	0·007
23								0·084	0·126	0·165	0·200	0·229	0·255	0·278	0·296	0·310	0·321	0·328	0·320	0·324	0·313	0·302
22								0·401	0·443	0·482	0·516	0·546	0·572	0·593	0·611	0·626	0·638	0·645	0·646	0·640	0·630	0·617
21								0·740	0·782	0·821	0·854	0·884	0·909	0·931	0·950	0·964	0·975	0·983	0·984	0·980	0·969	0·956
20													1·267	1·288	1·307	1·322	1·333	1·340	1·341	1·337	1·327	1·312
19													1·643	1·664	1·682	1·697	1·708	1·716	1·717	1·713	1·702	1·688
18													2·037	2·058	2·076	2·091	2·103	2·110	2·111	2·106	2·095	2·083
17													2·449	2·470	2·489	2·505	2·516	2·522	2·523	2·519	2·510	2·495
16													2·877	2·898	2·916	2·931	2·943	2·949	2·950	2·946	2·937	2·923
15													3·321	3·342	3·360	3·374	3·385	3·391	3·392	3·388	3·380	3·367
14													3·770	3·800	3·817	3·831	3·842	3·849	3·850	3·846	3·837	3·825
13													4·252	4·273	4·291	4·305	4·315	4·321	4·322	4·318	4·310	4·299
12													4·737	4·758	4·776	4·791	4·800	4·807	4·808	4·804	4·796	4·783
11													5·235	5·256	5·274	5·288	5·298	5·303	5·304	5·300	5·292	5·281
10													5·742	5·763	5·781	5·795	5·805	5·811	5·812	5·808	5·800	5·788
9													6·260	6·281	6·299	6·313	6·323	6·329	6·330	6·326	6·318	6·306
8													6·787	6·808	6·826	6·840	6·850	6·857	6·858	6·854	6·846	6·833

The sign is + above the horizontal dividing line and − below it.

Lat. \ Long.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°
30°	1·282	1·298	1·320	1·347	1·378	1·414	1·453	1·497	1·545	1·597	1·654	1·716	1·782	1·853	1·928	2·008	2·092	2·180	2·273	2·360	2·471	2·577
29	1·127	1·144	1·166	1·193	1·224	1·259	1·296	1·341	1·389	1·441	1·498	1·559	1·625	1·695	1·770	1·847	1·930	2·017	2·109	2·205	2·306	2·410
28	0·948	0·964	0·986	1·012	1·043	1·078	1·117	1·159	1·207	1·258	1·315	1·375	1·440	1·509	1·582	1·660	1·742	1·828	1·919	2·015	2·115	2·218
27	0·744	0·759	0·780	0·805	0·837	0·872	0·911	0·953	1·000	1·051	1·107	1·167	1·232	1·301	1·373	1·450	1·531	1·616	1·706	1·801	1·900	2·003
26	0·516	0·532	0·553	0·579	0·609	0·643	0·682	0·724	0·770	0·821	0·878	0·936	0·999	1·067	1·139	1·216	1·296	1·380	1·470	1·564	1·663	1·765
25	0·266	0·282	0·304	0·329	0·359	0·392	0·430	0·472	0·518	0·568	0·623	0·684	0·747	0·814	0·886	0·961	1·040	1·123	1·213	1·306	1·403	1·515
24	0·007	0·010	0·033	0·056	0·086	0·119	0·157	0·199	0·244	0·294	0·349	0·411	0·477	0·546	0·618	0·696	0·775	0·856	0·943	1·036	1·133	1·233
23	0·302	0·284	0·263	0·238	0·210	0·177	0·140	0·098	0·053	0·002	0·053	0·114	0·182	0·251	0·323	0·400	0·480	0·561	0·648	0·740	0·836	0·936
22	0·617	0·600	0·579	0·555	0·527	0·494	0·457	0·415	0·370	0·321	0·267	0·204	0·136	0·066	0·008	0·085	0·163	0·244	0·331	0·423	0·519	0·619
21	0·956	0·937	0·916	0·892	0·865	0·832	0·796	0·754			0·606	0·542	0·474	0·404	0·330	0·253	0·174	0·093	0·007	0·084	0·180	0·280
20	1·312	1·295	1·274	1·250	1·222	1·189	1·153	1·113			0·965	0·900	0·832	0·761	0·687	0·610	0·532	0·451	0·364	0·273	0·177	0·079
19	1·688	1·671	1·650	1·626	1·599								1·208	1·137	1·063	0·986	0·908	0·827	0·741	0·650	0·554	0·456
18	2·083	2·065	2·044	2·020	1·993								1·603	1·532	1·458	1·380	1·302	1·221	1·135	1·044	0·948	0·850
17	2·495	2·477	2·456	2·432	2·405								2·015	1·944	1·870	1·792	1·714	1·633	1·547	1·457	1·362	1·266
16	2·923	2·905	2·884	2·860	2·833								2·444	2·373	2·298	2·220	2·142	2·061	1·975	1·885	1·791	1·693
15																	2·586	2·505	2·419	2·329	2·236	2·138
14																	3·044	2·963	2·877	2·787	2·694	2·597
13																	3·517	3·437	3·351	3·262	3·169	3·072
12																	4·002	3·922	3·837	3·747	3·654	3·558
11																	4·500	4·420	4·334	4·244	4·151	4·055
10																	5·007	4·926	4·841	4·752	4·659	4·564
9																	5·523	5·442	5·358	5·270	5·177	5·083
8																	6·048	5·967	5·884	5·797	5·704	5·611

The sign is + above the horizontal dividing line and − below it.

Case I.—δa = 1 km.

Values of v_g in seconds.

TABLE XXX.

Long.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	Long.
Lat.	Positive																		Negative				Lat.
40°													4·348	3·581	2·815	2·048	1·280						40°
39													4·298	3·530	2·781	2·022	1·262						39
38													4·249	3·498	2·748	1·997	1·245						38
37													4·201	3·458	2·716	1·973	1·230						37
36									7·083	6·352	5·620	4·887	4·154	3·420	2·685	1·951	1·216	0·481	0·254	0·980	1·723	2·458	36
35									7·007	6·284	5·560	4·834	4·100	3·383	2·656	1·930	1·203	0·476	0·251	0·978	1·704	2·431	35
34									6·933	6·218	5·502	4·785	4·067	3·348	2·629	1·911	1·192	0·471	0·248	0·968	1·687	2·406	34
33									6·863	6·155	5·446	4·736	4·025	3·314	2·602	1·891	1·179	0·466	0·246	0·958	1·670	2·382	33
32	12·384	11·688	10·992	10·293	9·595	8·895	8·195	7·494	6·794	6·093	5·391	4·688	3·985	3·281	2·578	1·873	1·167	0·462	0·243	0·949	1·654	2·359	32
31	12·267	11·577	10·887	10·194	9·502	8·809	8·116	7·422	6·728	6·034	5·339	4·643	3·948	3·251	2·554	1·855	1·157	0·458	0·241	0·940	1·630	2·337	31
30	12·154	11·471	10·787	10·100	9·414	8·727	8·040	7·353	6·665	5·977	5·288	4·600	3·911	3·221	2·530	1·838	1·146	0·453	0·239	0·931	1·624	2·316	30
29	12·044	11·366	10·689	10·007	9·327	8·646	7·965	7·284	6·603	5·921	5·239	4·557	3·874	3·100	2·506	1·821	1·135	0·449	0·237	0·923	1·608	2·294	29
28	11·938	11·267	10·594	9·919	9·244	8·568	7·893	7·218	6·544	5·868	5·192	4·515	3·838	3·160	2·483	1·804	1·124	0·445	0·234	0·914	1·593	2·272	28
27	11·837	11·170	10·503	9·833	9·164	8·494	7·824	7·156	6·487	5·817	5·147	4·476	3·805	3·133	2·461	1·788	1·114	0·441	0·232	0·906	1·579	2·252	27
26	11·740	11·078	10·416	9·751	9·088	8·424	7·759	7·096	6·433	5·769	5·104	4·439	3·773	3·106	2·440	1·773	1·105	0·438	0·230	0·898	1·566	2·233	26
25	11·646	10·990	10·332	9·674	9·016	8·357	7·698	7·030	6·381	5·722	5·062	4·402	3·742	3·081	2·420	1·758	1·096	0·434	0·228	0·891	1·553	2·215	25
24								6·985	6·331	5·677	5·022	4·367	3·713	3·058	2·402	1·745	1·088	0·430	0·227	0·884	1·542	2·190	24
23								6·933	6·284	5·634	4·984	4·335	3·686	3·036	2·385	1·733	1·081	0·427	0·225	0·878	1·531	2·183	23
22								6·883	6·238	5·593	4·948	4·303	3·659	3·014	2·368	1·721	1·072	0·424	0·224	0·872	1·520	2·168	22
21								6·834	6·194	5·554	4·913	4·274	3·634	2·904	2·352	1·709	1·066	0·422	0·222	0·866	1·510	2·153	21
20													3·610	2·974	2·337	1·698	1·058	0·419	0·220	0·860	1·409	2·138	20
19													3·586	2·954	2·321	1·686	1·052	0·416	0·219	0·854	1·490	2·124	19
18													3·562	2·934	2·305	1·675	1·044	0·413	0·218	0·849	1·470	2·110	18
17													3·539	2·915	2·290	1·664	1·038	0·411	0·216	0·843	1·470	2·096	17
16													3·516	2·805	2·274	1·652	1·030	0·408	0·215	0·837	1·460	2·082	16
15													3·493	2·875	2·258	1·641	1·023	0·405	0·213	0·831	1·440	2·067	15
14													3·470	2·856	2·243	1·630	1·016	0·402	0·212	0·826	1·439	2·052	14
13													3·448	2·838	2·228	1·619	1·010	0·400	0·210	0·820	1·430	2·039	13
12													3·426	2·820	2·214	1·608	1·003	0·397	0·209	0·815	1·421	2·026	12
11													3·404	2·802	2·200	1·598	0·996	0·394	0·208	0·810	1·411	2·013	11
10													3·382	2·784	2·186	1·588	0·990	0·392	0·206	0·804	1·402	2·000	10
9													3·360	2·766	2·172	1·578	0·984	0·390	0·205	0·799	1·393	1·987	9
8													3·338	2·748	2·158	1·568	0·978	0·388	0·203	0·794	1·384	1·974	8

Long.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°	Long.
Lat.	Negative																						Lat.
30°	2·316	3·007	3·697	4·387	5·076	5·764	6·452	7·140	7·827	8·514	9·201	9·888	10·574	11·260	11·943	12·625	13·306	13·988	14·660	15·340	16·029	16·700	30°
29	2·294	2·978	3·662	4·345	5·028	5·710	6·392	7·073	7·754	8·435	9·116	9·797	10·477	11·157	11·834	12·511	13·187	13·863	14·538	15·214	15·888	16·562	29
28	2·272	2·950	3·628	4·306	4·983	5·659	6·334	7·010	7·684	8·359	9·034	9·710	10·385	11·050	11·731	12·403	13·073	13·744	14·413	15·083	15·752	16·421	28
27	2·252	2·925	3·597	4·268	4·939	5·610	6·279	6·949	7·617	8·286	8·956	9·626	10·295	10·964	11·630	12·297	12·963	13·628	14·292	14·957	15·621	16·285	27
26	2·233	2·900	3·566	4·232	4·898	5·563	6·227	6·891	7·554	8·217	8·881	9·546	10·210	10·873	11·534	12·196	12·857	13·518	14·177	14·836	15·495	16·153	26
25	2·215	2·876	3·537	4·198	4·858	5·517	6·176	6·835	7·494	8·152	8·811	9·470	10·128	10·786	11·442	12·098	12·753	13·409	14·064	14·719	15·373	16·026	25
24	2·199	2·855	3·510	4·165	4·819	5·474	6·129	6·783	7·437	8·091	8·745	9·400	10·054	10·705	11·357	12·008	12·659	13·308	13·955	14·603	15·251	15·900	24
23	2·183	2·834	3·484	4·134	4·783	5·432	6·082	6·732	7·381	8·031	8·679	9·327	9·977	10·625	11·272	11·919	12·563	13·207	13·850	14·494	15·137	15·780	23
22	2·168	2·814	3·459	4·104	4·748	5·393	6·038	6·684	7·329	7·973	8·617	9·256	9·900	10·545	11·190	11·831	12·472	13·110	13·747	14·385	15·024	15·663	22
21	2·153	2·795	3·435	4·075	4·715	5·355	5·996	6·637			8·554	9·186	9·827	10·468	11·107	11·744	12·380	13·016	13·649	14·282	14·916	15·550	21
20	2·138	2·776	3·412	4·048	4·683	5·319	5·955	6·591			8·495	9·117	9·754	10·390	11·024	11·658	12·291	12·923	13·551	14·181	14·812	15·443	20
19	2·124	2·758	3·390	4·021	4·653								9·681	10·315	10·946	11·574	12·204	12·829	13·454	14·082	14·708	15·336	19
18	2·110	2·740	3·368	3·995	4·623								9·609	10·237	10·866	11·493	12·116	12·738	13·361	13·983	14·602	15·227	18
17	2·096	2·721	3·345	3·969	4·593								9·540	10·165	10·788	11·408	12·029	12·647	13·268	13·886	14·502	15·122	17
16	2·082	2·703	3·323	3·944	4·563								9·471	10·091	10·708	11·327	11·945	12·559	13·176	13·791	14·404	15·018	16
15																	11·860	12·471	13·085	13·698	14·306	14·917	15
14																	11·775	12·384	12·993	13·604	14·200	14·815	14
13																	11·691	12·298	12·904	13·509	14·111	14·714	13
12																	11·610	12·213	12·816	13·417	14·016	14·616	12
11																	11·530	12·130	12·729	13·329	13·923	14·518	11
10																	11·450	12·047	12·643	13·238	13·831	14·421	10
9																	11·370	11·964	12·558	13·149	13·739	14·325	9
8																	11·290	11·882	12·474	13·061	13·648	14·230	8

Case I.—$\delta a = 1$ km.

Values of w_g in seconds.

TABLE XXXI.

Long.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	Long.
Lat.	Positive																		Negative				Lat.
40°													2·251	1·854	1·457	1·060	0·663						40°
39													2·187	1·801	1·415	1·029	0·643						39
38													2·125	1·750	1·374	0·999	0·623						38
37													2·065	1·700	1·335	0·970	0·605						37
36									3·422	3·060	2·716	2·362	2·008	1·654	1·299	0·944	0·589	0·233	0·123	0·478	0·833	1·188	36
35									3·329	2·986	2·642	2·298	1·953	1·608	1·263	0·918	0·578	0·227	0·119	0·465	0·811	1·156	35
34									3·239	2·906	2·572	2·237	1·902	1·566	1·230	0·894	0·558	0·221	0·116	0·453	0·790	1·126	34
33									3·157	2·833	2·507	2·180	1·853	1·526	1·198	0·870	0·543	0·215	0·113	0·441	0·760	1·097	33
32	5·587	5·277	4·965	4·653	4·338	4·023	3·708	3·393	3·078	2·762	2·445	2·126	1·807	1·488	1·169	0·849	0·529	0·210	0·110	0·430	0·750	1·069	32
31	5·453	5·151	4·847	4·541	4·233	3·926	3·618	3·311	3·004	2·696	2·387	2·075	1·763	1·452	1·140	0·828	0·516	0·204	0·108	0·420	0·731	1·043	31
30	5·325	5·080	4·733	4·435	4·134	3·834	3·533	3·233	2·933	2·633	2·331	2·027	1·723	1·418	1·113	0·809	0·504	0·200	0·105	0·410	0·715	1·019	30
29	5·203	4·914	4·624	4·332	4·039	3·746	3·452	3·159	2·866	2·573	2·278	1·981	1·684	1·386	1·088	0·790	0·493	0·195	0·103	0·401	0·699	0·996	29
28	5·087	4·804	4·520	4·234	3·948	3·662	3·375	3·080	2·802	2·515	2·227	1·937	1·647	1·356	1·065	0·774	0·482	0·191	0·100	0·392	0·683	0·974	28
27	4·976	4·699	4·421	4·142	3·863	3·584	3·303	3·023	2·742	2·461	2·179	1·896	1·612	1·328	1·043	0·758	0·473	0·187	0·099	0·384	0·669	0·954	27
26	4·871	4·600	4·328	4·056	3·783	3·510	3·235	2·960	2·685	2·410	2·134	1·856	1·579	1·301	1·023	0·743	0·463	0·183	0·097	0·377	0·656	0·935	26
25	4·773	4·507	4·241	3·975	3·708	3·440	3·171	2·902	2·632	2·362	2·091	1·819	1·547	1·275	1·003	0·729	0·454	0·180	0·095	0·370	0·644	0·917	25
24	4·681	4·421	4·161	3·900	3·637	3·374	3·111	2·847	2·582	2·316	2·050	1·784	1·518	1·251	0·984	0·715	0·446	0·177	0·093	0·363	0·632	0·900	24
23								2·794	2·534	2·273	2·013	1·752	1·490	1·228	0·965	0·702	0·438	0·174	0·091	0·356	0·620	0·884	23
22								2·746	2·490	2·234	1·978	1·721	1·464	1·206	0·948	0·689	0·430	0·170	0·090	0·350	0·609	0·868	22
21								2·699	2·448	2·106	1·944	1·691	1·438	1·185	0·931	0·677	0·423	0·167	0·088	0·343	0·598	0·853	21
20													1·414	1·165	0·916	0·666	0·415	0·164	0·087	0·338	0·588	0·838	20
19													1·393	1·147	0·901	0·655	0·409	0·162	0·085	0·332	0·579	0·825	19
18													1·373	1·131	0·888	0·645	0·403	0·160	0·084	0·328	0·571	0·813	18
17													1·355	1·116	0·876	0·636	0·397	0·157	0·083	0·323	0·563	0·802	17
16													1·338	1·102	0·865	0·628	0·392	0·155	0·082	0·319	0·556	0·792	16
15													1·322	1·089	0·856	0·622	0·388	0·153	0·081	0·315	0·550	0·784	15
14													1·307	1·077	0·847	0·616	0·384	0·152	0·080	0·312	0·544	0·775	14
13													1·293	1·066	0·838	0·609	0·380	0·150	0·079	0·308	0·538	0·767	13
12													1·279	1·054	0·829	0·603	0·376	0·149	0·078	0·305	0·532	0·759	12
11													1·267	1·044	0·821	0·597	0·372	0·147	0·078	0·303	0·527	0·751	11
10													1·256	1·035	0·813	0·591	0·369	0·146	0·077	0·300	0·523	0·745	10
9													1·246	1·026	0·806	0·586	0·366	0·145	0·076	0·297	0·519	0·740	9
8													1·238	1·019	0·800	0·581	0·362	0·143	0·076	0·296	0·516	0·736	8

Long.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°	Long.
Lat.	Negative																						Lat.
30°	1·019	1·323	1·628	1·933	2·237	2·540	2·841	3·141	3·441	3·741	4·042	4·342	4·641	4·938	5·234	5·528	5·822	6·113	6·404	6·692	6·980	7·267	30°
29	0·996	1·294	1·592	1·889	2·185	2·481	2·775	3·068	3·361	3·654	3·948	4·241	4·533	4·824	5·113	5·401	5·687	5·972	6·255	6·538	6·820	7·100	29
28	0·974	1·265	1·556	1·847	2·137	2·426	2·713	3·000	3·286	3·573	3·860	4·146	4·431	4·716	4·999	5·281	5·561	5·840	6·117	6·394	6·669	6·943	28
27	0·954	1·239	1·524	1·808	2·092	2·374	2·655	2·936	3·216	3·497	3·776	4·055	4·334	4·613	4·890	5·166	5·441	5·714	5·986	6·257	6·526	6·794	27
26	0·935	1·215	1·493	1·771	2·048	2·324	2·600	2·875	3·150	3·425	3·698	3·971	4·244	4·515	4·786	5·057	5·327	5·595	5·862	6·127	6·390	6·653	26
25	0·917	1·191	1·463	1·735	2·006	2·277	2·548	2·818	3·088	3·356	3·624	3·892	4·159	4·425	4·691	4·956	5·220	5·482	5·742	6·001	6·260	6·518	25
24	0·900	1·168	1·435	1·702	1·968	2·234	2·500	2·765	3·020	3·293	3·556	3·821	4·084	4·342	4·598	4·854	5·109	5·361	5·612	5·863	6·112	6·359	24
23	0·884	1·147	1·409	1·671	1·932	2·193	2·454	2·714	2·974	3·233	3·402	3·794	4·047	4·303	4·556	4·807	5·057	5·303	5·547	5·791	6·034	6·277	23
22	0·868	1·126	1·384	1·641	1·898	2·154	2·410	2·667	2·922	3·177	3·431	3·763	4·015	4·266	4·514	4·760	5·005	5·246	5·486	5·726	5·964	6·202	22
21	0·853	1·106	1·359	1·612	1·865	2·118	2·370	2·622			3·373	3·739	3·985	4·230	4·472	4·715	4·956	5·193	5·430	5·665	5·900	6·135	21
20	0·838	1·088	1·337	1·587	1·836	2·085	2·333	2·580			3·318	3·712	3·954	4·197	4·436	4·674	4·909	5·142	5·376	5·610	5·840	6·071	20
19	0·825	1·071	1·317	1·563	1·809								3·926	4·166	4·400	4·635	4·865	5·096	5·324	5·555	5·781	6·009	19
18	0·813	1·056	1·299	1·541	1·783								3·902	4·135	4·366	4·596	4·825	5·055	5·280	5·508	5·730	5·950	18
17	0·802	1·042	1·281	1·520	1·758								3·876	4·108	4·335	4·562	4·789	5·016	5·238	5·463	5·680	5·896	17
16	0·792	1·029	1·265	1·500	1·735								3·857	4·084	4·308	4·534	4·756	4·978	5·198	5·418	5·631	5·847	16
15																	4·725	4·942	5·158	5·375	5·587	5·803	15
14																	4·696	4·910	5·124	5·338	5·548	5·762	14
13																	4·668	4·880	5·092	5·303	5·511	5·718	13
12																	4·643	4·853	5·062	5·271	5·477	5·683	12
11																	4·618	4·826	5·033	5·239	5·444	5·645	11
10																	4·595	4·802	5·008	5·212	5·415	5·614	10
9																	4·574	4·780	4·985	5·188	5·389	5·586	9
8																	4·556	4·761	4·965	5·167	5·368	5·564	8

Case II.—δb = 1 km. Values of u_g in seconds. *TABLE XXXII.*

Lat. \ Long.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	Lat.
40°													10·294	10·295	10·297	10·298	10·297						40°
39													9·855	9·856	9·858	9·859	9·858						39
38													9·384	9·385	9·386	9·386	9·385						38
37													8·882	8·883	8·883	8·883	8·882						37
36									8·346	8·347	8·348	8·349	8·350	8·350	8·351	8·351	8·351	8·350	8·350	8·350	8·351	8·351	36
35									7·789	7·790	7·790	7·790	7·791	7·791	7·791	7·791	7·792	7·792	7·792	7·792	7·792	7·791	35
34									7·204	7·204	7·204	7·204	7·203	7·203	7·203	7·204	7·204	7·205	7·205	7·205	7·204	7·204	34
33									6·593	6·593	6·592	6·592	6·591	6·591	6·590	6·590	6·590	6·590	6·590	6·590	6·590	6·590	33
32	5·963	5·962	5·962	5·961	5·960	5·959	5·958	5·957	5·956	5·955	5·954	5·953	5·952	5·951	5·950	5·950	5·950	5·949	5·949	5·949	5·950	5·950	32
31	5·305	5·303	5·301	5·299	5·297	5·296	5·294	5·292	5·291	5·290	5·288	5·287	5·286	5·285	5·284	5·283	5·282	5·281	5·281	5·281	5·282	5·283	31
30	4·619	4·616	4·614	4·611	4·608	4·606	4·604	4·602	4·600	4·598	4·596	4·594	4·593	4·592	4·591	4·590	4·589	4·588	4·587	4·588	4·589	4·590	30
29	3·908	3·904	3·901	3·897	3·893	3·890	3·887	3·884	3·882	3·880	3·878	3·876	3·874	3·872	3·871	3·870	3·869	3·868	3·867	3·868	3·869	3·870	29
28	3·170	3·166	3·161	3·156	3·152	3·148	3·144	3·141	3·138	3·135	3·132	3·130	3·128	3·126	3·125	3·124	3·123	3·122	3·121	3·122	3·123	3·124	28
27	2·409	2·403	2·398	2·392	2·386	2·381	2·377	2·373	2·369	2·366	2·363	2·361	2·358	2·356	2·355	2·354	2·353	2·352	2·351	2·352	2·353	2·354	27
26	1·623	1·617	1·611	1·604	1·597	1·591	1·585	1·581	1·577	1·574	1·571	1·567	1·564	1·562	1·560	1·559	1·558	1·557	1·556	1·557	1·558	1·559	26
25	0·815	0·808	0·800	0·793	0·786	0·779	0·772	0·767	0·763	0·759	0·755	0·751	0·748	0·745	0·742	0·741	0·740	0·739	0·738	0·739	0·740	0·742	25
24								0·069	0·074	0·079	0·083	0·087	0·091	0·094	0·097	0·099	0·101	0·102	0·103	0·102	0·100	0·098	24
23								0·926	0·932	0·938	0·943	0·947	0·951	0·954	0·957	0·960	0·962	0·964	0·965	0·963	0·961	0·958	23
22								1·804	1·811	1·817	1·823	1·828	1·832	1·836	1·839	1·842	1·845	1·847	1·848	1·846	1·843	1·840	22
21								2·705	2·712	2·719	2·725	2·731	2·735	2·739	2·743	2·747	2·751	2·752	2·753	2·751	2·748	2·745	21
20													3·658	3·662	3·666	3·670	3·674	3·677	3·678	3·675	3·672	3·668	20
19													4·601	4·605	4·609	4·613	4·617	4·620	4·622	4·619	4·615	4·611	19
18													5·560	5·565	5·570	5·574	5·578	5·581	5·583	5·580	5·576	5·571	18
17													6·539	6·545	6·550	6·554	6·558	6·561	6·563	6·560	6·556	6·551	17
16													7·534	7·540	7·546	7·551	7·555	7·558	7·559	7·556	7·552	7·547	16
15													8·545	8·552	8·558	8·563	8·567	8·570	8·571	8·568	8·564	8·559	15
14													9·570	9·577	9·583	9·588	9·592	9·595	9·596	9·593	9·589	9·584	14
13													10·609	10·618	10·625	10·630	10·634	10·637	10·638	10·635	10·631	10·626	13
12													11·662	11·671	11·678	11·684	11·688	11·691	11·692	11·689	11·685	11·680	12
11													12·727	12·736	12·743	12·749	12·753	12·757	12·758	12·755	12·751	12·746	11
10													13·802	13·811	13·818	13·824	13·828	13·832	13·833	13·830	13·825	13·820	10
9													14·886	14·895	14·902	14·908	14·912	14·916	14·917	14·914	14·909	14·904	9
8													15·978	15·987	15·994	16·000	16·004	16·008	16·009	16·006	16·001	15·996	8

The sign is − above the horizontal dividing line and + below it.

Lat. \ Long.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°	Lat.
30°	4·590	4·591	4·592	4·594	4·596	4·598	4·600	4·602	4·604	4·606	4·608	4·610	4·613	4·616	4·618	4·621	4·624	4·627	4·630	4·633	4·636	4·639	30°
29	3·870	3·872	3·874	3·876	3·878	3·880	3·882	3·884	3·886	3·889	3·892	3·895	3·899	3·903	3·906	3·910	3·914	3·918	3·922	3·926	3·931	3·936	29
28	3·124	3·126	3·128	3·130	3·132	3·135	3·137	3·140	3·143	3·146	3·150	3·155	3·160	3·164	3·169	3·174	3·179	3·184	3·189	3·194	3·200	3·206	28
27	2·354	2·356	2·358	2·360	2·363	2·366	2·368	2·371	2·375	2·380	2·385	2·390	2·396	2·402	2·407	2·413	2·419	2·425	2·432	2·439	2·446	2·453	27
26	1·559	1·561	1·563	1·566	1·569	1·573	1·576	1·579	1·584	1·589	1·595	1·601	1·608	1·615	1·621	1·627	1·634	1·642	1·650	1·658	1·667	1·676	26
25	0·742	0·744	0·747	0·750	0·753	0·757	0·761	0·766	0·771	0·777	0·783	0·790	0·798	0·805	0·813	0·821	0·829	0·837	0·846	0·855	0·866	0·877	25
24	0·098	0·095	0·092	0·089	0·085	0·080	0·075	0·070	0·064	0·057	0·050	0·044	0·035	0·024	0·014	0·003	0·008	0·019	0·031	0·044	0·057	0·070	24
23	0·958	0·955	0·952	0·948	0·944	0·939	0·934	0·928	0·921	0·914	0·906	0·906	0·896	0·885	0·874	0·863	0·852	0·841	0·828	0·815	0·802	0·788	23
22	1·840	1·837	1·834	1·830	1·825	1·819	1·813	1·807	1·800	1·791	1·782	1·789	1·778	1·767	1·756	1·745	1·734	1·722	1·709	1·695	1·681	1·667	22
21	2·745	2·741	2·737	2·732	2·727	2·721	2·715	2·708			2·681	2·694	2·684	2·673	2·661	2·649	2·637	2·624	2·611	2·597	2·583	2·568	21
20	3·668	3·664	3·659	3·654	3·648	3·642	3·635	3·628			3·599	3·619	3·608	3·596	3·584	3·572	3·559	3·546	3·532	3·518	3·503	3·488	20
19	4·611	4·607	4·602	4·596	4·590								4·551	4·539	4·527	4·515	4·502	4·489	4·474	4·459	4·443	4·427	19
18	5·571	5·566	5·561	5·555	5·548								5·512	5·500	5·487	5·474	5·461	5·447	5·432	5·416	5·400	5·384	18
17	6·551	6·546	6·541	6·534	6·526								6·492	6·480	6·467	6·454	6·441	6·426	6·410	6·394	6·378	6·361	17
16	7·547	7·542	7·536	7·529	7·521								7·489	7·476	7·463	7·450	7·436	7·421	7·405	7·388	7·371	7·353	16
15																	8·448	8·432	8·415	8·398	8·380	8·361	15
14																	9·472	9·456	9·439	9·422	9·403	9·384	14
13																	10·513	10·497	10·479	10·460	10·441	10·421	13
12																	11·565	11·548	11·530	11·511	11·492	11·471	12
11																	12·630	12·613	12·594	12·574	12·554	12·532	11
10																	13·706	13·687	13·667	13·647	13·626	13·604	10
9																	14·793	14·774	14·753	14·731	14·709	14·686	9
8																	15·890	15·870	15·849	15·827	15·804	15·781	8

The sign is − above the horizontal dividing line and + below it.

Case II.—$\delta b = 1$ km.

Values of v_g in seconds.

TABLE XXXIII.

Long.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	Long.
Lat.	Negative																		Positive				Lat.
40°													0·497	0·410	0·323	0·233	0·143						40°
39													0·509	0·419	0·330	0·239	0·148						39
38													0·519	0·427	0·336	0·244	0·152						38
37													0·527	0·434	0·341	0·248	0·155						37
36									0·903	0·811	0·719	0·626	0·533	0·440	0·346	0·252	0·157	0·062	0·033	0·128	0·222	0·316	36
35									0·910	0·818	0·725	0·631	0·537	0·443	0·348	0·253	0·158	0·063	0·033	0·129	0·224	0·319	35
34									0·915	0·822	0·729	0·635	0·541	0·446	0·351	0·255	0·159	0·063	0·033	0·129	0·225	0·321	34
33									0·919	0·826	0·732	0·638	0·543	0·448	0·353	0·257	0·161	0·064	0·033	0·130	0·227	0·323	33
32	1·675	1·582	1·488	1·394	1·299	1·204	1·110	1·016	0·922	0·828	0·734	0·640	0·545	0·450	0·355	0·259	0·162	0·064	0·034	0·131	0·228	0·325	32
31	1·678	1·584	1·490	1·396	1·301	1·206	1·112	1·018	0·923	0·828	0·734	0·640	0·545	0·450	0·355	0·259	0·162	0·064	0·034	0·131	0·228	0·325	31
30	1·678	1·584	1·490	1·398	1·301	1·206	1·112	1·018	0·928	0·828	0·734	0·640	0·545	0·450	0·355	0·250	0·162	0·064	0·034	0·131	0·228	0·325	30
29	1·676	1·583	1·489	1·394	1·299	1·204	1·110	1·016	0·922	0·827	0·733	0·639	0·544	0·449	0·354	0·258	0·161	0·064	0·033	0·130	0·227	0·324	29
28	1·671	1·578	1·484	1·380	1·294	1·199	1·105	1·011	0·918	0·824	0·730	0·636	0·542	0·447	0·352	0·256	0·160	0·064	0·033	0·130	0·226	0·322	28
27	1·663	1·571	1·478	1·383	1·288	1·193	1·099	1·005	0·912	0·819	0·726	0·632	0·538	0·444	0·349	0·254	0·159	0·063	0·033	0·129	0·225	0·320	27
26	1·655	1·563	1·470	1·376	1·281	1·186	1·092	0·999	0·906	0·814	0·721	0·628	0·534	0·441	0·347	0·252	0·157	0·062	0·033	0·128	0·223	0·318	26
25	1·644	1·554	1·462	1·368	1·274	1·179	1·085	0·993	0·900	0·808	0·716	0·623	0·530	0·438	0·345	0·251	0·157	0·062	0·033	0·128	0·222	0·315	25
24								0·986	0·894	0·802	0·710	0·618	0·526	0·434	0·342	0·249	0·155	0·062	0·032	0·126	0·220	0·313	24
23								0·978	0·886	0·795	0·703	0·613	0·522	0·431	0·340	0·247	0·154	0·061	0·032	0·125	0·218	0·311	23
22								0·969	0·878	0·787	0·696	0·607	0·517	0·428	0·337	0·245	0·153	0·061	0·032	0·125	0·217	0·309	22
21								0·959	0·869	0·779	0·690	0·601	0·513	0·424	0·334	0·243	0·152	0·060	0·032	0·124	0·215	0·306	21
20													0·508	0·420	0·331	0·241	0·150	0·060	0·031	0·122	0·213	0·303	20
19													0·502	0·415	0·327	0·238	0·148	0·050	0·031	0·121	0·210	0·299	19
18													0·495	0·409	0·323	0·235	0·147	0·058	0·031	0·119	0·207	0·204	18
17													0·486	0·402	0·317	0·230	0·144	0·057	0·030	0·117	0·203	0·289	17
16													0·476	0·393	0·310	0·226	0·141	0·056	0·029	0·114	0·199	0·283	16
15													0·464	0·383	0·302	0·220	0·137	0·054	0·029	0·112	0·194	0·276	15
14													0·452	0·373	0·293	0·213	0·133	0·053	0·028	0·109	0·189	0·269	14
13													0·439	0·362	0·285	0·207	0·129	0·051	0·027	0·105	0·183	0·261	13
12													0·426	0·351	0·276	0·201	0·125	0·050	0·026	0·102	0·177	0·253	12
11													0·411	0·339	0·266	0·194	0·121	0·048	0·025	0·098	0·171	0·244	11
10													0·395	0·325	0·255	0·186	0·116	0·046	0·024	0·094	0·164	0·234	10
9													0·378	0·310	0·248	0·177	0·111	0·044	0·023	0·089	0·157	0·223	9
8													0·358	0·294	0·230	0·167	0·105	0·042	0·022	0·086	0·150	0·212	8

Long.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°	Long.
Lat.	Positive																						Lat.
30°	0·325	0·421	0·516	0·610	0·704	0·798	0·893	0·988	1·082	1·176	1·271	1·366	1·460	1·554	1·648	1·742	1·836	1·929	2·023	2·114	2·206	2·208	30
29	0·324	0·420	0·515	0·609	0·703	0·797	0·892	0·986	1·080	1·174	1·269	1·364	1·459	1·553	1·647	1·740	1·833	1·926	2·019	2·112	2·204	2·296	29
28	0·323	0·417	0·512	0·606	0·700	0·794	0·888	0·982	1·076	1·170	1·265	1·360	1·455	1·549	1·642	1·735	1·828	1·921	2·014	2·106	2·199	2·292	28
27	0·320	0·414	0·508	0·602	0·696	0·790	0·883	0·976	1·070	1·163	1·259	1·354	1·449	1·543	1·635	1·727	1·819	1·911	2·004	2·098	2·191	2·284	27
26	0·318	0·411	0·505	0·598	0·692	0·785	0·878	0·970	1·063	1·156	1·251	1·346	1·441	1·535	1·626	1·717	1·808	1·900	1·993	2·087	2·180	2·273	26
25	0·315	0·409	0·502	0·594	0·687	0·779	0·872	0·964	1·057	1·150	1·244	1·338	1·432	1·525	1·616	1·706	1·796	1·888	1·981	2·074	2·167	2·260	25
24	0·313	0·406	0·498	0·589	0·681	0·773	0·866	0·958	1·050	1·142	1·235	1·328	1·421	1·514	1·606	1·698	1·791	1·882	1·972	2·063	2·155	2·246	24
23	0·311	0·403	0·494	0·584	0·675	0·766	0·858	0·950	1·042	1·134	1·226	1·318	1·411	1·503	1·596	1·687	1·777	1·867	1·957	2·047	2·138	2·229	23
22	0·309	0·400	0·490	0·579	0·669	0·759	0·850	0·941	1·032	1·123	1·214	1·308	1·401	1·492	1·584	1·674	1·763	1·852	1·941	2·029	2·119	2·210	22
21	0·306	0·396	0·485	0·574	0·663	0·751	0·841	0·931			1·201	1·299	1·390	1·480	1·572	1·661	1·749	1·838	1·926	2·013	2·102	2·191	21
20	0·303	0·392	0·480	0·568	0·656	0·743	0·831	0·919			1·185	1·280	1·379	1·469	1·559	1·648	1·735	1·823	1·910	1·996	2·084	2·171	20
19	0·299	0·388	0·475	0·562	0·649								1·369	1·458	1·546	1·634	1·721	1·807	1·893	1·979	2·065	2·151	19
18	0·294	0·383	0·468	0·554	0·639								1·358	1·446	1·533	1·620	1·705	1·790	1·875	1·959	2·043	2·127	18
17	0·289	0·376	0·460	0·544	0·628								1·347	1·433	1·519	1·605	1·689	1·772	1·855	1·938	2·021	2·103	17
16	0·283	0·368	0·450	0·532	0·615								1·335	1·420	1·505	1·588	1·670	1·751	1·833	1·915	1·996	2·077	16
15																	1·650	1·730	1·811	1·892	1·971	2·050	15
14																	1·630	1·708	1·787	1·865	1·943	2·019	14
13																	1·610	1·686	1·763	1·839	1·914	1·988	13
12																	1·589	1·663	1·737	1·811	1·883	1·954	12
11																	1·566	1·638	1·710	1·781	1·850	1·918	11
10																	1·542	1·612	1·682	1·750	1·815	1·880	10
9																	1·517	1·585	1·652	1·717	1·780	1·841	9
8																	1·491	1·557	1·621	1·684	1·744	1·802	8

Case II.—$\delta b = 1$ km. Values of w_g in seconds. *TABLE XXXIV.*

Long.	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	Long.
Lat.	Positive																		Negative				Lat.
40°													0·216	0·177	0·139	0·102	0·066						40°
39													0·194	0·159	0·125	0·091	0·058						39
38													0·171	0·140	0·110	0·080	0·050						38
37													0·147	0·121	0·094	0·068	0·043						37
36									0·214	0·190	0·167	0·144	0·122	0·100	0·078	0·056	0·035	0·014	0·007	0·028	0·049	0·072	36
35									0·171	0·151	0·133	0·115	0·097	0·079	0·062	0·044	0·027	0·010	0·006	0·022	0·039	0·056	35
34									0·127	0·112	0·097	0·084	0·071	0·058	0·045	0·032	0·020	0·008	0·004	0·016	0·028	0·040	34
33									0·081	0·071	0·061	0·052	0·044	0·036	0·028	0·020	0·012	0·005	0·002	0·010	0·018	0·026	33
32	0·091	0·082	0·074	0·067	0·061	0·055	0·049	0·042	0·036	0·030	0·025	0·021	0·017	0·014	0·011	0·007	0·004	0·001	0·001	0·003	0·006	0·009	32
31	0·004	0·000	0·004	0·006	0·007	0·008	0·009	0·010	0·011	0·013	0·013	0·012	0·010	0·008	0·006	0·005	0·003	0·001	0·001	0·003	0·004	0·006	31
30	0·084	0·084	0·082	0·080	0·076	0·071	0·067	0·064	0·061	0·057	0·052	0·046	0·040	0·032	0·025	0·010	0·013	0·005	0·003	0·010	0·017	0·023	30
29	0·173	0·167	0·160	0·152	0·144	0·135	0·126	0·118	0·110	0·101	0·091	0·080	0·069	0·057	0·044	0·033	0·021	0·009	0·004	0·017	0·029	0·041	29
28	0·263	0·252	0·241	0·229	0·216	0·201	0·187	0·173	0·159	0·145	0·130	0·114	0·098	0·081	0·064	0·047	0·030	0·012	0·006	0·024	0·042	0·059	28
27	0·354	0·338	0·321	0·305	0·287	0·268	0·249	0·230	0·211	0·191	0·170	0·150	0·128	0·106	0·084	0·062	0·039	0·016	0·008	0·032	0·055	0·078	27
26	0·448	0·426	0·404	0·382	0·360	0·337	0·313	0·288	0·263	0·237	0·211	0·185	0·159	0·132	0·104	0·076	0·048	0·019	0·010	0·039	0·068	0·096	26
25	0·546	0·519	0·401	0·463	0·435	0·407	0·378	0·347	0·316	0·285	0·253	0·222	0·190	0·158	0·126	0·092	0·058	0·023	0·012	0·047	0·082	0·115	25
24								0·408	0·371	0·334	0·296	0·259	0·222	0·184	0·146	0·107	0·067	0·027	0·014	0·055	0·094	0·133	24
23								0·469	0·426	0·383	0·341	0·298	0·255	0·212	0·168	0·123	0·077	0·031	0·016	0·063	0·109	0·153	23
22								0·530	0·483	0·433	0·384	0·336	0·288	0·239	0·188	0·137	0·086	0·034	0·018	0·070	0·122	0·173	22
21								0·594	0·540	0·485	0·430	0·376	0·322	0·267	0·210	0·153	0·096	0·038	0·020	0·078	0·136	0·193	21
20													0·356	0·294	0·232	0·169	0·106	0·042	0·022	0·086	0·150	0·212	20
19													0·391	0·322	0·254	0·185	0·116	0·046	0·024	0·094	0·164	0·232	19
18													0·426	0·351	0·276	0·201	0·126	0·050	0·026	0·102	0·178	0·253	18
17													0·462	0·381	0·300	0·219	0·137	0·054	0·020	0·111	0·193	0·275	17
16													0·500	0·412	0·325	0·237	0·148	0·059	0·031	0·121	0·209	0·297	16
15													0·539	0·444	0·350	0·255	0·159	0·063	0·033	0·129	0·225	0·320	15
14													0·578	0·477	0·375	0·273	0·170	0·067	0·035	0·138	0·241	0·343	14
13													0·618	0·510	0·400	0·291	0·182	0·072	0·038	0·148	0·257	0·366	13
12													0·658	0·543	0·426	0·310	0·104	0·077	0·040	0·157	0·274	0·390	12
11													0·698	0·576	0·453	0·320	0·206	0·083	0·043	0·167	0·291	0·414	11
10													0·739	0·609	0·479	0·349	0·218	0·087	0·046	0·177	0·308	0·439	10
9													0·780	0·642	0·505	0·369	0·230	0·002	0·049	0·187	0·325	0·464	9
8													0·821	0·675	0·531	0·389	0·242	0·097	0·052	0·197	0·342	0·489	8

The signs below the horizontal dividing line are opposite to those above it.

Long.	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°	Long.
Lat.	Positive																						Lat.
30°	0·023	0·030	0·037	0·044	0·050	0·055	0·059	0·063	0·066	0·070	0·074	0·078	0·081	0·083	0·084	0·084	0·083	0·081	0·079	0·074	0·069	0·063	30°
29	0·041	0·052	0·065	0·077	0·088	0·098	0·107	0·116	0·124	0·132	0·141	0·150	0·158	0·165	0·171	0·176	0·180	0·184	0·186	0·187	0·186	0·185	29
28	0·059	0·076	0·093	0·110	0·125	0·140	0·155	0·169	0·183	0·197	0·211	0·224	0·237	0·249	0·260	0·270	0·279	0·288	0·295	0·301	0·305	0·309	28
27	0·078	0·100	0·122	0·143	0·164	0·184	0·205	0·225	0·244	0·263	0·281	0·299	0·317	0·334	0·350	0·365	0·380	0·394	0·406	0·417	0·427	0·436	27
26	0·096	0·123	0·150	0·177	0·203	0·229	0·255	0·281	0·305	0·330	0·353	0·376	0·398	0·420	0·442	0·463	0·483	0·502	0·520	0·536	0·551	0·565	26
25	0·115	0·148	0·180	0·212	0·244	0·275	0·307	0·338	0·368	0·398	0·427	0·455	0·482	0·510	0·537	0·564	0·590	0·614	0·636	0·657	0·677	0·697	25
24	0·133	0·172	0·210	0·248	0·285	0·322	0·360	0·396	0·432	0·468	0·502	0·528	0·568	0·605	0·643	0·680	0·717	0·753	0·788	0·828	0·858	0·891	24
23	0·153	0·198	0·242	0·285	0·327	0·370	0·413	0·457	0·499	0·539	0·579	0·527	0·572	0·616	0·659	0·703	0·745	0·786	0·826	0·866	0·907	0·947	23
22	0·173	0·223	0·273	0·321	0·369	0·418	0·467	0·515	0·563	0·611	0·658	0·524	0·575	0·625	0·675	0·724	0·772	0·818	0·864	0·911	0·957	1·003	22
21	0·193	0·249	0·305	0·359	0·413	0·468	0·523	0·577			0·737	0·523	0·580	0·636	0·691	0·747	0·801	0·853	0·905	0·957	1·010	1·062	21
20	0·212	0·274	0·336	0·398	0·459	0·520	0·581	0·642			0·818	0·522	0·585	0·648	0·708	0·769	0·828	0·888	0·947	1·006	1·065	1·124	20
19	0·232	0·300	0·369	0·438	0·505								0·590	0·659	0·725	0·791	0·858	0·925	0·991	1·056	1·121	1·185	19
18	0·253	0·328	0·403	0·478	0·552								0·595	0·670	0·743	0·816	0·890	0·963	1·035	1·107	1·177	1·248	18
17	0·275	0·356	0·437	0·518	0·600								0·601	0·682	0·762	0·842	0·922	1·001	1·050	1·158	1·235	1·311	17
16	0·297	0·385	0·472	0·560	0·648								0·609	0·695	0·782	0·868	0·954	1·040	1·126	1·210	1·293	1·377	16
15																	0·989	1·081	1·172	1·264	1·354	1·444	15
14																	1·024	1·123	1·221	1·319	1·416	1·513	14
13																	1·059	1·165	1·270	1·375	1·479	1·582	13
12																	1·095	1·208	1·320	1·431	1·542	1·653	12
11																	1·132	1·252	1·371	1·490	1·608	1·725	11
10																	1·169	1·297	1·423	1·549	1·674	1·798	10
9																	1·207	1·342	1·476	1·610	1·742	1·872	9
8																	1·245	1·388	1·531	1·672	1·811	1·948	8

Case III.—$u_0=1''$.

Values of u, v, w in seconds.

TABLE XXXV*.

Lat.	Long.	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°
Values of u (*positive*).														
29°–8°		0·977	0·973	0·969	0·964	0·959	0·954	0·949	0·944	0·938	0·932	0·926	0·919	0·912
Values of v.														
29°	Positive	0·119	0·128	0·137	0·147	0·156	0·165	0·175	0·184	0·193	0·202	0·211	0·219	0·228
28		0·114	0·123	0·132	0·141	0·150	0·158	0·167	0·176	0·185	0·194	0·202	0·211	0·219
27		0·109	0·118	0·126	0·135	0·144	0·152	0·160	0·169	0·177	0·185	0·195	0·202	0·210
26		0·104	0·113	0·121	0·129	0·137	0·145	0·153	0·161	0·169	0·177	0·185	0·193	0·201
25		0·100	0·108	0·116	0·124	0·131	0·138	0·146	0·153	0·161	0·169	0·176	0·185	0·192
24		0·095	0·103	0·110	0·118	0·125	0·133	0·140	0·147	0·155	0·162	0·169	0·176	0·183
23		0·091	0·098	0·105	0·112	0·119	0·126	0·134	0·141	0·147	0·154	0·161	0·168	0·175
22		0·087	0·093	0·100	0·107	0·114	0·120	0·127	0·134	0·140	0·147	0·153	0·160	0·166
21		0·082	0·089	0·095	0·102	0·108	0·114	0·120	0·127	0·133	0·140	0·146	0·152	0·158
20		0·078	0·084	0·090	0·096	0·102	0·108	0·114	0·120	0·126	0·132	0·138	0·144	0·150
19		0·074	0·079	0·085	0·091	0·097	0·102	0·108	0·114	0·120	0·125	0·131	0·136	0·142
18		0·069	0·075	0·080	0·086	0·091	0·097	0·102	0·107	0·113	0·118	0·123	0·129	0·134
17		0·065	0·071	0·076	0·081	0·086	0·091	0·096	0·101	0·106	0·111	0·116	0·121	0·126
16		0·061	0·066	0·071	0·076	0·081	0·085	0·090	0·095	0·100	0·104	0·109	0·114	0·118
15		0·057	0·062	0·066	0·071	0·075	0·080	0·084	0·089	0·093	0·098	0·102	0·106	0·110
14		0·053	0·058	0·062	0·066	0·070	0·074	0·079	0·083	0·087	0·091	0·095	0·099	0·103
13		0·049	0·053	0·057	0·061	0·065	0·069	0·073	0·076	0·080	0·084	0·088	0·091	0·095
12		0·045	0·049	0·053	0·056	0·060	0·063	0·067	0·070	0·074	0·077	0·081	0·084	0·088
11		0·041	0·045	0·048	0·051	0·055	0·058	0·061	0·064	0·068	0·071	0·074	0·077	0·080
10		0·038	0·041	0·044	0·047	0·050	0·053	0·055	0·058	0·061	0·064	0·067	0·070	0·073
9		0·034	0·037	0·040	0·043	0·046	0·048	0·050	0·052	0·055	0·057	0·060	0·063	0·065
8		0·030	0·033	0·036	0·039	0·042	0·044	0·045	0·047	0·049	0·051	0·053	0·056	0·058
Values of w.														
29°	Positive	0·245	0·264	0·283	0·303	0·322	0·341	0·360	0·379	0·398	0·416	0·435	0·453	0·471
28		0·242	0·261	0·281	0·300	0·319	0·338	0·357	0·375	0·394	0·412	0·431	0·449	0·467
27		0·240	0·259	0·278	0·297	0·316	0·334	0·353	0·372	0·390	0·408	0·427	0·444	0·462
26		0·238	0·257	0·276	0·294	0·313	0·332	0·350	0·369	0·387	0·405	0·423	0·441	0·458
25		0·236	0·255	0·273	0·292	0·311	0·329	0·347	0·365	0·383	0·401	0·419	0·437	0·454
24		0·234	0·253	0·271	0·290	0·308	0·326	0·344	0·363	0·380	0·398	0·416	0·433	0·451
23		0·232	0·251	0·269	0·287	0·306	0·324	0·342	0·360	0·378	0·395	0·413	0·430	0·448
22		0·231	3·249	0·267	0·285	0·304	0·322	0·339	0·357	0·375	0·392	0·410	0·427	0·444
21		0·229	0·247	0·265	0·284	0·302	0·319	0·337	0·355	0·372	0·390	0·407	0·424	0·441
20		0·228	0·246	0·264	0·282	0·300	0·317	0·335	0·352	0·370	0·387	0·404	0·421	0·438
19		0·227	0·244	0·262	0·280	0·298	0·315	0·333	0·351	0·368	0·385	0·402	0·419	0·436
18		0·225	0·242	0·261	0·278	0·296	0·313	0·331	0·348	0·366	0·383	0·400	0·416	0·433
17		0·224	0·241	0·259	0·277	0·295	0·312	0·329	0·347	0·364	0·381	0·398	0·414	0·431
16		0·222	0·240	0·258	0·275	0·293	0·310	0·327	0·345	0·362	0·379	0·395	0·412	0·429
15		0·221	0·239	0·256	0·274	0·291	0·309	0·326	0·343	0·360	0·377	0·393	0·410	0·427
14		0·220	0·238	0·255	0·273	0·290	0·307	0·324	0·341	0·358	0·375	0·392	0·408	0·425
13		0·219	0·237	0·254	0·271	0·289	0·306	0·323	0·340	0·357	0·373	0·390	0·406	0·423
12		0·218	0·236	0·253	0·270	0·288	0·305	0·322	0·339	0·355	0·372	0·388	0·405	0·421
11		0·218	0·235	0·252	0·270	0·287	0·304	0·321	0·338	0·354	0·371	0·387	0·403	0·420
10		0·217	0·234	0·251	0·269	0·286	0·303	0·320	0·337	0·353	0·369	0·386	0·402	0·418
9		0·216	0·233	0·250	0·268	0·285	0·302	0·319	0·336	0·352	0·368	0·385	0·401	0·417
8		0·215	0·232	0·249	0·267	0·284	0·301	0·318	0·335	0·351	0·367	0·384	0·400	0·416

* Extension of tables XVII, XVIII for Burma and Assam.

Case IV.—$w_0=1''$. Values of u, v, w in seconds. *TABLE XXXVI**.

Lat.	Long.	90°	91°	92°	93°	94°	95°	96°	97°	98°	99°	100°	101°	102°
Values of u (*negative*).														
29°–8°		0·196	0·212	0·227	0·243	0·258	0·273	0·288	0·303	0·318	0·333	0·348	0·363	0·377
Values of v.														
29°	Positive	0·085	0·082	0·081	0·078	0·076	0·074	0·071	0·068	0·065	0·062	0·059	0·056	0·052
28		0·065	0·063	0·061	0·059	0·057	0·055	0·052	0·049	0·046	0·043	0·040	0·037	0 033
27		0·045	0·043	0·041	0·039	0·037	0·035	0·032	0·030	0·027	0·024	0·021	0·018	0·014
26		0·026	0·024	0·022	0·020	0.018	0·016	0·013	0·011	0·008	0 005	0·003	0·000	0·003
25		0·007	0·005	0·004	0·002	0·000	0·002	0·004	0·006	0·009	0·012	0·014	0·018	0·020
24	Negative	0·012	0·013	0·015	0·017	0·019	0·021	0·023	0·025	0·028	0·031	0·033	0·036	0·039
23		0·030	0·032	0·033	0·035	0·037	0·039	0·041	0·043	0·046	0·048	0·051	0·054	0·057
22		0·048	0·050	0·051	0·053	0·055	0·057	0·059	0·061	0·063	0·065	0·068	0·071	0·074
21		0·066	0·068	0·069	0·071	0·072	0·074	0·076	0·078	0·080	0·082	0·085	0·088	0·090
20		0·084	0·085	0·086	0·088	0·090	0·091	0·093	0·095	0·097	0·099	0·102	0·104	0·106
19		0·100	0·102	0·104	0·105	0·107	0·108	0·110	0·112	0·114	0·116	0·118	0·120	0·122
18		0·118	0·119	0·121	0·122	0·124	0·125	0·127	0·129	0·130	0·132	0·134	0·136	0·138
17		0·135	0·136	0·138	0.139	0·140	0·142	0·143	0·145	0·146	0·148	0·150	0·152	0·154
16		0·151	0·152	0·154	0·155	0·157	0·158	0·159	0·161	0·163	0·164	0·166	0·168	0·170
15		0·167	0·169	0·170	0·172	0·173	0·174	0·175	0·177	0·178	0·179	0·181	0·183	0·185
14		0·184	0·185	0·187	0·188	0·189	0·190	0·192	0·193	0·194	0·196	0·197	0·199	0·200
13		0·201	0·202	0·204	0·205	0·206	0·207	0·208	0·209	0·210	0·212	0·213	0·214	0·215
12		0·216	0·217	0·219	0·220	0·221	0·222	0·223	0·224	0·225	0·227	0·228	0·229	0·230
11		0·232	0·233	0·235	0·236	0·237	0·238	0·239	0·240	0·241	0·242	0·243	0·244	0·245
10		0·248	0·249	0·251	0·252	0·252	0·253	0·254	0·255	0·256	0·257	0·258	0·259	0·260
9		0·264	0·265	0·266	0·267	0·267	0·268	0·269	0·270	0·271	0·272	0·273	0·274	0·275
8		0·279	0·280	0·281	0·282	0·282	0·283	0·284	0 285	0·286	0·287	0·288	0·289	0·290
Values of w.														
29°	Positive	1·019	1·015	1·011	1·006	1·001	0·996	0·990	0·984	0·978	0·972	0·965	0·958	0·950
28		1·009	1·005	1·001	0·997	0·992	0·987	0·982	0·976	0·970	0·963	0·956	0·949	0·942
27		1·000	0·997	0·992	0·988	0·983	0·978	0·972	0·966	0·960	0·954	0·947	0·940	0·933
26		0·992	0·988	0·984	0·979	0·974	0·969	0·963	0·958	0·952	0·945	0·939	0·932	0·925
25		0·984	0·980	0·975	0·971	0·966	0·961	0·956	0·950	0·944	0·938	0·931	0·925	0 918
24		0·976	0·972	0·968	0·963	0·958	0·954	0·948	0·943	0·937	0·931	0·924	0·917	0·910
23		0·969	0·965	0·961	0·956	0·951	0·947	0·942	0·936	0·930	0·924	0·918	0·911	0·904
22		0·962	0·958	0·954	0·949	0·944	0·939	0·934	0.928	0·923	0·917	0·910	0·904	0·897
21		0·955	0·951	0·947	0·943	0·938	0·934	0·928	0·923	0·917	0·911	0·905	0·898	0·891
20		0·949	0·945	0·941	0·937	0·932	0·927	0·922	0·916	0·911	0·904	0·898	0·892	0·885
19		0·943	0·939	0·935	0·931	0·926	0·921	0·916	0·911	0·905	0·899	0·893	0·886	0·879
18		0·938	0·934	0·930	0·926	0·921	0·916	0·911	0·906	0·900	0·894	0·888	0·882	0·875
17		0·933	0·929	0·925	0·921	0·916	0·912	0·907	0·901	0·896	0·890	0·883	0·877	0·870
16		0·928	0·924	0·920	0·916	0·911	0·907	0·902	0·896	0·891	0·885	0·879	0·872	0·865
15		0·924	0·920	0·916	0·911	0·907	0·902	0·897	0·892	0·886	0·880	0·874	0·868	0·861
14		0·920	0·916	0·912	0·908	0·903	0·898	0·893	0·888	0·883	0·877	0·870	0·864	0·857
13		0·916	0·912	0·908	0·904	0·899	0·894	0·889	0·884	0·879	0·873	0·867	0·861	0·854
12		0·913	0·909	0·905	0·901	0·896	0·892	0·887	0·881	0·876	0·870	0·864	0·858	0·851
11		0·909	0·905	0·901	0·897	0·892	0·888	0·883	0·878	0·872	0·866	0·860	0·854	0·847
10		0·906	0·902	0·898	0·894	0·890	0·885	0·880	0·875	0·869	0·863	0·857	0·851	0·844
9		0·903	0·899	0·895	0·891	0·887	0·882	0·877	0·872	0·866	0·860	0·854	0·848	0·841
8		0·900	0·896	0·892	0·888	0·884	0·879	0·874	0·869	0·863	0·857	0·851	0·845	0·838

* Extension of tables XIX, XX for Burma and Assam.

It will be noticed that in tables XXXI, XXXIV discontinuities in the values of w_g in the neighbourhood of lat. 20°, long. 91°, 92° are easily apparent. More careful examination of tables XXIX, XXX, XXXII, XXXIII reveals similar but much less marked discontinuities in the values of u_g and v_g. These are inevitable in view of the method by which the quantities have been found, and the differences are in agreement with those which may be computed by equations (42)—(47) of Chapter I following the two paths (viz. by direct geodesic and by two geodesics through the second origin) to such a point as $L = 91\frac{2}{3}°$, $\lambda = 20°$. The amounts are not however sufficiently large to be of practical importance: and moreover they do not actually occur to the same extent in the actual triangulation of India as in the tables, for the tables have been extended somewhat beyond the triangulation limits for facility of subsequent interpolation. It may be mentioned in passing that the azimuth of a ray of length 40 miles is altered by an amount of order $0''\cdot 1$ when its terminal latitude or longitude is altered by an amount of order $0''\cdot 001$: so that the taking out of azimuth to more than one place of decimals is not really defensible when the coordinates are given to only three places. In the present instance the ordinary procedure of the department has been followed and three places of decimals have been kept, with the idea that at any time the latter two of these may be disregarded.

CHAPTER IV.

Geometrical change from one Spheroid of Reference to another.

1. In selecting a spheroid of reference for the geoid there is no doubt as to the direction of the polar axis; for this is the axis about which heavenly bodies appear to rotate. Hence all possible spheroids of reference are defined by the size of their axes and the position of their centres.

Consider two such spheroids. Let the semi-axes of one be a, b and of the other $a' = a + da$, $b' = b + db$. Select the origin of coordinates at the centre of the first spheroid and let the coordinates of the centre of the second be $a\alpha$, $a\beta$, $b\gamma$ where α, β, γ are small quantities.

In relating a point on a geoid to the spheroid the natural course seems to be to draw the normal through the point to the spheroid and to find out the coordinates of the point where this normal meets the spheroid. So long as the spheroid and geoid are not widely different this normal may, without appreciable error be considered as the vertical to the geoid and also as a straight line. For supposing there is a plumb-line deflection of 1 minute and a separation of the geoid and spheroid by 300 feet, the divergence of the normal from the vertical only amounts to about one inch which only affects coordinates by 0·001 of a second. It is accordingly satisfactory to relate a point on the geoid to one on the spheroid by merely producing the vertical of the geoid until it meets the spheroid. Considering then the relation between the points thus obtained on two reference spheroids corresponding to a point on the geoid, it is clear that all these points may with sufficient accuracy be regarded as being on a straight line, this straight line being normal to one of the three surfaces, whichever is most convenient.

To any triangle formed by three points on the geoid there is a corresponding triangle on any reference spheroid. The angles of these triangles are not identical. Those on the spheroid have different spheroidal excesses. The angles of a triangle observed on the geoid accordingly require correcting before they can be properly applied to a spheroid of reference. If this is properly done then this point relationship given above will hold. It is a fault in reduction of most, if not all, survey observations that geoidal and spheroidal angles have been treated as identical.

2. The coordinates of a point P on the first spheroid may be represented by

$$a \cos \phi \cos L, \qquad a \cos \phi \sin L, \qquad b \sin \phi$$

while those of a related point P' on the second spheroid may be represented by

$$a\alpha + a' \cos \phi' \cos L', \qquad a\beta + a' \cos \phi' \sin L', \qquad b\gamma + b' \sin \phi'$$

where $\phi' = \phi + d\phi, \quad L' = L + dL$. It is necessary to find expressions for $d\phi$ and dL. It is customary to decide on a point on a spheroid of reference as origin. All spheroids of reference are supposed to pass through this. Suppose that the origin lies in the plane of xz, so that L vanishes at the origin. In notation of previous chapters $dL = v$, $d\phi = u_1$. At the origin these quantities reduce to v_0 and ${}_0u_1$. The value of v_0 is not obtained directly, but is derivable after azimuth has been decided on by the relation

$$w_0 = v_0 \sin \lambda_0 \quad \dots \dots \dots \dots \quad (1)$$

This of course does not show the error of longitude of the origin: it merely shows by how much it will be changed if the azimuth is changed on the supposition of a plumb-line deflection in prime vertical.

Since the origin on either spheroid is identical in position the expression for its coordinates may be equated. Hence putting $\phi = \phi_0$ and $L = 0$

$$\left.\begin{aligned} a \cos \phi_0 &= a\alpha + a' \cos \phi_0' \cos v_0 \\ 0 &= a\beta + a' \cos \phi_0' \sin v_0 \\ b \sin \phi_0 &= b\gamma + b' \sin \phi_0' \end{aligned}\right\} \quad \dots \dots \dots \dots \quad (2)$$

Neglecting second order quantities and substituting from (1) for v_0 (2) may be written

$$\left.\begin{aligned} \alpha + \frac{da}{a} \cos \phi_0 - {}_0u_1 \sin \phi_0 &= 0 \\ \beta + w_0 \frac{\cos \phi_0}{\sin \lambda_0} &= 0 \\ \gamma + \frac{db}{b} \sin \phi_0 + {}_0u_1 \cos \phi_0 &= 0 \end{aligned}\right\} \quad \dots \dots \dots \dots \quad (3)$$

These equations serve to determine α, β, γ in terms of the axes changes and changes at the origin: if the quantities $\frac{da}{a}, \frac{db}{b}$ are multiplied by cosec 1″ the results are expressed in seconds.

3. Two further conditions are obtained by expressing the fact that the normal to the first spheroid at any point passes through the related point on the other spheroid.

The normal at P is

$$\frac{x - a \cos \phi \cos L}{\dfrac{\cos \phi \cos L}{a}} = \frac{y - a \cos \phi \sin L}{\dfrac{\cos \phi \sin L}{a}} = \frac{z - b \sin \phi}{\dfrac{\sin \phi}{b}}$$

and the conditions that P' should lie on this are

$$\frac{a\alpha + d\,(a \cos \phi \cos L)}{\dfrac{\cos \phi \cos L}{a}} = \frac{a\beta + d\,(a \cos \phi \sin L)}{\dfrac{\cos \phi \sin L}{a}} = \frac{b\gamma + d\,(b \sin \phi)}{\dfrac{\sin \phi}{b}}$$

whence

$$\frac{\alpha}{\cos \phi \cos L} + \frac{da}{a} - \tan \phi \,.\, u_1 - \tan L \,.\, v = \frac{\beta}{\cos \phi \sin L} + \frac{da}{a} - \tan \phi \,.\, u_1 + \cot L \,.\, v$$

$$= (1 - e^2) \left\{ \frac{\gamma}{\sin \phi} + \frac{db}{b} + \cot \phi \,.\, u_1 \right\} \quad . \quad . \quad (4)$$

From the first of equations (4)

$$v\,(\tan L + \cot L) = \sec\phi\left(\frac{\alpha}{\cos L} - \frac{\beta}{\sin L}\right)$$

or

$$v = \sec\phi\,(\alpha\sin L - \beta\cos L) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

Eliminating v from (4) it follows that

$$\left(\frac{\alpha}{\cos\phi\cos L} + \frac{da}{a} - u_1\tan\phi\right)\cot L + \left(\frac{\beta}{\cos\phi\,\sin L} + \frac{da}{a} - u_1\tan\phi\right)\tan L$$

$$= (1-e^2)\,(\tan L + \cot L)\left(\frac{\gamma}{\sin\phi} + \frac{db}{b} + u_1\cot\phi\right)$$

whence

$$(\alpha\cos L + \beta\sin L)\sec\phi + \frac{da}{a} - u_1\tan\phi = (1-e^2)\left(\frac{\gamma}{\sin\phi} + \frac{db}{b} + u_1\cot\phi\right)$$

and expressing the results in seconds this may be written

$$u_1(1-e^2\cos^2\phi) = (\alpha\cos L + \beta\sin L)\sin\phi - (1-e^2)\gamma\cos\phi + \sin\phi\cos\phi\left\{\frac{da}{a} - (1-e^2)\frac{db}{b}\right\}\operatorname{cosec} 1'' \quad . \quad . \qquad (6)$$

The relation between u_1 and u is given by (16) of Chap. III, and in terms of da and db is

$$u = \left(1 + \frac{e^2}{2}\cos 2\phi\right)u_1 + \tfrac{1}{2}\left(\frac{da}{a} - \frac{db}{b}\right)\sin 2\lambda\operatorname{cosec} 1'' \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

The quantities α, β, γ, $\frac{da}{a}$, $\frac{db}{b}$ all enter linearly into the equations. Their several effects can accordingly be computed separately and combined afterwards in any desired way. Cases corresponding to each of the four quantities $\frac{da}{a}$, $\frac{db}{b}$, ${}_0u_1$ and w_0 will now be considered.

Case (i) $da = 1$ km. ${}_0u_1 = 0$, $(u_0 = 12''\cdot063)$

From (3)

$$\alpha + A\cos\phi_0 = 0 \qquad \text{where } A = \frac{da}{a}\operatorname{cosec} 1'' = 32''\cdot3437$$

$$\beta = \gamma = 0$$

$$\therefore \quad \alpha = -\,29''\cdot536$$

From (5) and (6)

$$\left.\begin{aligned} v &= \alpha\sec\phi\sin L \\ u_1\,(1 - e^2\cos^2\phi) = u_1\cdot\frac{\sin^2\phi}{\sin^2\lambda} &= \alpha\sin\phi\cos L + \tfrac{1}{2}A\sin 2\phi \\ u &= (1 + \frac{e^2}{2}\cos 2\phi)u_1 + \tfrac{1}{2}A\sin 2\lambda \\ &= \sin 2\lambda\left(\frac{u_1}{\sin 2\phi} + \tfrac{1}{2}A\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (8)$$

Case (ii) $db = 1$ km. ${}_0u_1 = 0$ $(u_0 = -\,12''\cdot1034)$

From (3) $\alpha = \beta = 0$

$$\gamma + B\sin\phi_0 = 0 \qquad \text{where } B = \frac{db}{b}\operatorname{cosec} 1'' = 32''\cdot4516$$

$$\therefore \quad \gamma = -\,13''\cdot2244$$

From (5) and (6)

$$\left.\begin{aligned} v &= 0 \\ u_1\,(1-e^2\cos^2\phi) &= -\,(1-e^2)\,(\gamma + B\sin\phi)\cos\phi \\ u &= \left(1+\frac{e^2}{2}\cos 2\phi\right)u_1 - \tfrac{1}{2}B\sin 2\lambda \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (9)$$

Case (iii) $\quad {}_0u_1 = 9''\cdot 978 \qquad (u_0 = 10'')$

From (8)

$$\begin{aligned} a - {}_0u_1 \sin\phi_0 &= 0 \\ \beta &= 0 \\ \gamma + {}_0u_1\cos\phi_0 &= 0 \\ \therefore \quad a &= 4''\cdot 0659 \qquad \gamma = -\,9''\cdot 1118 \end{aligned}$$

From (5) and (6)

$$\left.\begin{aligned} v &= a\sec\phi\sin L \\ u_1\,(1-e^2\cos^2\phi) &= a\sin\phi\cos L - (1-e^2)\,\gamma\cos\phi \\ u &= \left(1+\frac{e^2}{2}\cos 2\phi\right)u_1 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (10)$$

Case (iv) $\quad w_0 = 1'' \qquad v_0 = w_0 \operatorname{cosec}\lambda_0 = 2''\cdot 447$

$$a = \gamma = 0$$

From (3) $\quad \beta + \dfrac{\cos\phi_0}{\sin\lambda_0} = 0 \qquad \therefore \quad \beta = -2''\cdot 2347$

From (5) and (6)

$$\left.\begin{aligned} v &= -\,\beta\sec\phi\cos L \\ u_1\,(1-e^2\cos^2\phi) &= \beta\sin L\sin\phi \\ u &= \left(1+\frac{e^2}{2}\cos 2\phi\right)u_1 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (11)$$

To find the azimuth change w, the following equation holds for all cases

$$v - v_0 = w\operatorname{cosec}\lambda - w_0\operatorname{cosec}\lambda_0 \quad \ldots\ldots\ldots \quad (12)$$

in which v_0 includes the entire origin change of longitude and is not restricted to that due to plumb-line deflection only. The equation follows from the fact that either side of it gives the difference between spheroidal and geoidal longitude. It is proved otherwise in the following chapter (*vide* equation (4)). With reference to the case IV (or (iv)) it will be noticed that the value of $u \propto \sin L \sin\phi$. The value found by the method of Chapter I was independent of ϕ. The two cases however are not geometrically similar. In the case of Chapter I an azimuth change of origin involves a twist about the normal at the origin. In the present case the fixed axis is the polar axis and any twist introduced to give any desired azimuth change is only a component of a twist round an axis parallel to the polar axis. This makes it clear why the effect on latitude of this azimuth change is zero at the equator, the equator being at right angles to the axis of twist.

The values of u, v, w have been computed for the four cases by means of equations (8) to (12). The values of λ and L are the same as those of tables XXVII, XXVIII, and hence it is easy to make a comparison between the values of u, v, w found by the method of the present chapter which may be denoted by u_r, v_r, w_r (related points on two spheroids) and u_g, v_g, w_g (found by following a geodesic). For this purpose values of $u_r - u_g$ &c. are exhibited in tables XXXVII—XL.

TABLE XXXVII.

Case I.—$\delta a = 1$ km.

L′ — L	0°	4°	8°	12°	16°	20°	24°
λ	Values of $(u_r - u_g)$ in seconds.						
38°	0·000	+0·010	+0·049	+0·117	+0·201	+0·315	+0·442
34	+0·011	+0·021	+0·053	+0·099	+0·165	+0·249	+0·347
30	+0·005	+0·014	+0·031	+0·061	+0·105	+0·154	+0·216
26	+0·005	+0·008	+0·013	+0·022	+0·038	+0·058	+0·076
22	−0·003	−0·010	−0·017	−0·028	−0·043	−0·061	−0·090
18	−0·004	−0·017	−0·038	−0·070	−0·117	−0·180	−0·255
14	+0·005	−0·012	−0·048	−0·103	−0·186	−0·290	−0·418
10	+0·035	+0·013	−0·040	−0·120	−0·239	−0·386	−0·576
	Values of $\pm(v_r - v_g)$* in seconds.						
38	0·000	+0·038	+0·060	+0·080	+0·067	−0·005	−0·116
34	0·000	+0·016	+0·026	+0·020	−0·019	−0·100	−0·234
30	0·000	+0·003	+0·003	−0·015	−0·071	−0·162	−0·312
26	0·000	+0·001	−0·005	−0·026	−0·090	−0·193	−0·351
22	0·000	−0·002	−0·003	−0·031	−0·091	−0·186	−0·346
18	0·000	+0·002	+0·004	−0·012	−0·061	−0·153	−0·203
14	0·000	+0·021	+0·033	+0·034	−0·001	−0·079	−0·204
10	0·000	+0·047	+0·082	+0·108	+0·100	+0·044	−0·066
	Values of $\pm(w_r - w_g)$* in seconds.						
38	0·000	+0·371	+0·735	+1·093	+1·436	+1·756	+2·058
34	0·000	+0·273	+0·542	+0·806	+1·058	+1·292	+1·509
30	0·000	+0·168	+0·320	+0·494	+0·641	+0·782	+0·913
26	0·000	+0·052	+0·107	+0·160	+0·205	+0·238	+0·265
22	0·000	−0·068	−0·130	−0·197	−0·267	−0·338	−0·411
18	0·000	−0·191	−0·384	−0·573	−0·760	−0·950	−1·136
14	0·000	−0·328	−0·648	−0·969	−1·285	−1·508	−1·903
10	0·000	−0·466	−0·928	−1·380	−1·830	−2·278	−2·707

TABLE XXXVIII.

Case II.—$\delta b = 1$ km.

0°	4°	8°	12°	16°	20°	24°	L′ — L
Values of $(u_r - u_g)$ in seconds.							λ
+0·078	+0·073	+0·043	−0·005	−0·062	−0·148	−0·246	38°
+0·017	+0·008	−0·012	−0·044	−0·003	−0·156	−0·234	34
+0·008	+0·001	−0·011	−0·032	−0·064	−0·105	−0·159	30
−0·004	−0·006	−0·010	−0·020	−0·032	−0·052	−0·075	26
+0·003	+0·008	+0·012	+0·017	+0·024	+0·029	+0·035	22
−0·001	+0·011	+0·025	+0·040	+0·060	+0·098	+0·139	18
−0·034	−0·022	+0·001	+0·034	+0·084	+0·144	+0·216	14
−0·114	−0·100	−0·065	−0·017	+0·060	+0·147	+0·259	10
Values of $\pm(v_r - v_g)$* in seconds.							
0·000	−0·068	−0·133	−0·201	−0·266	−0·317	−0·371	38
0·000	−0·030	−0·064	−0·101	−0·127	−0·152	−0·168	34
0·000	−0·008	−0·024	−0·037	−0·043	−0·044	−0·043	30
0·000	−0·003	−0·009	−0·015	−0·005	−0·004	+0·015	26
0·000	−0·001	−0·011	−0·011	−0·008	−0·007	+0·011	22
0·000	−0·008	−0·025	−0·043	−0·052	−0·059	−0·056	18
0·000	−0·033	−0·069	−0·107	−0·147	−0·173	−0·196	14
0·000	−0·070	−0·137	−0·217	−0·285	−0·348	−0·400	10
Values of $\pm(w_r - w_g)$* in seconds.							
0·000	−0·388	−0·777	−1·170	−1·566	−1·960	−2·363	38
0·000	−0·280	−0·564	−0·853	−1·143	−1·438	−1·740	34
0·000	−0·171	−0·341	−0·523	−0·700	−0·889	−1·090	30
0·000	−0·053	−0·114	−0·177	−0·251	−0·325	−0·416	26
0·000	+0·067	+0·124	+0·182	+0·228	+0·268	+0·288	22
0·000	+0·190	+0·379	+0·558	+0·728	+0·885	+1·030	18
0·000	+0·324	+0·642	+0·953	+1·265	+1·542	+1·815	14
0·000	+0·463	+0·920	+1·366	+1·805	+2·233	+2·636	10

TABLE XXXIX.

Case III.—$u_0 = 1''$

L′ — L	0°	4°	8°	12°	16°	20°	24°
λ	Values of $(u_r - u_g)$ in seconds.						
38°	−0·031	−0·029	−0·024	−0·015	−0·002	+0·014	+0·033
34	−0·016	−0·014	−0·009	+0·001	+0·014	+0·030	+0·050
30	−0·006	−0·004	+0·002	+0·011	+0·025	+0·042	+0·062
26	−0·001	+0·002	+0·007	+0·017	+0·031	+0·048	+0·070
22	0·000	+0·002	+0·008	+0·018	+0·033	+0·051	+0·072
18	−0·005	−0·002	+0·004	+0·014	+0·029	+0·047	+0·070
14	−0·014	−0·012	−0·005	+0·005	+0·021	+0·040	+0·063
10	−0·029	−0·026	−0·020	−0·008	+0·007	+0·027	+0·051
	Values of $\pm(v_r - v_g)$* in seconds.						
38	0·000	+0·019	+0·037	+0·055	+0·073	+0·091	+0·108
34	0·000	+0·013	+0·026	+0·038	+0·051	+0·064	+0·075
30	0·000	+0·008	+0·016	+0·023	+0·030	+0·037	+0·044
26	0·000	+0·002	+0·005	+0·007	+0·009	+0·011	+0·014
22	0·000	−0·003	−0·004	−0·006	−0·009	−0·012	−0·014
18	0·000	−0·007	−0·014	−0·022	−0·029	−0·035	−0·042
14	0·000	−0·011	−0·023	−0·035	−0·046	−0·057	−0·068
10	0·000	−0·017	−0·032	−0·049	−0·065	−0·081	−0·096
	Values of $\pm(w_r - w_g)$* in seconds.						
38	0·000	+0·067	+0·133	+0·198	+0·262	+0·326	+0·387
34	0·000	+0·065	+0·130	+0·194	+0·257	+0·319	+0·380
30	0·000	+0·065	+0·128	+0·191	+0·253	+0·315	+0·375
26	0·000	+0·064	+0·127	+0·191	+0·252	+0·313	+0·372
22	0·000	+0·065	+0·128	+0·191	+0·253	+0·313	+0·371
18	0·000	+0·065	+0·129	+0·102	+0·254	+0·315	+0·373
14	0·000	+0·065	+0·129	+0·193	+0·256	+0·317	+0·378
10	0·000	+0·066	+0·131	+0·196	+0·260	+0·323	+0·383

TABLE XL.

Case IV.—$w_0 = 1''$

0°	4°	8°	12°	16°	20°	24°	L′ — L
Values of $\pm(u_r - u_g)$* in seconds.							λ
0·000	+0·031	+0·064	+0·095	+0·128	+0·158	+0·188	38°
0·000	+0·023	+0·047	+0·069	+0·093	+0·116	+0·138	34
0·000	+0·013	+0·028	+0·042	+0·056	+0·071	+0·084	30
0·000	+0·004	+0·009	+0·013	+0·019	+0·024	+0·028	26
0·000	−0·006	−0·011	−0·016	−0·021	−0·025	−0·030	22
0·000	−0·016	−0·031	−0·047	−0·061	−0·076	−0·090	18
0·000	−0·027	−0·053	−0·078	−0·103	−0·127	−0·151	14
0·000	−0·038	−0·074	−0·110	−0·145	−0·180	−0·214	10
Values of $(v_r - v_g)$ in seconds.							
+0·083	+0·078	+0·062	+0·037	+0·001	−0·045	−0·100	38
+0·040	+0·035	+0·020	−0·005	−0·038	−0·085	−0·140	34
+0·013	+0·009	−0·006	−0·031	−0·066	−0·110	−0·164	30
+0·001	−0·004	−0·019	−0·044	−0·078	−0·122	−0·176	26
+0·002	−0·003	−0·017	−0·043	−0·077	−0·121	−0·184	22
+0·014	+0·008	−0·006	−0·031	−0·066	−0·110	−0·164	18
+0·036	+0·031	+0·016	−0·010	−0·044	−0·080	−0·143	14
+0·068	+0·064	+0·047	+0·020	−0·014	−0·059	−0·114	10
Values of $(w_r - w_g)$ in seconds.							
+0·586	+0·585	+0·580	+0·573	+0·564	+0·550	+0·536	38
+0·407	+0·406	+0·403	+0·398	+0·393	+0·382	+0·371	34
+0·235	+0·235	+0·233	+0·231	+0·226	+0·221	+0·214	30
+0·075	+0·074	+0·073	+0·073	+0·072	+0·070	+0·068	26
−0·082	−0·082	−0·082	−0·081	−0·079	−0·078	−0·079	22
−0·234	−0·233	−0·232	−0·229	−0·225	−0·220	−0·214	18
−0·384	−0·383	−0·380	−0·376	−0·370	−0·362	−0·350	14
−0·533	−0·532	−0·528	−0·522	−0·512	−0·501	−0·486	10

* + or − according as point is west or east of origin.

4. The differences $u_r - u_g$ and $v_r - v_g$ are never sufficiently great to have any important effect on geodetic results. In the case of $w_r - w_g$ larger values are met with. The deflections of plumb-line in the prime vertical are affected by the amount $(w_r - w_g)\cot\lambda$, a quantity which may be as much as several seconds. In the practical case however where, according to the most recent determinations $\delta a = 0{\cdot}924$km and $\delta b = 0{\cdot}743$km : the combined effect of Cases I and II in these proportions are not large, the two cases tending to cancel each other; as may be seen from table XLI below, in which the values of $(w_r - w_g)$ cotλ are also given.

TABLE XLI.

λ	L′−L	Cot λ	0·924 × Case I	0·743 × Case II	Combined effect	Discrepancy in plumb-line deflection
38°	24°	1·280	1″·902	−1″·756	0″·146	0″·187
34	24	1·483	1·394	−1·293	0·101	0·150
30	24	1·732	0·844	−0·810	0·034	0·059
26	24	2·050	0·245	−0·309	−0·064	−0·131
22	24	2·475	−0·380	0·214	−0·166	−0·411
18	8	3·078	−0·355	0·282	−0·073	−0·225
14	4	4·011	−0·303	0·241	−0·062	−0·249
10	4	5·671	−0·431	0·344	−0·087	−0·493

The only case in which the discrepancy in prime vertical deflection would be considerable occurs in low latitudes and this case does not concern Indian Triangulation as in these latitudes there are no great longitude differences: for the case of Burma special treatment, as given in Chapter III, is in any case necessary.

The conclusion is that either this method or that of the preceding chapter could be used with practically satisfactory results. The discrepancies however indicate how far theoretical accuracy has been departed from in failing to project geoidal angles on to the spheroid of reference before introducing them in the computations.

The figures in tables XXXVII—XL may be noticed to be a little irregular. This is doubtless due to the fact that the computations of Chapter III were not made with sufficient accuracy to ensure the last figure always being correct. This was not considered to be sufficiently important to justify the extra labour which would have been necessary. The results are fully accurate enough for all practical purposes to which they can be put.

5. Three methods of finding the change in coordinates due to any proposed changes of the axes of the spheroid and the latitude and azimuth at the origin have now been given. That of Chapter I gives a means of computing these along any path defined by a relation between λ and L. Chapter III gives the results for the special case when the path selected is the geodesic through the origin and the point at which the changes are required: and in the present chapter the geometrical relation between corresponding points on two spheroids is worked out. All these methods give somewhat different results. The latter two have the advantage over the former of being free from any ambiguity due to multiple values and inconsistency: and the differences met with between them are not of amount sufficiently large to be troublesome. The reason for their discrepancy is examined in the following chapter: and the conclusion is arrived at, from theoretical considerations, in view of the methods by which the observations of the triangulation of India have been reduced, that **the method of calculation along geodesics as set forth in Chapter III is the correct one to use.** The detailed tables XXIX—XXXIV permit this to be done readily.

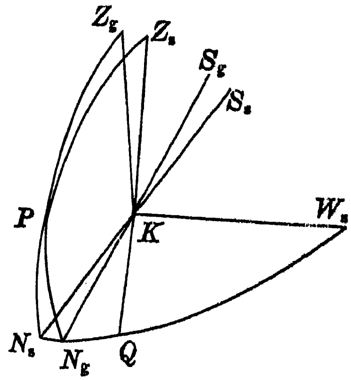
CHAPTER V.

Laplace's Equation and the Choice of a Spheroid of Reference.

1. When a large survey is begun one of the first essentials is the selection of a point as origin. The coordinates of this point have to be decided on. The longitude is of little consequence and the meridian through the point may be taken as that from which all deduced longitudes are measured. The latitude and azimuth can be observed astronomically: but their geodetic values depend on the plumb-line deflection existing at the point. Plumb-line deflection is of course merely the deviation of the vertical from the normal to some assumed figure of reference. In chosing the origin it must be eventually decided whether to consider the deflection there as nil—in other words, chosing a spheroid of reference parallel to the geoid at the origin—or whether considerations of topographical features and irregularities of density justify the adoption of certain values of the deflection in the two components. The question of height above geoid of the point selected as origin also arises. If an error of 10 feet is made in this it is practically equivalent to assuming a spheroid with axes 10 feet different from those actually selected. In the case of the origin of the Indian Survey there is no reason to suppose an error of nearly so much, and so no further consideration will be given to this point here.

Having decided on the origin O, it is next necessary to decide on a figure of reference. This will generally be referred to as the "spheroid" in opposition to the "geoid" or sea level equipotential surface of the earth. It is not implied by this that the figure of reference must be a spheroid, though the almost universal practice is to take a spheroid as a reference figure.

2. Chains of triangulation may now be computed *rigidly* if proper corrections given below in § 16 are applied: and the coordinates, latitude, longitude, height and azimuth at a distant point K may be deduced. Suppose that astronomical azimuth and latitude are observed at K and also that the arc OK is observed as a telegraphic longitude arc. Let λ, L be the geodetic latitude and longitude of K and let A be the geodetic azimuth at K of some reference mark: these are quantities brought up by triangulation. Let η, ξ be the plumb-line deflection at K in meridian and prime vertical (positive for southerly and westerly deflections of the plumb bob) referred to the selected spheroid of reference. These quantities obviously differ for different spheroids of reference.



In the figure suffixes s, g refer to spheroidal and geoidal points respectively: or in other words points derived from triangulation and star observations respectively. P is the pole and Z the zenith. The astronomic azimuth of a point Q is clearly $A+\xi \tan \lambda$ being S_gKS_s greater than the geodetic azimuth. Let suffix zero denote quantities appertaining to the origin of the survey: so that the values of η_0 ξ_0 have been decided in some way or other.

3. First consider the longitude observations. They depend on the interval of time between the meridians of O and K as shown by star transits. If the zenith at a station is displaced in the prime vertical, the meridian is also displaced as a result, the direction of the pole being fixed. Time stars are observed when they transit the plane $Z_g P$ instead of the plane $Z_s P$. With a westerly deflection the zenith is moved towards the east and the result is that stars are observed too soon by the angle $Z_gPZ_s = \xi \sec \lambda$. If ξ is expressed in seconds of arc, the star time as given by the local meridian is early by $\xi \sec \lambda/15$ seconds of time. Now of the two stations O, K, if O is the more westerly and T is the time interval between the transits of a star at the two meridians, then the time interval between the two spheroidal meridians is

$$T-\frac{\xi \sec \lambda}{15}+\frac{\xi_0 \sec \lambda_0}{15}$$

and this should be the same as $\frac{L-L_0}{15}$, or the difference of longitude in time, *on that spheroid on which ξ, ξ_0 represent the plumb-line deflections in prime vertical at K and O.*

The quantity T is an observation quantity; suppose its error is δT: also suppose the error in longitude generated in the triangulation is δL. It follows that

$$\xi_0 \sec \lambda_0 - \xi \sec \lambda = L-L_0-\delta L-15\ (T-\delta T) \qquad (1)$$

Now consider the azimuth observations. Let A' be the astronomically observed azimuth which has an error $\delta A'$: A and δA being the geodetic azimuth computed on the spheroid and its error. Then

$$\left.\begin{aligned} A'-\delta A-A+\delta A &= \xi \tan\lambda \\ A_0'-\delta A_0-A_0 &= \xi_0 \tan\lambda_0 \end{aligned}\right\} \qquad (2)$$

Eliminating ξ, ξ_0 between (1) and (2) it follows that

$$(A_0'-\delta A'_0-A_0)\ \mathrm{cosec}\lambda_0-(A'-\delta A'-A+\delta A)\ \mathrm{cosec}\lambda = L-L_0-\delta L-15\ (T-\delta T) \qquad (3)$$

which is an elaborated form of Laplace's equation.

4. Suppose now that the computations had been carried out on a slightly different spheroid. *If this had been done rigorously* the quantity δL, being itself a small quantity will not be changed appreciably, while A', $\delta A'$, A_0', $\delta A_0'$, T, δT are all quantities which are not affected by the change in spheroid. The only quantities in (3) which change appreciably are A, $L-L_0$ and λ. The λ terms are multiplied by small coefficients and their variations can be neglected. Hence differentiating (3) for change of spheroid it follows that in the notation of Chapter I where u v w represent changes in latitude, longitude and azimuth

$$w \ \mathrm{cosec}\, \lambda - w_0 \ \mathrm{cosec}\, \lambda_0 = v - v_0 \qquad (4)$$

which is equation (12) of Chapter IV.

It might be expected that this equation would be in accordance with those found in Chapter I. As was noticed there, however, the quantities u, v, w are many-valued, a separate set of values appertaining to each route along which the integration is performed. Equation (4) on the other hand is free from any ambiguity and accordingly cannot be in accord with the equations of Chapter I. If numerical quantities are substituted it is at once clear that the relation (4) is not satisfied. Consider the values of

$$v = v_x - f(v_x - v_y) \text{ and } w = w_x - f'(w_x - w_y)$$

where f and f' are fractional quantities. These expressions are the values of v, w computed along routes intermediate to those of v_x and v_y. Taking case where $\delta a = 1$ km., $\lambda = 30^0$, $L = 66^0$, $v_0 = w_0 = 0$ from the tables VII—X it follows that

$$v \sin 30^0 = 4\cdot027 - \cdot013\ f$$

and

$$w = 3\cdot784 - \cdot504\ f$$

which cannot be made equal by any positive fractional values of f and f'. In the same way the tables of Chapter III show that the relation (4) is not satisfied along a geodesic.

5. It has generally been considered that azimuth and longitude observations both give the same information, namely deflection of the plumb-line in prime vertical, and nothing more: and in so far as the results differ by the two methods the reason is that the observations are burdened by errors. Clarke states* that "the observations of the difference of longitude gives "us no information that is not also given by the observation for azimuth". With this principle Colonel Sir Sidney Burrard† has used the longitude observations of India to correct azimuth observations for the accumulation of error due to triangulation, considering the differences of the resulting plumb-line deflection found by the two observations to be entirely accounted for by observation error in the triangulation.

6. The explanation of these apparent inconsistencies was not discovered for some time. Equation (4) is perfectly correct *if the triangulation is properly computed.* The ordinary process of computation is not quite correct. Angles are measured by means of a theodolite and reduced to the horizontal plane of the geoid. This is not quite the same thing in general as the horizontal plane of the spheroid. If the computation is to be effected on the spheroid (on which all the various formulæ are based) the observed angles should be projected on to the selected spheroid of reference, and so will differ according to what spheroid is selected. The actual amount by which the geoidal angle must be altered to get the spheroidal angle depends on two things (vide § 16 below)

(1) the deflection of the plumb-line or inclination of the geoidal (astronomic) vertical to the spheroidal (triangulated) vertical.

(2) the inclination to the horizontal of the rays between which the geoidal angle is measured.

The first of these quantities varies appreciably with change of spheroid and accordingly the correction to the geoidal angle varies according to the spheroid used. The actual case under consideration is represented in symbols by supposing δL in (1) to contain not only the error due to faulty observations but also the error due to failure to correct the geoidal angles to spheroidal angles. This is purely a computation error. The actual "grinding" process has treated these errors as errors of observation.

This perhaps explains why Laplace's equation is in general not satisfied so well as the probable errors of the several observations on which its formation depends would cause to be expected.

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* "Geodesy" by Col. A.R. Clarke, p. 291.

† Appendix No. 5 of G.T. Volume XVIII. "On the azimuth observations of the G.T.S. of India".

7. It also explains how it is that the different values of u, v, w arise as noticed in Chapter I. In this case the fact that the closing errors of circuits will differ from one spheroid to another *unless all the geoidal angles are reduced to spheroidal angles* makes the distribution of closing errors have a different effect according as the spheroid is altered.

Equation (4) may be re-written to meet the actual case as follows:

$$[w \operatorname{cosec} \lambda - v] = \Delta L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where ΔL is the change in computation error of longitude difference due to the treatment of spheroidal and geoidal angles as identical. The fact that ΔL is not zero would be of more serious importance in the question of change of spheroid had equation (3) been used in all possible cases as a condition for the series of triangulation to satisfy. When the main Indian triangulation was adjusted the longitude arcs either were not available or else were ignored, so that Laplace's condition was not imposed on the triangulation. In correcting the azimuth observations, Colonel Sir Sidney Burrard introduced the condition for the first time in India.

8. Clarke's statement quoted in § 5 was deduced from an equation which arises in his work: but it may be seen to be true without any analysis. The longitude observation fixes the meridian plane at a point, that is the plane through the zenith and the pole, by taking the time of stars transiting this plane. It obviously does no more than fix this plane with relation to another. The azimuth observation practically draws the great circle through the pole and zenith and locates where this cuts the horizon, by means of a horizontal angle measured from a fixed point. The position of the pole and the place of observation being already given the fixing of one other point suffices to fix the meridian plane. Thus longitude and azimuth observations both merely fix the position of the meridian plane and nothing more. The inclination of this plane to the meridian plane deducible from triangulation is the deflection in prime vertical.

9. None the less equation (3) does definitely give some information as to the error of computation generated in the triangulation, and to this extent Clarke's statement needs modification. When the practical case is considered from equation (3) it may be seen that the identity of plumb-line deflection, whether derived from longitude observations or azimuth observations, affords some information concerning the slightly faulty method of computing from geoidal angles instead of from spheroidal angles. For split up the error δL into $\delta_1 L$ due to faulty observation and $\delta_2 L$ due to faulty computation. Suppose next that the spheroid of reference is changed so that it is necessary to substitute $A+w$ for A and $L+v$ for L. The quantity $\delta_1 L$ remains unaltered, but $\delta_2 L$ obviously is a variable according to the spheroid used and from (3) it follows that

$$(A'_0 - A_0 - w_0) \operatorname{cosec} \lambda_0 - (A' - A - w) \operatorname{cosec} \lambda = L - L_0 + v - v_0 - 15T - \delta_2 L + \Delta E \quad . \quad . \quad . \quad (6)$$

in which the only variables are w, w_0, $v - v_0$ and $\delta_2 L$, and ΔE is the combined and fixed effect of observation errors. It is possible to form sixteen equations of the form (6) from the longitude and azimuth observations of India. Expressing $w, w_0, v - v_0$ in terms of $\delta a, \delta b, u_0$ and w_0 it is possible to solve these equations for $\delta a, \delta b, u_0, w_0$ so as to make $\Sigma\,(\Delta E - \delta_2 L)^2$ a minimum: *i.e.* since ΔE is equally likely to be positive or negative $\Sigma\,\overline{\Delta E}|^2 + \Sigma\,\overline{\delta_2 L}|^2$ is a minimum. But as the value of ΔE is not being varied, this implies that $\Sigma\,\overline{\delta_2 L}|^2$ is a minimum. Now when the spheroid differs widely from the geoid it is clear that the computation errors increase: and conversely when the spheroid approximates more closely to the geoid these errors diminish. The fact that $\Sigma\,\overline{\delta_2 L}|^2$ is made a minimum affords one criterion for the spheroid being in close agreement with the geoid for the *area* over which the triangulation of India extends. It is of course possible to consider what spheroid suits the actual deflections best: but this is an entirely different point of view from that indicated above, and deals only with the actual localities in which the deflections are measured: and moreover is burdened by the errors of computation involved in treating geoidal and spheroidal angles as identical.

10. The interest in the method is chiefly theoretical. The quantities to be dealt with are very small: and in most cases the effects of observation error may well mask those due to the computation error. Sixteen equations of the form (6) are given below. These can be solved for δa, δb, u_0, w_0 or, treating δa, δb as known, for u_0, w_0 only. It was not anticipated that the former course would give reliable values of δa, δb but the solution was none the less made. Afterwards the solution of u_0, w_0 only taking the latest values of δa, δb was performed. Referring to these two solutions as A and B, one difficulty of the application of (6) arises in A, but to a very much less extent in B. This difficulty is the selection of the route along which u, v, w in terms of δa, δb shall be determined. The actual courses of the triangulation series are numerous, and the case seems to be best met by taking the geodesic solution of Chapter III, for this in general leads to a medial path through the triangulation. In solution B it so happens that the δa and δb terms very nearly cancel one another. The form of the equations is as follows

$$(v_1\sin\lambda - w_1)\,\delta a + (v_2\sin\lambda - w_2)\,\delta b + (v_3\sin\lambda - w_3)\,u_0 + (v_4\sin\lambda - w_4)\,w_0$$
$$+A' - A - (A'_0 - A_0)\sin\lambda\,\operatorname{cosec}\lambda_0 - \sin\lambda\,(15T - L + L_0) = 0 \quad . \; . \; . \; . \; . \; . \quad (7)$$

in which $A_0' - A_0 = 1''\cdot 29 - w_0$ (*vide* Chapter I §4).

The sixteen arcs from Kalianpur give rise to sixteen equations which are exhibited in the table.

TABLE XLII.

Azimuth station	Longitude station	Coordinates of Azimuth station		Coefficients of				$A' - A$	$15T - L + L_0$	Absolute term A $-1\cdot 29 \operatorname{cosec}\lambda_0 \sin\lambda$ $+A' - A$ $-(15T - L + L_0)\sin\lambda$	Residual A	Absolute term B	Residual B
		λ	L	δa	δb	u_0	w_0						
Karachi Observatory	Karachi T. O.	24° 49′ 50″	67° 1′ 35″	+0″·060	−0″·059	+0″·168	+0″·042	− 1″·4	+ 0″·5	− 2″·9	− 1″·6	− 2″·9	− 3″·0
Dehra Dun Observatory (old)	Dehra Dun Longitude Station	30 19 57	78 3 35	−0·017	+0·018	−0·005	+0·242	−11·9	−25·7	− 0·5	− 2·5	− 0·5	− 2·3
Quetta T. O.	Quetta T. O.	30 11 57	67 0 32	+0·457	−0·459	+0·160	+0·251	− 4·4	+ 2·4	− 7·2	− 3·4	− 7·1	− 8·8
Calcutta Base-line, S. end	Calcutta	22 30 56	88 22 54	+0·118	−0·118	−0·172	−0·043	− 8·9	−11·0	− 5·9	− 5·4	− 5·9	− 5·8
Orejhar	Fyzabad T. O.	26 46 56	82 12 8	−0·085	+0·085	−0·071	+0·107	− 4·1	− 0·5	− 5·3	− 7·4	− 5·3	− 6·2
Jalpaiguri	Jalpaiguri	26 31 17	88 44 13	−0·199	+0·196	−0·172	+0·109	− 4·7	−20·4	+ 3·0	− 1·0	+ 3·0	+ 2·0
Nagarkhana	Chittagong T. O.	22 22 58	91 48 30	+0·226	−0·051	−0·227	−0·043	− 8·7	−11·7	− 5·4	− 0·1	− 5·2	− 5·1
Bolarum P.W.D. Office	Bolarum	17 30 13	78 31 11	+0·046	−0·045	−0·014	−0·257	− 1·1	− 3·5	− 1·0	+ 1·4	− 1·0	+ 0·9
Vizagapatam Base-line, N. end	Waltair	18 1 3	83 13 43	+0·268	−0·267	−0·092	−0·235	− 1·4	− 3·3	− 1·4	+ 2·7	− 1·4	+ 0·2
Karaundi	Jubbulpore T. O.	23 10 40	79 59 43	+0·017	−0·021	−0·038	−0·036	− 4·0	−10·2	− 1·2	− 1·1	− 1·2	− 1·0
Colaba Observatory	Bombay	18 53 49	72 48 49	−0·198	+0·109	+0·080	−0·199	+ 1·0	− 6·8	+ 2·2	+ 2·2	+ 2·2	+ 3·7
Deesa T. O.	Deesa T. O.	24 15 30	72 11 6	+0·003	−0·003	+0·086	+0·009	− 4·6	+ 3·6	− 7·4	− 6·9	− 7·4	− 7·4
Mangalore	Mangalore	12 52 14	74 50 43	−0·261	+0·261	+0·048	−0·434	− 2·8	− 2·0	− 3·1	− 2·2	− 3·1	+ 0·1
Bangalore Base-line, S.W. end	Bangalore	13 0 41	77 35 0	−0·007	+0·007	+0·003	−0·430	− 5·3	+ 2·9	− 6·7	− 3·4	− 6·7	− 3·6
St. Thomas' Mount Trestle	Madras	13 0 15	80 11 41	+0·233	−0·233	−0·044	−0·429	− 4·0	− 7·2	− 3·1	+ 2·4	− 3·1	0·0
Kudankulam Observatory	Nagarkoil	8 10 22	77 41 27	+0·005	−0·007	0·000	−0·615	− 7·7	+ 1·8	− 8·4	− 3·6	− 8·4	− 3·9
							Sum of squares		...	360·23	203·33	359·88	292·30
							Square root of mean square		...	4·74	3·56	4·74	4·27

The solution *A*, *i.e.* the most probable value of δa, δb, u_0, w_0, is

$$\left.\begin{aligned} \delta a &= 33\cdot 08 \text{ km} \\ \delta b &= 22\cdot 63 \text{ km} \\ u_0 &= +6''\cdot 10 \\ w_0 &= -7''\cdot 71 \end{aligned}\right\} \quad \cdots\cdots\cdots \quad A$$

to which correspond the residuals under "Residual *A*" in the table. If 0·92363 km and 0·74273 km are substituted for δa and δb (*vide* Chapter I §3) the following most probable values of u_0 and w_0 are arrived at (solution *B*) :—

$$\left.\begin{aligned} u_0 &= +1''\cdot 01 \\ w_0 &= -7''\cdot 28 \end{aligned}\right\} \quad \cdots\cdots\cdots \quad B$$

and the corresponding residuals are shown in the table under heading "Residual *B*".

12. Solution *A* is obviously of no practical use as the values of δa and δb are much larger than it is possible could be correct. Solution *B* is not unreasonable. A southerly deflection at Kalianpur has previously been inferred, the estimated amount being 4″. The value of w_0 indicates an easterly deflection of 16″·2. The value computed from the topography, but taking no account of compensation is 10″·7 (*vide* Prof. Paper 13, p. 116). The solutions however have been given more as illustrations of a principal than for their numerical values. The residuals show that the solution is not highly successful in satisfying the equations : yet the values of u_0, w_0 derived from *B* are reasonable and the residuals might fairly be attributed to observation errors.

13. *The choice of a figure of reference for the geoid.* In surveying a surface such as the geoid, in the first place of unknown form, it is necessary at the outset to decide on some figure of reference to which measurements may be referred. This figure of reference may be of any form whatever—a particular case would be any set of three orthogonal planes. In traverse operations a single plane is chosen, on the assumption that for a limited area the geoid is not much different from a plane: and it would be possible to extend the application of the plane of reference by the introduction of a third coordinate, namely that at right angles to the plane. If at each triangulation station the direction of the geoidal vertical is determined by means of latitude and longitude observations the data is sufficient to enable the position of any point to be expressed by means of its three coordinates, quite independently of the shape of the geoid. The statement of the position of numerous points on the geoid in fact determines the shape of the geoid. If however the position of the several points are referred to a figure which approximates in shape and position to the geoid, the actual shape of the geoid is much more readily grasped by the small deviations it exhibits from the well known reference figure. The choice of such a figure is extremely useful and greatly decreases the labour of calculation of the positions of points on the geoid. The closer the approximation between it and the geoid the smaller are the quantities which express the difference of the two surfaces: and, as a result, when these quantities appear in formulæ as square and power terms they may be neglected in many of the computations which arise. It is important none the less to recognise that the two surfaces cannot always be treated as identical, and to examine each case thoroughly. Moreover it is to be borne in mind that the complexity of computation will be much increased if a very complex figure of reference is selected. A balance must be struck between the two considerations, and it has been customary to adopt a spheroid. This is at the same time a comparatively simple figure for computation and also a fairly close approximation to the geoid. This choice need not at all imply that no other geometrical figure can be found which approximates more closely to the geoid. Suppose even that the geoid was in actual fact an ellipsoid (not of revolution) not very different from a spheroid. It would still be strictly accurate to refer it to a spheroid of reference:

and it is probable that this would be the simplest course to follow in dealing with the results of any one survey, for instance the Indian Survey. Or the geoid might be referred with strict accuracy to a sphere: but in this case the residuals in a vertical direction might be inconveniently large.

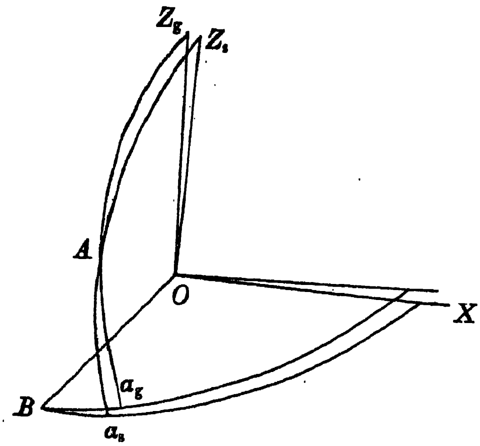
14. This does not appear to have been quite the point of view usually taken, seeing that much energy has been devoted to finding the spheroid which best fits the whole earth. The origin of this research was doubtless the desire to uphold the Newtonian theory that the earth, being a revolving gravitating mass, should approximate in form to an oblate spheroid: rather than to the prolate spheroid which early measurements led the French school to believe in. This question was finally settled in favour of the Newtonian theory by the measurements of the arcs in Peru and Lapland: and the matters now to be investigated are the relatively minor deviations of the geoid from the oblate spheroid. Given a ready means of converting coordinates from one spheroid to another, each survey may properly select the spheroid most suitable to its own requirements. In any case the several large surveys of the world are expressed in terms of different spheroids, and for purposes of intercomparison it is necessary to develop a method of changing from one spheroid to another. An interesting question is to consider how closely the several spheroids, which best fit the respective surveys, agree *inter se*: to account for any differences: and to see whether a theory of density distribution can be found which will bring all these spheroids into agreement. The same question may be considered by taking the surveys on the spheroids they happen to have been reduced on and afterwards expressing the results in terms of a single spheroid and the local differences of the geoid from this general spheroid. Even if this general spheroid is so selected as to make the differences from the geoid a minimum it still remains only a convenient figure of reference and a more or less close approximation to the geoid.

15. In the case of triangulation the usual procedure is as follows: horizontal angles are measured on the geoid, that is to say a theodolite is set up and levelled so that its horizontal circle is tangential to a level surface of the geoid. Spheroidal excess, calculated from the assumed spheroid, is applied to these angles. Further computations of the latitude and longitude of the points of triangulation are then carried out as though the spheroid and geoid were identical.

Now in certain disturbed districts the geoid is of considerably different curvature from the adopted spheroid: and the excess over 180° of the sum of the three angles of a triangle observed on the geoid is not the same as that computed from the spheroid. On account of the relative smallness of this excess in triangles of the size which occur in triangulation, this difference is not of great importance, though it gives rise to the two entirely different methods of Chapters III and IV. But if the rays observed have a considerable elevation, such as 5°, a very appreciable error is introduced, as will shortly be explained. It is necessary to be more precise. The most natural way of relating a point on the geoid to the spheroid is by giving the coordinates (latitude and longitude) of that point of the spheroid the normal—or more strictly for large distances the orthogonal confocal hyperbola— at which passes through the point on the geoid; and by stating the height of the geoid above the spheroid measured along this normal as well as the angle between this normal and the normal to the geoid (deflection of the plumb-line) and the azimuth of the plane containing the two normals.

Defining the position of a geoidal point in this way for the present, the separation of the geoid and spheroid need not be considered. To each point on the geoid there is a corresponding point on the spheroid: and consequently to each geoidal triangle a spheroidal triangle corresponds. It is with such spheroidal triangles that computations of latitude and longitude etc. really deal, the formulæ being deduced from properties of the spheroid. Consider then the relation between the angles of a geoidal triangle and the corresponding spheroidal triangle.

16. Suppose a theodolite is set up at a point O and levelled in the ordinary way: at this point two zeniths may be distinguished, Z_g that of the geoid and Z_s that of the spheroid, the former being indicated by the direction of the theodolite when the altitude is set to 90°.



The plane Z_gOZ_s is the plane of deflection and the line OB at right angles to this plane is parallel to both spheroid and geoid and is chosen as axis of Y: so that OB may be regarded indifferently as belonging to the spheroid or the geoid. Consider another point A and draw the great circles $Z_s\,A\,a_s$ and $Z_g\,A\,a_g$. Then $O\,a_s$ and Oa_g are the traces of the ray OA on the spheroid and geoid respectively. Suppose that the horizontal angle between OA and OB is required. Observation by the theodolite gives the angle $a_g\ OB$: but the angle required for computation on the spheroid is $a_s\ OB$. Denote by a the geoidal angle of elevation of A and by z the azimuthal angle X_gOa_g, corresponding quantities for the spheroid being $a+\delta a$, $z+\delta z$. From triangles $Z_g\,BA$ and $Z_s BA$

$$\cos AB = \sin a \cos Z_gB - \cos a \sin Z_gB \cos(z_g-90^\circ) = \sin(a+\delta a)\cos Z_sB - \cos(a+\delta a)\sin Z_sB\cos(z_s-90^\circ)$$

But $Z_gB = Z_sB = 90^\circ$; hence

$$\cos a \sin z = \cos\ (a+\delta a)\ \sin\ (z+\delta z) \quad \ldots \quad (1)$$

i.e.

$$\tan a.\,\delta a = \cot z.\,\delta z \quad \ldots \quad (2)$$

neglecting second order terms.

Now δz, δa are the corrections which should be applied to geoidal quantities to correct them into spheroidal quantities and make them suitable for spheroidal formulæ. δa is $Aa_s - Aa_g$ and is approximately $\epsilon \cos z$, where ϵ is the total plumb-line deflection which is in the plane $OX_g\,Z_g$: so that (2) may be written

$$\delta z = \epsilon \tan a \sin z \quad \ldots \quad (3)$$

The correction δz accordingly is greatest when $z=90°$, that is when the observed object is in the plane of no deflection, and its magnitude in this case is $\epsilon \tan a$. Now values of ϵ up to one minute have been observed: and if at the same time a ray of elevation of 4° is observed, the horizontal angle may need a correction of 4 seconds—a very appreciable quantity in geodetic triangulation. The figures here given are roughly applicable to a ray through Jharipani (Dehra Dun district) where the deflection exceeds one minute and considerable angles of depression occur. In the case of a triangle at two of whose corners there is no deflection while a considerable deflection occurs at the third, a large triangular error will be apparent. More usually however the deflection is not so widely different at the three corners and the angular errors partially compensate one another in the sum, thus masking the error, but leaving the triangle distorted.

17. It is of interest to note that deflections have a corresponding effect on the measurements of base lines. Suppose that an element ds is measured along a line inclined at an angle a to the geoidal horizontal and $a+\delta a$ to the spheroidal vertical. Its reduced length is generally taken as $ds \cos a$, whereas reduced to the spheroid it is $ds \cos(a+\delta a)$, so that a correction

of $-ds \sin\alpha.\delta\alpha$ is required. The error on the whole line is $\int ds.\sin\alpha\,\delta\alpha$. This is equal to $s\sin\alpha_m\,\delta\alpha_m$ where α_m and $\delta\alpha_m$ are values which occur at some part of the line. If α is fairly constant $s\sin\alpha$ is approximately the difference of level of the two ends of the base and the error is approximately $(h_2-h_1)\,\delta\alpha_m$. Owing to h_2-h_1 being small compared with the length this is only liable to affect the length by a quantity of as much as 1 in 10^6 in extreme cases.

18. Deflections are usually stated in terms of their westerly and southerly components, ξ, η. It is clear that the effect of either component on a ray can be computed independently and then the two results combined. In the case of a ray of azimuth A it follows from (3) that a correction to the geoidal azimuth of amount δA is required where

$$\delta A=(-\xi\cos A+\eta\sin A)\tan\alpha \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Consider now the case of a traverse. Denote the successive points by 1, 2, 3 . . n: let α_n be the angle of elevation of $n+1$ from n and let β_n be the elevation of $n-1$ from n. Also let A_n be the azimuth of $n, n+1$ and B_n that of $n, n-1$ and c_n be the arc subtended at centre of earth by $n\ n+1$

$$\left.\begin{array}{ll}\text{Then} & \alpha_n+\beta_{n+1}=-c_n \\ \text{and} & A_n=\beta_{n+1}+180^\circ-K_n\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where K_n is the convergency.

This traverse may be regarded as the flank of a series of triangulation: and in proceeding along it, the accumulation of azimuth error will be estimated. Now the flank of a triangulation series may, without much loss of generality be considered to proceed along a great circle of the earth (or a geodesic to be more precise). The great circles on the earth which are most conveniently considered are the meridians: but it is clear that by changing the system of coordinates to which points are referred any great circle may be regarded as a meridian of a different system of coordinates. It will accordingly be sufficient to consider the case of a meridian (not necessarily one of the system with the axes of rotation as pole). Along such a meridian the azimuthal angle A is zero or 180°. Suppose then that the traverse 1 2 3 . . n lies on this meridian and that μ_n is the component of the plumb-line deflection at n in a direction perpendicular to this meridian (but not necessarily east and west as the meridian may be any great circle).

The correction to the angle at n will now be

$$\delta C_n=\mu_n\,(\tan\alpha_n+\tan\beta_n) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which may be written sufficiently accurately for the present purpose

$$\delta C_n=\mu_n\,(\alpha_n+\beta_n) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

since α_n and β_n seldom if ever are so large as 5° in triangulation of a geodetic kind.

Let δh_n be the height of $n+1$ above n: then very approximately, if R is the radius of the earth

$$\delta h_n = Rc_n\,(\alpha_n+\tfrac{1}{2}c_n) = -Rc_n\,(\beta_{n+1}+\tfrac{1}{2}\,c_n) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$\delta C_n = \frac{\mu_n}{R}\left(\frac{\delta h_n}{c_n}-\frac{\delta h_{n-1}}{c_{n-1}}\right)-\tfrac{1}{2}\,\mu_n\,(c_{n-1}+c_n) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The accumulated azimuth error of the side $n, n+1$ is accordingly C_n where

$$C_n = \frac{1}{R}\,\Sigma\,\frac{\delta h_n}{c_n}\,(\mu_n-\mu_{n+1}) - \tfrac{1}{2}\,\Sigma\mu_n\,(c_{n-1}+c_n) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$= {}_1C_n + {}_2C_n.$$

Some attention to detail of the limits of this summation is necessary to obtain the precise value in a particular case: but the present object is to discuss the accumulation of the error and so this detail need not be considered now. It is clear that the first expression on the right hand side of (10) is not liable to great increase: for δh_n is equally likely to be positive or negative

as also is $\mu_n - \mu_{n+1}$ (considering that μ_n is the deflection at right angles to the line $n, n+1$). The most probable value of the expression ${}_1C_n$ is

$$\frac{\sqrt{n}}{R} \cdot \frac{\delta h}{c} (\mu - \mu')$$

where $\frac{\delta h}{c}$ and $\mu - \mu'$ are values intermediate to the extreme values met with. To get an idea of the magnitude which this might reasonably reach after 25 sides put $\frac{\delta h}{cR} = \frac{1}{50}$ corresponding to an angular elevation of more than 1° and $\mu - \mu' = 10''$. The value is then $5 \times \frac{1}{50} \times 10'' = 1''$. Now angular elevations of 1° are average, but changes of deflection of as much as 10″ in the distance between two stations are not usual, though occasionally much bigger changes occur. It is felt then that the estimate of 1″ for 25 rays is fair and that the danger of accumulation of error from this term is not considerable. The second term of (10) remains and its magnitude is liable to be somewhat greater. It may be written

$${}_2C_n \doteq -\mu_m \Sigma c$$

where μ_m is some value intermediate to the extreme values of μ met with: Σc is merely the whole arc subtended by the terminal stations at the earth's centre. Taking $\Sigma c = \frac{1}{10}$ radian which corresponds to a series about 400 miles long we get

$${}_2C_n = -\tfrac{1}{10}\mu_m$$

In a series along the first range of the Himalayas deflections at right angles to this range of as much as 40″ are of common occurrence. If $\mu_m = 40''$

$${}_2C_n = -4''$$

Now this is an error of magnitude about what might possibly occur in a single angle at which $\mu = 60''$ and $\tan a = \frac{1}{15}$: so that the conclusion may be drawn that the danger of failing to correct observed angles for deflection of the plumb-line is almost confined to the angles themselves and is not liable to produce a cumulative error of azimuth, if the angles were utilised as in a traverse. The effect however may be felt in a way different from that considered above owing to the distribution of triangular error. Each triangle which contains a station where the deflection differs considerably from those at the other stations is liable to be deformed when the angles are adjusted to equal two right angles plus the spherical excess calculated on the spheroid. The amount of this deformation and the effect on computed coordinates of the stations of the triangulation do not appear to be such as can be estimated for a general case. Its effect in the actual triangulation of India is mixed up with the effect of error of observation and its amount is in general considerably less, as appears from the solution of the modified Laplace equations given above in §§ 10—12.

19. In observations for azimuth the result of using a point at considerable angular elevation about the station of observation as a reference point seems to have always been ignored. Yet the same geometrical fact which causes the horizontal trace of the ray through the pole to be displaced in azimuth also gives rise to an azimuthal deflection of the horizontal trace of the ray through the reference point, unless it happens that the ray is in the azimuth at right angles to that of geoidal deflection. Suppose that at any point the southerly and westerly deflections are η, ξ respectively. The horizontal (spheroidal) trace of the ray through the pole and geoidal zenith will be deflected in azimuth by $+\xi \tan \lambda$, λ being the altitude of the pole. The horizontal trace of a ray in azimuth A and angular elevation a will be deflected by

$$-\tan a\,(\xi \cos A - \eta \sin A) \qquad \textit{vide } (4)$$

The difference between astronomic and geodetic azimuth is accordingly

$$\xi \tan \lambda + (\xi \cos A - \eta \sin A) \tan a$$

instead of the simpler expression $\xi \tan\lambda$ usually taken, which is more closely approximated to as a decreases.

20. The advantages and disadvantages of the methods of correction worked out in Chapters I, III, IV may now be considered. The method of Chapter I in which u_y and v_x are taken as the changes of latitude and longitude has the justification and weaknesses referred to on pages 10-12. In a triangulation system where the bulk of the triangulation is along parallels and meridians this solution would be satisfactory were it not for the azimuths. The azimuth computed by the corrections at the ends of a ray of triangulation along a parallel differs by an appreciable amount from those for a ray along a meridian, and at first sight it appears that the difference is the necessary correction to the angle contained by these two rays. This however is not satisfactory as it is clear that the longitudinal and meridional series must be a little bent and that the whole error should not be forced into the junction angles. It would be equivalent to putting all the angular closing errors of a traverse which followed approximately the sides of a rectangle into the four angles at the corners of the rectangle. Moreover the final azimuth will not agree with the longitude as laid down in Laplace's equation. Laplace's equation might be adopted as a mode of determining the azimuth changes: but obviously the result would be inconsistent with the latitude and longitude changes found viz. u_y, v_x. In fact it appears that this method could rightly be applied merely to the junction points of the triangulation series. After changes for these points had been found, the corresponding changes along the series might be adjusted as is done in closing a traverse. This would involve a consideration in detail of all the series and has the disadvantage of being most laborious, and when done it is inconvenient in that the solution for the case of a further change of axes would have to be taken up right from the beginning. It might be supposed to be advantageous in that it takes cognisance of the actual form of the triangulation: but seeing that it is based on a method which is not entirely justifiable there seems to be little advantage in this partial approach to accuracy in the final stages of the reduction.

21. The method of computation along the geodesics at once gets rid of the difficulty of dual values of the changes. It is obvious however that the values obtained for the changes vary according as one origin or another is selected: for the closing errors in a triangle formed by joining any point and two selected origins exist just as much here as in the first method. This closing error however does not occur all at one point as in that method, but is satisfactorily distributed. There is some trouble in computing the geodesics: but this is of minor importance seeing that it has been done once for all and correction tables have been made out from which the coordinates of any point may be deduced by interpolation. These tables permit of the changes due to any desired changes in the elements a, b and latitude and azimuth at the origin being made immediately and admit of further changes being subsequently made when this becomes desirable.

Both this and the first method are based on the idea of the accuracy of the ratios of the sides in the triangulation: this is almost independent of small changes in the spheroid and the consequent minute changes in the spherical excess of any observed triangle. It may be noted here that in the case of an equilateral triangle with observed angles of equal weight the ratios are unaffected by the amount of spherical excess as this would be distributed equally. But the angles of the geoid have been used in place of spheroidal angles and from this some disturbance must have arisen.

While then the ratios of the sides may be regarded as practically perfect so far as corrections due to size of spheroid are concerned, it must at the same time be remembered that the observation errors have a cumulative effect on the ratio of a side to the original base as the side considered is separated more widely from the base: and the treatment of the observed angles as applicable to the spheroid without correction will aggravate this. The magnitude of the errors

so developed is indicated where closure has been made on additional base lines. It is clear that in a network of triangulation these additional bases may be reached from the origin by various routes and the length of these routes must accordingly be duly considered.

The following figures are taken from the circuits and base-lines of the N.W. Quadrilateral* in which there are 5 circuits and 4 measured bases.

TABLE XLIII.

(1) Number of equation	(2) Logarithmic closing error $\times 10^6$	(3) Number of triangles	$(2)^2 \div (3)$
1	4·40	51	0·380
2	6·82	96	0·465
3	7·19	36	1·438
4	7·96	95	0·666
5	16·38	123	2·180
6	12·46	185	0·841
7	15·09	88	2·586
8	0·53	138	0·002
			Sum $=8·558$
			Mean $=1·070$
			Square root of Mean $=1·035$

It appears that the mean error per triangle in side ratio is 1·035 in the 6th place of logs which corresponds to an error of one part in 420,000 showing that a high order of accuracy has been attained.

22. In the third method, that of geometrical transformation, the idea of the spheroid as merely a figure of reference is used as a basis for the argument. It is free from the difficulties of multiple values, one for each route traversed, and gives a definite set of values for the changes at any point. Being geometrically correct it naturally satisfies the Laplace condition: but it does not keep the constancy of side ratios, though the departure from constancy is not serious. No attempt is made to correct for the distance between geoid and spheroid which in conjunction with large deflections such as have so far been discovered would make very small changes in the coordinates. It is to be remembered however that the original geoidal triangles have been applied without angular correction to the old spheroid of reference, although considerable corrections must have been necessary in some cases. To put this matter right now, deflections at many stations would need to be observed: and to make use of the information that might be gained by observation, it would be necessary to re-grind the whole triangulation of India. It is the object of the present investigation to avoid this immense piece of work: but as has just been pointed out, it could not be undertaken until many deflection observations had been made. Had the corresponding corrections been made in the first instance, which would have been possible if comparatively rough latitudes and azimuths had been observed at each triangulation station, the method of change of coordinates explained in Chapter IV would have been absolutely correct. The fact that this was not done is the source of the present difficulty and

* *Vide* G.T. Volume II of the Survey of India, pages 303, 304.

practically disposes of the usefulness of this method. Further the Laplace equations should by right have been applied in the original grinding: they were not. This omission also makes an objection to any method of computing short of regrinding. And so the geometrical accuracy of the method of Chapter IV is vitiated. It is useless to insist on a method which strictly accords with Laplace's equations when the original quantities which are to be corrected fail to satisfy those equations.

23. As remarked above in §7 the reason of the multiple values of u, v, w according to traverse route followed is that in the computations no attempt has been made to correct observed geoidal angles to angles on the particular spheroid which is selected as a reference figure. Had these corrections been applied the method of geometrical change explained in Chapter IV would have given the changes u_r, v_r, w_r which would then have been applicable on changing from one spheroid of reference to another. But seeing that no such corrections were applied, and that the closing errors of circuits were dispersed and treated as errors of observations it is clear that this method is not strictly applicable. The portion of the closing errors due to this lack of correction to the angles is small compared with those due to errors of observation: so in the main no great fault was committed. A greater fault was the neglect of closing on the longitude arcs, or in other words applying Laplace's longitude equation. What is at first sight naturally regarded as a defect of the methods of Chapters I and III is that equation (4) of this chapter is not satisfied. But when it is considered that Laplace's condition, of which equation (4) is an immediate consequence, was not enforced on the computation of the original triangulation, it is clear that there is nothing to be gained by now enforcing equation (4) on to the small changes to be applied an account of change of spheroid. A preferable course is to make these changes and then apply Laplace's condition to the final result as Sir Sidney Burrard has done in his discussion of the Indian azimuth observations. It is concluded then that the two objections to the methods of Chapters I, III cited above have little weight in view of the slight inaccuracies of method by which the Indian triangulation has been reduced. It remains then to decide merely on what route should be followed in deducing the changes of coordinates by the method of Chapter I. This method is applicable to any route if the "closing errors" are applied as explained at the end of that chapter: and in Chapter III although the results are obtained in a special way, yet these results might have been obtained by the method of Chapter I. It is clear that no route can be laid down as rigorously correct and that the best that can be done is to select a route which appears to be the best. Suppose there were four triangulation series all of equal merit forming a square. Then the route which should be followed from one corner to another is the diagonal, and this produces a result intermediate to those which would be found by following either pair of sides. If one pair of sides was distinctly better triangulation than the other, the best route would doubtless be one closer to the good sides than the bad sides. But in the case of a great network of triangulation it is too complicated to go into such detail and so the diagonal would be selected. Now the geodesic corresponds fairly closely in the case of a spheroid to the diagonal of any square or rectangle, and it gives a satisfactory medial path among the triangulation and medial results for the changes deduced. This choice is slightly arbitrary, but seems the best that can be made. Another arbitrary choice is that of the origin from which changes are computed on the point from which all the selected geodesics radiate. It is apparent from the theory of the "closing errors" that different values would be deduced for the change according as this central point is selected; but the differences are not really appreciable in comparison with the errors due to faulty observation: Kalianpur is very centrally situated as regards India, and as it is the origin of the triangulation it appears that it would merely be an unnecessary complication to select a slightly different point for the point from which the geodesics radiate. It would be useless to go into any great refinement as to

the theoretically best centre for this purpose because it would be constantly disturbed as new triangulation is added to the Indian system.

It appears then that, without there being any rigidly accurate reason for adopting the method and results of Chapter III, yet that it meets the present case quite satisfactorily and has no defect of appreciable magnitude: and that any defect of theoretical precision would be present in any alternative method which might be proposed. The conclusion quoted at the end of Chapter IV is accordingly reiterated, namely that **the method of calculation along geodesics through Kalianpur as set forth in Chapter III is the correct one to use.**

CHAPTER VI.

Strength and Adjustment of Triangulation. Mechanical Analogy.

A criterion of strength of triangulation series.

1. If a mechanical network, which is analogous to a triangulation series in the sense explained in § 10 below, replaces each series in a system of triangulation a mechanical framework is formed. Each mesh in this framework corresponds to a circuit in the triangulation and needs straining to close in a way exactly analogous to the need of adjustment of triangulation circuits. Now in general in Indian triangulation, series follow approximately straight lines and these are generally more or less along meridians or parallels. Consider four series which form a circuit *A B C D*, and their mechanical analogues. To effect closing at *A* in the mechanical framework strains must be applied: and it seems fairly clear that the strains which will be caused in the side *A B* will be of the same nature all along this side, but will differ essentially from those caused in *B C*. An example would be that the strains in *A B* would be such as to increase the length *A B* while those in *B C* would be to slightly curve *B C*. On this account it appears desirable to consider strains of a particular type as existing throughout *A B*: but not existing to the same extent in *B C*. The side *A B* is thought of as having uniform strength which differs from the uniform strength of *B C*. Reverting to the triangulation series it may be remarked that in one series the same strength is aimed at throughout, angles being observed with similar precision and figures of the same type selected as far as possible. When topographical conditions change entirely, as must essentially occur on the passage from plain to hilly country, the series should be considered in sections.

In considering one series of a circuit, it is only necessary to think of a "route" formed by those sides which persist in the general direction of the series, but bearing in mind that the length of each side is expressed in terms of the previous side, and that in any adjustment its relative length to this previous side is the quantity which is to be slightly varied; and similarly its azimuth is relative to the previous side. It will not be very far from the truth in effect if these several sides are for simplicity regarded as of equal length l and practically in the same direction. Suppose the angle between the r^{th} and $\overline{r+1}|^{\text{th}}$ side, originally practically 180°, is changed by the small angle η_r: and that the ratio of the length of the $\overline{r+1}|^{\text{th}}$ side to the r^{th}, originally unity, is changed to $1+\epsilon_r$ Expressed in terms of the base, the r^{th} side changes in length in the ratio $\prod_1^r (1+\epsilon)-1 \doteqdot \sum_1^r \epsilon$ and in direction by $\sum_1^r \eta$. These quantities ϵ and η may be chosen to give any possible small changes in the length and azimuth of the terminal side of the series. Consider the displacements in the terminal

point $r+1$. That in the direction of the series is clearly

$$l\epsilon_1 + l\,(\epsilon_1+\epsilon_2) + \quad . \quad . \quad . \quad . \quad + l\,(\epsilon_1 + \quad . \quad . \quad . \quad \epsilon_r) = \frac{l}{r}\left(r\epsilon_1 + \overline{r-1}|\,\epsilon_2 + \quad . \quad . \quad . \quad . \quad . \right)$$

where s is the length of the series and accordingly $s = rl$.

The most probable value of this is

$$\frac{s\epsilon}{r}\left(r^2 + \overline{r-1}|^2 + \quad . \quad . \quad . \quad . \quad . \right)^{\frac{1}{2}} = \frac{s\epsilon}{r}\sqrt{\frac{r\,(r+1)\,(2r+1)}{6}} \doteqdot s\epsilon\sqrt{\frac{r}{3}} \quad . \quad . \quad . \quad (1)$$

if r is large, ϵ being the most probable value of any of the quantities ϵ_r. Similarly the displacement of the terminal point r in the direction at right angles to the series is

$$l\eta_1 + l\,(\eta_1+\eta_2) + \quad . \quad . \quad . \quad . \quad . \quad \doteqdot s\eta\sqrt{\frac{r}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Both the quantities ϵ and η depend on the probable error of an angle (adjusted by the triangular conditions) in the series. General Ferrero introduced the quantity "m" as a criterion of triangulation,

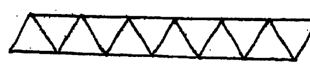
where
$$m = \sqrt{\frac{\Sigma\Delta^2}{3\,n}}$$

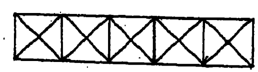
Δ being the triangular error of any triangle and n the number of triangles considered. This quantity "m" is accordingly the error of mean square of one angle of a triangle. The probable error of an observed angle is $\cdot 6745\ m$. The probable error of an angle adjusted to satisfy the triangular condition is

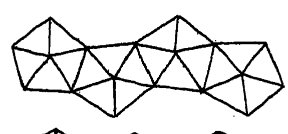
$$a = \sqrt{\tfrac{2}{3}} \times \cdot 6745\ m = \cdot 551\ m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

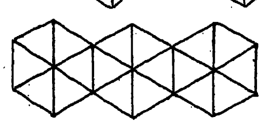
The formulæ (1) and (2) may accordingly be expressed by saying that the probable displacement of the terminal point of a series of given length in any direction varies as $m/\sqrt{l}$.

They accordingly show the advantages of figures with long sides. It remains to consider the effect of various types of figures in the series, simple triangles, braced quadrilaterals, central pentagons, hexagons etc. Only regular figures are considered and the sides of each are taken equal to l. To complete a series of each type of given length s suppose there are $n_3\ n_4\ n_5$ etc. figures, simple triangles, braced quadrilaterals, pentagons, etc. It is clear that the gains in distance in the required direction are


Triangles


Quadrilaterals


Pentagons


Hexagons

$n_3 l \cos 60°$	in the case of simple triangles
$n_4 l$	. . . . quadrilaterals
$n_5 l\ (\cos 18° + \frac{1}{2}\cos 54°)$	. . . . pentagons †
$2 n_6 l\ \cos\ 30°$	. . . . hexagons

and as these must all be equal to s

$$\therefore \quad \tfrac{1}{2}\,n_3 = n_4 = \frac{n_5}{1\cdot 245} = \frac{n_6}{1\cdot 732}$$

Now the probable errors in the determination of a terminal side after a given number of figures of each of the kind mentioned are in the ratio

$$0\cdot 82 \ : \ 1 \ : \ 1\cdot 21 \ : \ 1\cdot 29\ *$$

so that at the end of each series *of the same length* the errors are in the ratio

$$0\cdot 82\sqrt{n_3} \ : \ \sqrt{n_4} \ : \ 1\cdot 21\sqrt{n_5} \ : \ 1\cdot 29\sqrt{n_6}$$

which reduce to
$$1\cdot 17 \ : \ 1 \ : \ 1\cdot 08 \ : \ 0\cdot 98$$

for the cases of triangles, quadrilaterals, pentagons and hexagons respectively.

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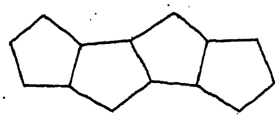
* *Vide* Account of the Operations of the G. T. Survey of India, Vol. II, p. 199.

† An alternative arrangement of the pentagon  gives a result practically the same.

Suppose there is a series composed of α simple triangles, β braced quadrilaterals, γ pentagons and δ hexagons, then the ratio of its terminal probable errors to those of a series of the same length composed of quadrilaterals is

$$\sqrt{\frac{\overline{1\cdot17}|^2\alpha + \beta + \overline{1\cdot08}|^2\gamma + \overline{\cdot98}|^2\delta}{\alpha + \beta + \gamma + \delta}} : 1$$

which may be approximately written $1+f : 1$

where
$$f = \tfrac{1}{12}\cdot\frac{2\alpha+\gamma}{\alpha+\beta+\gamma+\delta} \quad \ldots \ldots \quad (3)$$

Heptagons, nonagons etc. occur rarely and may be treated as pentagons. Octagons, decagons etc. may be treated as hexagons. Combining this result with (1) and (2) the quantity

$$M = (1+f)\, m \sqrt{\frac{18}{l}} \quad \ldots \ldots \quad (4)$$

is formed in which 18, the average length in miles of sides in the Indian triangulation, is introduced, and "m" is General Ferrero's expression for error of mean square of an angle and l is the average length of side expressed in miles in the series under consideration.

2. This quantity M takes cognizance not only of the probable error of the angles in the triangulation but also of the length of side and type of figure. For a given length of triangulation it gives a relative idea of the errors likely to occur in series of different precision and type: for example if there are several series of the same length, say 300 miles each, for which values M_1, M_2, M_3 . . . have been found by (4), then the probable errors of northing or easting of the terminal point are approximately in ratio $M_1 : M_2 : M_3$ . . . and the same is true of the probable errors of length or azimuth of the terminal side. "M" gives a criterion of the value of triangulation considering in proper proportion the excellence of observation and the success in chosing well-proportioned figures which has been attained: "m" only gauges the excellence of observation.

The deduction of the quantity M is confessedly based an approximations and simplifications. It would not be expected to be very accurate if applied to badly conditioned figures, and it is not intended that this should be done. In geodetic triangulation such figures are exceptional and figures approximately symmetrical largely predominate: and in these cases M is a practically useful criterion of the excellence or strength of the series.

3. All the triangulation of India has been classified according to values of M (*vide* table XLIV) and the order of merit of the several series deduced. The series are arranged in chronological order and designated by a serial number. Reference to any series can generally be made more conveniently by use of its serial number than by the rather long and frequently artificial names which have been applied. A consideration of the list shows that the principal and secondary triangulation ranges fairly continuously from very high class work in the best of which No. 76 North Baluchistan Series $m'' = 0\cdot221$ and $M = 0\cdot17$; to the least successful secondary triangulation No. 65 Siam Branch in which $m = 3''\cdot711$ and $M = 4\cdot34$. The mean square (*vide* note at foot of table) value of M for the triangulation which was utilised in the grinding of the Indian network is $1\cdot04$: that for the whole triangulation $1\cdot51$. In some cases so called secondary triangulation proves better than poor principal triangulation: in general there is no marked gap between the two classes. This classification of triangulation into principal and secondary is accordingly dropped after the completion of the series and both are classed as "geodetic" triangulation and placed according to the values of M yielded by them. The further distinction in Indian triangulation is between "geodetic" and "minor" triangulation. The former is always rigorously computed taking account of spherical excess. The latter, which is generally very much rougher, disregards spherical excess.

TABLE XLIV.

Values of "*m*" and "M" for all Geodetic Series of the Indian Triangulation.

No	Name of Series	Seasons	$\pm m$	l	Number of Independent Figures: 3-sided	4-sided	5-sided	6-sided	7-sided	8-sided	9-sided	10-sided	11-sided	12-sided	Compound	f	$\pm$M	Order of Merit
1	South Pārasnāth Mer.	1831-39	3·308	24·0	6	...	...	...	...	...	...	...	...	...	2	·139	3·26	92
2	Budhon Meridional ...	1833-43	2·242	19·2	25	1	2	4	...	...	...	...	...	...	...	·135	2·46	86
3	*Amūa Meridional* ...	1834-38	1·647	18·9	34	...	...	...	...	...	...	...	...	...	...	·167	1·88	77
4	*Rangīr Meridional* ...	1834-64	1·643	20·6	32*	...	...	...	...	...	...	...	...	...	...	·167	1·79	72
5	*Calcutta Longitudinal*	1834-69	0·369	26·6	...	...	4	2	2	2	1	1	1	...	...	·051	0·32	8*t*
6	*Great Arc Meridional, Section 24°-30°* ...	1835-66	0·708	22·2	10	...	2	2	...	...	...	...	...	1	4	·109	0·71	36*b*
7	*Bombay Longitudinal*	1837-63	0·844	27·6	1	1	2	2	...	...	...	...	...	...	2	·077	0·74	38
8	*Great Arc Meridional, Section 18°-24°* ...	1838-41	0·567	21·3	17	3	4	...	...	1	...	...	...	...	3	·118	0·59	28*b*
9	*Great Arc Meridional, Section 8°-18°* ...	1840-74	0·390	23·7	1	4	2	5	3	1	...	...	...	...	3	·054	0·36	13*b*
10	*Singi Meridional* ...	1842-62	1·187	24·9	18	3	...	...	...	...	...	...	...	...	2	·131	1·14	51
11	*South Konkan Coast*	1842-67	2·176	29·6	16	3	...	...	...	...	...	...	...	...	...	·140	1·93	79
12	*Karāra Meridional* ...	1843-45	1·507	16·4	21	1,‖1	...	...	...	...	...	...	...	1	...	·146	1·81	73
13	*North Malūncha Mer.*	1844-46	1·266	18·4	6	...	...	1	2	...	...	...	...	...	...	·130	1·42	60
14	*Chendwār Meridional*	1844-69	0·841	15·1	17	1	1	1	...	...	...	...	...	...	...	·146	1·06	45
15	*Gora Meridional* ...	1845-47	0·973	15·6	23	...	...	...	...	...	...	...	...	...	1	·161	1·21	52*b*
16	*Calcutta Meridional*...	1845-48	1·173	8·5	45	...	...	...	...	...	...	...	...	...	...	·167	1·99	82
17	South Malūncha Mer.	1845-53	1·606	15·7	11	2	...	...	...	...	...	...	...	...	...	·141	1·97	81
18	*Khānpisura Meridional*	1845-62	1·227	27·3	...	4	1	1	2	...	...	...	...	...	1	·071	1·07	46
19	*Gurwāni Meridional*...	1846-47	1·165	13·8	32	...	...	...	...	...	...	...	...	...	...	·167	1·55	65
20	*North-East Lon.* ...	1846-55	0·446	11·1	96	2	1	3	...	1	...	...	...	...	...	·156	0·65	31*b*
21	*Hurīlāong Meridional*	1848-52	1·502	14·3	20	...	...	1	...	...	...	...	...	...	1	·142	1·92	78
22	*North-West Himalaya*	1848-53	0·641	25·3	...	6	1	2	...	...	...	...	...	...	3	·021	0·55	26
23	*Gurhāgarh Meridional*	1848-62	0·914	13·6	70	...	1	2	...	1	...	...	...	...	1	·157	1·21	52*b*
24	*East Coast* ...	1848-63	0·608	16·9	22	6	...	3	...	1	...	...	...	...	2	·116	0·70	34*b*
25	*Karāchi Longitudinal*	1849-53	0·558	15·8	...	10	2	10	2	2	...	...	...	...	1	·015	0·60	30
26	*Abu Meridional* ...	1851-52	0·617	15·9	1	...	...	3	...	...	...	...	...	...	...	·042	0·68	33
27	*North Pārasnāth Mer.*	1851-52	0·895	12·6	20	...	...	...	...	...	...	...	...	...	...	·167	1·25	54
28	*Kāthiāwār Meridional*	1852-56	0·990	17·4	7	3	1	1	...	...	...	...	...	...	3	·101	1·11	49
29	*Gujarāt Longitudinal*	1852-62	0·859	14·2	31	...	2	...	...	...	...	...	...	...	...	·157	1·12	50
30	Kāthiāwār Minor Lon.	1853	1·481	23·1	1	5	...	...	...	...	...	...	...	...	...	·028	1·34	58
31	Sābarmati Secondary	1853-54	1·348	5·3	15	2	...	...	...	...	...	...	...	...	...	·147	2·84	88
32	*Great Indus* ...	1853-61	0·359	12·7	...	9	1	12	2	2	...	3	...	1	...	·008	0·43	20*b*
33	*Rahūn Meridional* ...	1853-63	0·327	14·1	...	2	...	15	2	...	...	...	...	...	...	·009	0·37	15*t*

Mer. = Meridional. Lon. = Longitudinal. * One centred △. ‖ Centred Quadrilateral.

TABLE XLIV.—(Contd.)

Values of "*m*" and "M" for all Geodetic Series of the Indian Triangulation.

No.	Name of Series	Seasons	$\pm m$	l	Number of Independent Figures: 3-sided	4-sided	5-sided	6-sided	7-sided	8-sided	9-sided	10-sided	11-sided	12-sided	Compound	f	$\pm$M	Order of Merit
34	*Assam Longitudinal* ...	1854-60	0·579	12·5	...	2	3	4	...	1	...	...	...	...	...	·025	0·71	36*b*
35	*Cutch Coast* ...	1855-58	0·986	12·5	...	5	6	...	...	1	...	...	...	...	1	·074	1·27	55*b*
36	Kashmīr Principal ...	1855-60	0·884	19·2	...	18	1	1	...	...	...	...	...	...	...	·004	0·86	40
37	*Jogī-Tīla Meridional*	1855-63	0·481	12·3	...	...		7	1	1	...	...	...	...	...	·009	0·59	28*b*
38	Sambalpur Lon. ...	1856-57	0·806	19·3	7	2		1	...	...	...	...	...	...	...	·117	0·87	41
39	(Cutch) Coast Line Sec.	1856-60	0·975	10·8	33	...		...	...	...	...	...	...	...	...	·167	1·47	61
40	Kāthiāwār Minor Meridional No. 1 ...	1858-59	0·930	9·1	13	1	...	...	...	...	...	...	...	...	...	·155	1·51	64
41	Kāthiāwār Minor Meridional No. 2 ...	1859-60	1·247	12·1	14	1	...	...	...	...	...	...	...	...	1	·145	1·75	70
42	Kāthiāwār Minor Meridional No. 3 ...	1859-60	0·969	10·0	17	3	1	...	...	...	...	...	...	...	...	·139	1 48	62
43	*Bider Longitudinal* ...	1859-72	0·311	22·0	...	1,‖1	2	1	...	3	...	...	...	...	1	·064	0·30	5*b*
44	*Eastern Frontier or Shillong Meridional*	1860-64	0·409	13·2	...	6	2	1	1	...	1	...	...	1	...	·028	0·49	23*b*
45	*Sutlej Meridional* ...	1861-63	0·346	10·6	50	...	...	...	...	...	...	...	...	...	...	·167	0·53	25
46	*Madras Mer. and Coast*	1861-68	0·426	21·6	...	8	2	6	3	2	...	...	...	...	...	·026	0·40	19
47	Kāthiāwār Minor Meridional No. 4 ...	1863-64	1·154	10·8	14	...	...	...	...	...	...	...	...	...	1	·157	1·73	69
48	*East Calcutta Lon.* ...	1863-69	0·379	10·7	32	...	...	2	...	...	...	...	...	...	...	·157	0·57	27
49	*Mangalore Meridional*	1863-73	0·440	20 7	1	1	1	4	2	...	...	...	...	...	2	·096	0·45	22
50	Kumaun and Garhwāl	1864-65	1·742	26·7	2	4	...	1	...	...	...	...	...	...	...	·048	1·50	63
51	Nāsik Secondary ...	1864-65	2·033	10·5	26	...	...	...	...	...	...	...	...	...	...	·167	3·12	91
52	Burma Coast ...	1864-82	0·380	19·8	14	18	5	5	1	2	...	...	...	...	4	·078	0·39	18
53	*Jabalpur Meridional*	1865-67	0·340	22·4	...	2	...	7	1	...	...	...	...	...	...	·008	0·31	7
54	*Madras Longitudinal*	1865-80	0·384	21·4	1	1	3	6	...	...	...	...	...	...	...	·038	0·37	15*t*
55	Assam Valley Triangulation ...	1867-78	1·690	9·5	46	5,‖3	1	...	...	...	...	...	...	...	...	·141	2·65	87
56	*Brahmaputra Mer.* ...	1868-74	0·564	12·0	...	...	1	6	...	1	...	...	...	...	1	·009	0·70	34*b*
57	Coimbatore Minor No. 1	1869-71	1·547	13·6	19	...	1	...	...	...	...	...	...	...	...	·163	2·07	83
58	*Bilāspur Meridional* ...	1869-73	0·302	15·7	...	6	4	6	...	...	...	...	...	...	...	·021	0·33	11
59	Cuddapah Minor ...	1871-72	0·826	17·6	8	1	...	...	...	...	...	...	...	...	...	·148	0·96	43*b*
60	Hyderabad Minor ...	1871-72	1·405	19·9	9	...	...	...	...	...	...	...	...	...	...	·167	1·56	66
61	Malabar Coast ...	1871, 74, 80	1·532	17·4	12	...	...	...	...	...	...	...	...	...	...	·167	1·82	74
62	Jodhpore Meridional	1873-76	0·291	15·6	...	3	1	7	1	...	1	...	...	...	...	·019	0·32	8*t*
63	*South East Coast* ...	1875-79	0·522	11·7	...	8	...	11	2	...	...	...	...	...	1	·007	0·65	31*b*

Mer. = Meridional. Lon. = Longitudinal. Sec. = Secondary. ‖ Centred Quadrilateral.

TABLE XLIV.—(Contd.)

Values of "*m*" and "M" for all Geodetic Series of the Indian Triangulation.

No.	Name of Series	Seasons	$\pm m$	l	3-sided	4-sided	5-sided	6-sided	7-sided	8-sided	9-sided	10-sided	11-sided	12-sided	Compound	f	$\pm$M	Order of Merit
64	Eastern Sindh Mer.	1876-81	0·244	13·0	...	3	2	5	2	...	...	...	...	...	...	·028	0·30	5*b*
65	Siam Branch Triangulation ...	1878-81	3·711	16·1	7	4	...	...	...	...	...	...	...	...	1	·107	4·34	94
66	Mandalay Meridional	1889-95	0·418	27·0	...	13,‖3	...	...	...	...	...	...	...	...	1	·009	0·35	12
67	Mong Hsat Secondary	1891-93	3·054	24·0	9	1	...	...	...	...	...	...	...	...	1	·139	3·01	89*b*
68	Manipur Longitudinal	1894-99	0·453	28·4	...	5,‖2	...	...	...	...	...	...	...	...	...	·000	0·36	13*b*
69	Makran Longitudinal	1895-97	0·285	23·9	2	2,‖1	1	...	...	...	...	...	...	...	...	·069	0·26	4
70	Mandalay Lon. ...	1899-1909	1·696	17·2	8	2	1	...	...	...	...	...	...	...	...	·129	1·96	80
71	Manipur Minor Mer.	1899-1902 } 1915-16 }	0·750	20·9	31	‖1	...	...	...	...	...	...	...	...	...	·161	0·81	39
72	Great Salween ...	1900-11	0·404	32·6	...	2,‖4	2	...	...	...	...	...	...	...	1	·056	0·32	8*t*
73	Kidarkanta Secondary	1902-03	1·323	15·2	3	1	...	...	...	...	...	...	...	...	...	·125	1·62	67
74	Kalāt Longitudinal ...	1904-08	0·365	39·7	...	6,‖5	...	1	...	...	...	...	...	...	...	·000	0·25	3
75	Baluchistān Triangulation ...	1908-09	1·348	33·2	...	1	...	...	...	...	...	...	...	...	1	·083	1·08	47*b*
76	North Baluchistān ...	1908-10	0·221	32·7	...	5,‖3	1	...	...	...	...	...	...	...	...	·009	0·17	1
77	Gilgit ...	1909-11	0 443	31·6	5	3,‖1	...	...	...	...	...	...	...	...	...	·093	0·37	15*t*
78	Khāsi Hills Secondary	1909-11	2·038	10·7	14	3	...	...	...	...	...	...	...	...	...	·137	3·01	89*b*
79	Mawkmai Secondary	1909-11	1·575	10·9	41	1	...	...	...	...	...	...	...	...	...	·163	2·35	85
80	Upper Irrawaddy ...	1909-11	0·596	30·6	4	5	...	...	...	...	...	...	...	...	...	·074	0·49	23*b*
81	Jaintia Hills Sec. ...	1910-11	0·986	6·9	23	...	...	...	...	...	...	...	...	...	...	·167	1·86	76
82	Bhīr Secondary ...	1911-12	0·794	17·4	24	...	...	...	...	...	...	...	...	...	...	·167	0·94	42
83	Ranchi Secondary ...	1911-12	1·840	15·2	13	...	...	...	...	...	...	...	...	...	...	·167	2·34	84
84	Villupurām Secondary	1911-12	1·184	10·9	18	...	...	...	...	...	...	...	...	...	...	·167	1·78	71
85	Sambalpur Meridional	1911-14	0·250	25·7	...	6,‖4	1	1	...	...	...	...	...	...	...	·007	0·21	2
86	Indo-Russian Connection ...	1912-13	2·790	10·9	11	7,‖2	...	...	...	...	...	...	...	...	...	·092	3·92	93
87	Khandwa Secondary	1912-13	0·999	15·2	22	...	...	...	...	...	...	...	...	...	...	·167	1·27	55*b*
88	Ashta Secondary ...	1913-15	1·048	15·3	21	...	...	...	...	...	...	...	...	...	...	·167	1·33	57
89	Buldāna Secondary ...	1913-14	0·304	12·3	18	...	...	...	...	...	...	...	...	...	...	·167	0·43	20*b*
90	Naldrug Secondary ...	1913-14	1·465	15·2	27	1	...	...	...	...	...	...	...	...	...	·161	1·85	75
91	Nāga Hills Secondary	1913-14	0·913	21·3	7	1	1	...	...	...	...	...	...	...	...	·139	0·96	43*b*
92	Middle Godaveri Sec.	1914-15	0·913	17·1	14	1	...	...	...	...	...	...	...	...	...	·156	1·08	47*b*
93	Kohima Secondary ...	1914-15	1·094	15·0	13	...	...	...	...	...	...	...	...	...	...	·167	1·39	59
94	Cachar ...	1914-15	1·077	10·5	10	...	...	...	...	...	...	...	...	...	...	·167	1·65	68

Mer. = Meridional. Lon. = Longitudinal. Sec. = Secondary. ‖ Centred quadrilateral.

For 42 Series entering the Simultaneous Grinding (shown in italics above)

$\Sigma M^2 = 45 \cdot 3591$ Mean Square $M = \pm \sqrt{\dfrac{45 \cdot 3591}{42}} = \pm 1 \cdot 04$

For Series up to No. 94 $\sum_{1}^{94} M^2 = 214 \cdot 3199$ Mean Square $M = \pm \sqrt{\dfrac{214.3199}{94}} = \pm 1 \cdot 51$

4. Replace each triangulation series by one of its flanks. The network is then nearly similar to a traverse network, with the addition that closure of the length of the last side is necessary as well as its azimuth and the position of its terminal point. The flank of any series is usually not far from straight: or else consists of two or more portions with approximately straight flanks. Consider each such portion separately and denote it by the name "*triangulation line*."

Each triangulation line is liable to be slightly bent and to have its length slightly altered in the course of adjustment. This is effected by the angle at each station, and by the ratio of successive sides (between stations) of the triangulation line being slightly changed. Triangulation lines may be of different strengths according to the series from which they are derived: but it will be assumed that the strength of any one triangulation line is uniform. In other words, if it is necessary to adjust the azimuth at the end of a triangulation line this would be done correctly by giving the angles at all its stations an equal change: and to adjust the length of terminal side it would be correct to change the ratios of successive sides each by equal percentages.

When several triangulation lines are concerned the angular adjustment at any station of one will in general be different from that at any station of any other on account of both the different strengths of the several triangulation lines as well as their directions. The question of strengths has been considered in some detail above (*vide* § 1) and can be taken into account by means of M.

Adjustments of latitude and longitude at the end of a triangulation line may also be effected by a combination of small changes of the angles at the stations and the ratios of sides between stations: but in these cases the most probable adjustment would not be that of changing all the angles and the successive side ratios by equal amounts. The actual difference of this latter course from the most probable one is not very great in triangulation lines of moderate length, and it may be deemed justifiable on the ground of simplicity to make the adjustment by adopting the latter. This would bring the four types of adjustment into one simple scheme: but the more general case will now be explained and the simple case can easily be deduced from this if desired by omission of certain terms.

5. Consider any triangulation line and let the successive stations along the line be denoted by the numbers $0, 1, 2, \ldots n$, there being altogether n sides in the line. Let A_r be the azimuth at r of $r+1$ and λ_r, L_r the latitude and longitude of r. Denote by c_r the length of the r^{th} side and by $\Delta\lambda_r$, ΔL_r, ΔA_r the increments of latitude, longitude and azimuth along this side $(r-1, r)$. Suppose that the angle at the station r is changed by η_r radians and the ratio of the $\overline{r+1}|^{th}$ to the r^{th} side to $\frac{c_{r+1}}{c_r}(1+\epsilon_r)$ and consider what changes will be caused thereby.

The following expressions hold approximately

$$\left.\begin{aligned} \Delta\lambda_r &= -\frac{c_r}{a}\cos A_{r-1} \\ \Delta L_r &= -\frac{c_r}{a}\sin A_{r-1}\sec\lambda_{r-1} \\ \Delta A_r &= -\frac{c_r}{a}\sin A_{r-1}\tan\lambda_{r-1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5)$$

in which $\Delta\lambda_r$, ΔL_r, ΔA_r are expressed in radians. The differences between ρ, ν the principal radii of curvature and a the mean radius are neglected as only approximate equations are required in what follows.

Differentiate (5) with regard to c_r, A_{r-1}, λ_{r-1}. Denoting the changes in latitude, longitude, back azimuth and forward azimuth at station r by u_r, v_r, w_r, $w_r+\eta_r$ it follows that the change in $\Delta\lambda_r$ is $u_r - u_{r-1}$ etc., so that

$$\left.\begin{aligned} u_r - u_{r-1} &= \Delta\lambda_r \left\{ \frac{\delta c_r}{c_r} - \tan A_{r-1} \left(w_{r-1} + \eta_{r-1} \right) \right\} \\ v_r - v_{r-1} &= \Delta L_r \left\{ \frac{\delta c_r}{c_r} + \cot A_{r-1} \left(w_{r-1} + \eta_{r-1} \right) + \tan \lambda_{r-1} u_{r-1} \right\} \\ w_r - w_{r-1} - \eta_{r-1} &= \Delta A_r \left\{ \frac{\delta c_r}{c_r} + \cot A_{r-1} \left(w_{r-1} + \eta_{r-1} \right) + \sec \lambda_{r-1} \operatorname{cosec} \lambda_{r-1} u_{r-1} \right\} \end{aligned} \right\} \quad . \quad . \quad . \quad (6)$$

In these equations $\cot A_{r-1}$ or $\tan A_{r-1}$ is liable to be inconveniently large: but this is always accompanied by either $\Delta\lambda_r$ or ΔL_r being correspondingly small. It is convenient to eliminate A_{r-1} by means of (5) and to write (6)

$$\left.\begin{aligned} u_r - u_{r-1} &= \Delta\lambda_r \frac{\delta c_r}{c_r} - \Delta L_r \cos\lambda_{r-1} \left(w_{r-1} + \eta_{r-1} \right) \\ v_r - v_{r-1} &= \Delta L_r \frac{\delta c_r}{c_r} + \Delta\lambda_r \sec\lambda_{r-1} \left(w_{r-1} + \eta_{r-1} \right) + \Delta L_r \tan\lambda_{r-1} u_{r-1} \\ w_r - w_{r-1} &= \Delta L_r \sin\lambda_{r-1} \frac{\delta c_r}{c_r} + \Delta\lambda_r \tan\lambda_{r-1} \left(w_{r-1} + \eta_{r-1} \right) + \Delta L_r \sec\lambda_{r-1} u_{r-1} \end{aligned} \right\} \quad . \quad (7)$$

In accordance with notation explained above, since $\frac{c_r}{c_0} = \frac{c_r}{c_{r-1}} \cdot \frac{c_{r-1}}{c_{r-2}} \quad . \quad . \quad . \quad . \quad . \quad \frac{c_1}{c_0}$ and $\frac{c_r}{c_{r-1}}$ is changed into $\frac{c_r}{c_{r-1}}(1+\epsilon_{r-1})$: then $\frac{c_r}{c_0}$ is changed into $\frac{c_r}{c_0} \prod_0^{r-1} (1+\epsilon_r)$. Hence $\frac{\delta c_r}{c_r} \doteqdot \mathrm{E} + \sum_0^{r-1} \epsilon$ where c_0, which may be regarded as the last side previous to the side 01, is supposed to change to $c_0(1+\mathrm{E}_0)$.

Now suppose that the changes in successive side ratios and angles are in arithmetic progression: *i.e.*

$$\epsilon_0 = \epsilon_1 - \epsilon' = \epsilon_2 - 2\epsilon' = \quad . \quad . \quad . \quad . \quad = \epsilon_r - r\epsilon' = \epsilon$$

and

$$\eta_0 = \eta_1 - \eta' = \eta_2 - 2\eta' = \quad . \quad . \quad . \quad . \quad = \eta_r - r\eta' = \eta$$

Then

$$\frac{\delta c_r}{c_r} = \mathrm{E} + r\epsilon + \frac{r(r-1)}{2}\epsilon'$$

E and H being the quantities relating to the side from which the triangulation line emanates.

Equations (7) may now be written

$$\left.\begin{aligned} u''_r - u''_{r-1} &= \left(\mathrm{E} + r\epsilon + \frac{r(r-1)}{2}\epsilon' \right) \operatorname{cosec} 1'' \Delta\lambda_r - \left(w''_{r-1} + \eta + (r-1)\eta' \right) \operatorname{cosec} \lambda_{r-1} \Delta L_r \\ v''_r - v''_{r-1} &= \left(\mathrm{E} + r\epsilon + \frac{r(r-1)}{2}\epsilon' \right) \operatorname{cosec} 1'' \Delta L_r + \left(w''_{r-1} + \eta + (r-1)\eta' \right) \sec \lambda_{r-1} \Delta\lambda_r \\ &\qquad + u''_{r-1} \tan \lambda_{r-1} \Delta L_r \quad . \quad . \quad . \quad . \\ w''_r - w''_{r-1} - \eta - \overline{r-1}|\,\eta' &= \left(\mathrm{E} + r\epsilon + \frac{r(r-1)}{2}\epsilon' \right) \operatorname{cosec} 1'' \sin\lambda_{r-1} \Delta L_r \\ &\qquad + \left(w''_{r-1} + \eta + (r-1)\eta \right) \tan\lambda_{r-1} \Delta\lambda_r + u''_{r-1} \sec\lambda_{r-1} \Delta L_r \quad . \quad . \end{aligned} \right\} \quad . \quad (8)$$

in which u, v, w, η are now expressed in seconds and $\Delta\lambda$, ΔL are in radians, and $w_0 = \mathrm{H}$.

To apply these equations it is necessary to know the values of E_0, ϵ, η'', u_0'', w_0'' and v_0''. The last quantity is simply additive to all values of v. The remaining five quantities give rise to five cases: for they can be considered separately and the results combined afterwards, since second order quantities are being neglected. By successive application of (8) the solution may be obtained in the form

$$\left.\begin{aligned} u_r &= (A_u \mathrm{E} + B_u \epsilon + G_u \epsilon') \operatorname{cosec} 1'' + C_u \eta + F_u \eta' + D_u u_0 + K_u w_0 \\ v_r - v_0 &= (A_v \mathrm{E} + B_v \epsilon + G_v \epsilon') \operatorname{cosec} 1'' + C_v \eta + F_v \eta' + D_v u_0 + K_v w_0 \\ w_r &= (A_w \mathrm{E} + B_w \epsilon + G_w \epsilon') \operatorname{cosec} 1'' + C_w \eta + F_w \eta' + D_w u_0 + K_w w_0 \end{aligned} \right\} \quad . \quad . \quad . \quad (9)$$

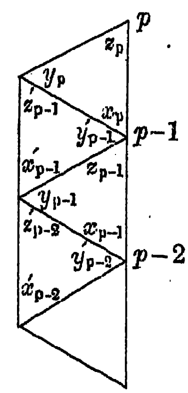
The coefficients A, B etc . . have to be determined for each triangulation line and then the latitude, longitude and azimuth changes are expressed by (9). The r^{th} side is changed in the ratio $1 + \mathrm{E} + r\epsilon + \dfrac{r\,(r-1)}{2}\epsilon' \;:\; 1$; so that the solution is complete.

It is convenient to denote the changes at the end of any, the m^{th}, triangulation line in latitude, longitude, azimuth and side by ${}_{\mathrm{m}}U \quad {}_{\mathrm{m}}V \quad {}_{\mathrm{m}}W \quad {}_{\mathrm{m}}\mathrm{E}$: then taking account of different values of ϵ, η which occur in different triangulation lines the change in latitude, longitude and azimuth of the r^{th} station of the m^{th} line may be written

$$\left.\begin{aligned}
{}_{\mathrm{m}}U_{\mathrm{r}} &= ({}_{\mathrm{m}}A_u\,{}_{\mathrm{m-1}}\mathrm{E} + {}_{\mathrm{m}}B_u\,\epsilon_{\mathrm{m}} + {}_{\mathrm{m}}G_u\,\epsilon_{\mathrm{m}}')\operatorname{cosec} 1'' + {}_{\mathrm{m}}C_u\,\eta_{\mathrm{m}} + {}_{\mathrm{m}}\mathrm{D}_u\,u_0 + {}_{\mathrm{m}}K_u\,w_0 \\
{}_{\mathrm{m}}V_{\mathrm{r}} - {}_{\mathrm{m-1}}V &= ({}_{\mathrm{m}}A_v\,{}_{\mathrm{m-1}}\mathrm{E} + {}_{\mathrm{m}}B_v\,\epsilon_{\mathrm{m}} + {}_{\mathrm{m}}G_v\,\epsilon_{\mathrm{m}}')\operatorname{cosec} 1'' + {}_{\mathrm{m}}C_v\,\eta_{\mathrm{m}} + {}_{\mathrm{m}}D_v\,u_0 + {}_{\mathrm{m}}K_v\,w_0 \\
{}_{\mathrm{m}}W_{\mathrm{r}} &= ({}_{\mathrm{m}}A_w\,{}_{\mathrm{m-1}}\mathrm{E} + {}_{\mathrm{m}}B_w\,\epsilon_{\mathrm{m}} + {}_{\mathrm{m}}G_u\,\epsilon_{\mathrm{m}}')\operatorname{cosec} 1'' + {}_{\mathrm{m}}C_w\,\eta_{\mathrm{m}} + {}_{\mathrm{m}}D_w\,u_0 + {}_{\mathrm{m}}K_w\,w_0 \\
{}_{\mathrm{m}}\mathrm{E}_{\mathrm{r}} &= {}_{\mathrm{m-1}}\mathrm{E} + r\epsilon_{\mathrm{m}} + \frac{r(r-1)}{2}\,\epsilon'_{\mathrm{m}}
\end{aligned}\right\} \quad (10)$$

Equations (10) give in the most general form the changes that are effected by introducing the alterations $\eta, \eta', \epsilon, \epsilon'$ different for each triangulation line.

6. We may now consider the probable relative values of ϵ and η (the latter expressed in radians) for this purpose treating ϵ' and η' as zero.. Take the case of a series of simple equilateral triangles and let $p-2,\ p-1,\ p$ be three successive stations on a flank: also let $x_{\mathrm{p}}\, y_{\mathrm{p}}\, z_{\mathrm{p}}$ etc. be changes which are applied to the several angles, as indicated in the figure. The ratio of the successive flank sides Q R/P Q being denoted by r, then



$$d \log r = \cot 60^{\circ}\,(x_{\mathrm{p-1}} + x'_{\mathrm{p-1}} + y_{\mathrm{p}} - y'_{\mathrm{p-1}} - z'_{\mathrm{p-1}} - z_{\mathrm{p}})$$

Hence

$$\begin{aligned}
\Sigma\, d \log r &= d \log \Pi\, r \\
&= \cot 60^{\circ}\left\{\sum_{0}^{\mathrm{p-1}}(x_{\mathrm{r}} + x'_{\mathrm{r}} - z_{\mathrm{r-1}} - z'_{\mathrm{r-1}}) + y_{\mathrm{p}} - y'_{0}\right\} \\
&\doteqdot p\epsilon.
\end{aligned}$$

Hence the most probable way of getting a particular value for the change in logarithm of the side is by making all the x^{s} equal to each other and of opposite sign to all the z^{s} which will also all be equal. The y^{s} do not come into the case at all except at the two ends, unless an azimuth change is also required.

For the azimuth it is clear that

$$\Sigma\,\eta_{\mathrm{p-1}} = \Sigma\,(x_{\mathrm{p}} + y'_{\mathrm{p-1}} + z_{\mathrm{p-1}}) = p\eta$$

is the azimuth change, since the η^{s} are all to be equal.

Now in the most probable distribution of changes obviously all the x^{s} are equal as are the y^{s} and z^{s}. Hence

$$p\epsilon = \frac{p}{\sqrt{3}}\left\{x + x' - z - z'\right\} + \frac{1}{\sqrt{3}}\,(y - y')$$

and

$$p\eta = p\,(x + y' + z).$$

Now

$$x + z = -\,y$$

$\therefore$

$$\left.\begin{aligned}
\eta &= y' - y \\
\epsilon &\doteqdot \frac{1}{\sqrt{3}}\left(x + x' - z - z'\right)
\end{aligned}\right\} \quad (11)$$

and

The probable values of η and ϵ are accordingly in the ratio of the most probable values of $\frac{y\sqrt{3}}{x-z}$ in which the quantities are subject to the relation

$$x+y+z=0$$

Following the usual plan of independent multipliers explained below (Chapter VII § 4) we have for the two cases (a) of y and (b) of $x-z$

$$a_1 = a_2 = a_3 = 1$$
$$b_1 = b_2 = b_3 = 0$$
$$\text{etc.}$$
$$u_1 = u_2 = u_3 = 1$$

and

$$3k_1 = 1 \text{ in case } (a); \quad 3k_1 = 0 \quad \text{in case} (b)$$

$\therefore$

$$u_F = 1-\tfrac{1}{3} = \tfrac{2}{3} \text{ in case } (a); \quad u_F = 2-0 = 2 \quad \text{in case } (b)$$

Hence the ratio of probable values of η and ϵ is unity, *i.e.*, η and ϵ have the same weight.

7. Now it is clear that in the adjustment of azimuth and side the most probable solution is obtained by introducing equal values of η, ϵ and by putting $\epsilon' = \eta' = 0$, all round a circuit composed of triangulation lines of equal strength: and if the triangulation lines are of different strengths the value of η, ϵ in any one is inversely proportional to the strength. With latitude and longitude it is clear that the directions of the traverse lines are of essential importance. Thus in the case of a triangulation line along a meridian a change in ϵ will give changes in latitude but no change in longitude. Moreover to get any desired change the most probable values of changes in angle and side ratio would not be equal at all the stations along a triangulation line: though it is probable that they would not alter much on one triangulation line if the circuit was composed of several such lines. Taking the case of a triangulation line along a meridian it is clear that to obtain a given change in longitude the change in the angles at the starting end of the line are more effective than equal changes in the angles near the closing point. If the sides were of equal length the most probable changes would be in arithmetical progression. The complexity of different changes at the various stations of a triangulation line on account of this has been deemed in general to overbalance the slight gain in theoretical accuracy of adjustment and at least in some preliminary adjustments which are about to be made (*e.g.* the incomplete Burma triangulation) the plan will be followed of making η and ϵ uniform along one triangulation line and putting $\eta' = \epsilon' = 0$. When the probable errors of position etc. are considered (*vide* Chapter VII.) it is believed that this plan will be considered fully satisfactory for many cases of geodetic application.

The equations (10) however show how the variation in changes along a triangulation line may be taken into account: and in closing a single triangulation line between two previously fixed sides they give the necessary number of quantities (four) at choice to satisfy the closing condition. It is clear from the theorem stated at the end of § 13 that for the general case of closure the changes at the several stations of a triangulation line should be in arithmetical progression this being a combination of the most probable adjustment firstly for log. side and azimuth and secondly for latitude and longitude. When the number of η^s and ϵ^s at choice is in considerable excess of the number of conditions to be satisfied it is believed that little gain in accuracy is obtained by introducing η'^s and ϵ'^s: and certainly this doubles the number of unknowns and greatly increases the labour of formation and solution of the normal equations. It also increases the complexity of the solution, and its subsequent application.

8. In the case of a network of circuits including Laplace stations and extra base lines, equations of form of (10) may be formed; and by equating the right hand sides to the several closing

errors which arise, four equations are formed for each circuit together with one extra for each extra base line or Laplace station. These may be used to determine the most probable values of η, ϵ (and η', ϵ' if it is thought desirable not to put these equal to zero for each triangulation line), due regard being paid to the strength of each triangulation line.

If $m = \sqrt{\frac{\Sigma\Delta^2}{3n}}$ where Δ is the triangular error of any one of the triangles of a series, then the probable value of η in the case of a series of simple triangles in which the triangular error has been dispersed is, by (11), $\sqrt{\frac{2}{3}}\, m\sqrt{2} \times \cdot 6745 = \cdot 779m$. Account may be taken of the series comprising quadrilaterals and other figures by the introduction of a factor $\frac{1+f}{1+\frac{1}{6}}$ (*see* § 1 above) so that the probable values of η and ϵ (which are equal) are both equal to

$$\cdot 779m\,\frac{1+f}{1+\frac{1}{6}} = 0\cdot 668m\,(1+f) = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

in which m is supposed to be expressed in radians.

The equations for η and ϵ accordingly have to be solved subject to the condition that the sum of the squares of the corrections multiplied by their weights is a minimum. In one triangulation line the sum of the squares of the corrections is

$$\Sigma\left(\eta^2+\epsilon^2\right) + \Sigma\left(\eta'^2+\epsilon'^2\right)r^2 = r\left(\eta^2+\epsilon^2\right) + \frac{r(r+1)(2r+1)}{6}\left(\eta'^2+\epsilon'^2\right)$$
$$\doteqdot r\left(\eta^2+\epsilon^2\right) + \frac{r^3}{3}\left(\eta'^2+\epsilon'^2\right)$$

when r is the number of sides in a triangulation line.

Hence the condition to be satisfied is that

$$\Sigma\left[\left\{ r\left(\eta^2+\epsilon^2\right) + \frac{r^3}{3}\left(\eta'^2+\epsilon'^2\right) \right\} \div m^2\left(1+f\right)^2\right] = \text{a minimum} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

the summation extending over all the triangulation lines.

Mechanical Analogy

9. Suppose that there is a network of trilateration, in which the lengths of all sides have been determined by direct measurement. Let l be the measured length of any side, $l+\delta l$ the adjusted value, and w the weight of the determination l. In making any adjustment of the network, for example to bring a terminal side into agreement with a line slightly differently placed, the principle of least squares demands that $\Sigma\, w\, \overline{\delta l}|^2$ shall be a minimum, all imposed conditions being satisfied.

Now consider a similar framework formed by rods of material obeying Hooke's law of extension and compression, and suppose these rods are freely jointed at their junctions. If this framework is in equilibrium without any strains in action, and, the first side being held fast, the last side is brought into a slightly different position and its length slightly changed, then the several rods will undergo compression or extension and their lengths will be slightly altered. If the unstrained length of any rod is l and its strained length is $l+\delta l$; and the force in action in it causing this strain is F: then the work done on the rod is $\frac{1}{2} F\,\delta l$. Now by Hooke's law

$$\frac{\delta l}{l} = \frac{F}{aE}$$

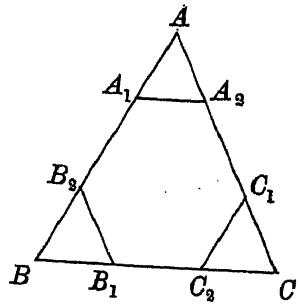
where E is Young's modulus and a is the cross section.

Hence

$$\tfrac{1}{2} F\delta l = \frac{aE}{2l}\cdot\overline{\delta l}|^2$$

and this represents the work done on the rod. The quantity $\frac{aE}{2l}$ may be varied by suitably chosing a: suppose it is made equal to w. Then the work done on the rod is $w\ \overline{\delta l}|^2$. The principle of least work immediately shows that for the set of strains applied to bring the last side into the desired position the total work done must be a minimum: whence $\Sigma\, w\ \overline{\delta l}|^2$ is a minimum. The solution accordingly is the same as that of the most probable adjustment of the similar trilateration network.

10. Now trilateration has never been carried out on a large scale for one obvious reason that no large tract of country is suitable for its execution by ordinary methods. Triangulation on the other hand extends over vast tracts and it is its adjustment which is of importance to geodesists. A mechanical analogy can be supposed for triangulation also. It is less simple than that just described for trilateration and to give it practical shape would be a matter of greater difficulty. Imagine a set of rods, the medial parts of which are laterally rigid, but which are longitudinally extensible without the application of (appreciable) force*. Let these rods be freely jointed to form a framework similar to a network of triangulation, the angles of the triangulation being maintained by rigid pieces. For example in the $\Delta\ A\,B\,C$ the rod $A\,B$ is freely extensible between A_1 and B_2: a cross piece $A_1\,A_2$ maintains the angle A at its proper value and so on. It will be seen then that such a triangle may be enlarged to any size but that it will always remain similar, unless the pieces $A_1\,A_2$, $B_1\,B_2$, $C_1\,C_2$ are changed in length. If these pieces $A_1\,A_2$ etc. are made of material obeying Hooke's law, by properly chosing their cross section it is possible to make them represent the "strength" of the angles A, B, C. When the system is deformed the work done on $A_1\,A_2$ will be proportional to the square of the change in the angle A and accordingly a condition of form $\Sigma\, w\ \overline{\delta\theta}|^2$ = minimum must be satisfied so that the total work done shall be a minimum. All geometrical conditions such as triangular conditions, central station conditions and side ratio conditions obviously cannot be avoided in the mechanical analogy, so that the solution of the mechanical problem is precisely the same as that of the triangulation adjustment according to the method of least squares.



11. It is clear that if the change in any observed quantity can be made to correspond to the extension of a rod which obeys Hooke's law; and if a system of such rods are linked up in such a way as to represent the geometrical conditions controlling the observations; then a mechanical analogy for the set of observations is obtainable. The governing fact is that the work done on any rod is proportional to the square of its extension: so that by substituting extension for error of observation the equation of minimum squares is transformed into the principle of least work.

12. Consider a framework representing a network of triangulation and suppose that it is held fast at one or more points. It may be necessary to bring a terminal side into agreement with a predetermined value and position. To do this four conditions have to be satisfied:

(1) The terminal side must be adjusted to the correct length.
(2) The terminal side must be adjusted to the correct azimuth.
(3) One extremity of the terminal side must be moved to the correct latitude.
(4) This extremity of the terminal side must be moved to the correct longitude.

Now these adjustments may be considered one by one. First strain the terminal side to the correct length and hold it so: then change its azimuth, etc. The adjustments may also be performed

* Approximations to these can readily be conceived, *e.g.* a rod sliding in a tube.

in any order and the final result is the same. This is immediately obvious mechanically, for it is clear that the final configuration due to small strains has nothing to do with the order in which they are applied.

13. The analogy thus proves an important theorem† in the adjustment of observations, namely that *provided all imposed conditions are maintained, the adjustment conditions may be introduced separately in any order, the previous adjustment conditions in each case being maintained; and the most probable complete adjustment is obtained after the last adjustment condition has been applied.* This enables the circuit adjustments of triangulation to be applied after the figural adjustments, as has been done in the Survey of India. A further theorem is also easily deducible. In the case of the closing of a simple triangulation circuit in which there are four closing conditions to satisfy (*i.e.* the case in which there are no additional base lines or independent azimuth determinations) denote the four closing quantities by X, Y, Z, U. To effect the closing X alone in the most probable manner, changes are concurrently introduced which affect the other quantities by amounts y_x z_x u_x : and similarly for the other quantities. From the mechanical analogy it is clear that adjustments as follows should be made :—

$$\begin{array}{lllll}
(1) & x & y_x & z_x & u_x \\
(2) & x_y & y & z_y & u_y \\
(3) & x_z & y_z & z & u_z \\
(4) & x_u & y_u & z_u & u
\end{array}$$

in which

$$\left.\begin{array}{l}
x + x_y + x_z + x_u = X \\
y_x + y + y_z + y_u = Y \\
z_x + x_y + z + z_u = Z \\
u_x + u_y + u_z + u = U
\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (14)$$

and the quantities with suffixes are geometrically related to the suffix quantity, *i.e.*

$$\left.\begin{array}{l}
\dfrac{y_x}{B_x} = \dfrac{z_x}{C_x} = \dfrac{u_x}{D_x} = x \\
\dfrac{x_y}{A_y} = \dfrac{z_y}{C_y} = \dfrac{u_y}{D_y} = y
\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (15)$$

etc.

where the coefficients A B C D depend only on the form of the triangulation and are independent of the closing errors. It accordingly follows that

$$\left.\begin{array}{l}
x + A_y y + A_z z + A_u u = X \\
B_x x + y + B_z z + B_u u = Y \\
C_x x + C_y y + z + C_u u = Z \\
D_x x + D_y y + D_z z + u = U
\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (16)$$

and by solving these the following equations may be obtained

$$\left.\begin{array}{l}
x = a_x X + a_y Y + a_z Z + a_u U \\
y = b_x X + b_y Y + b_z Z + b_u U \\
z = c_x X + c_y Y + c_z Z + c_u U \\
u = d_x X + d_y Y + d_z Z + d_u U
\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (17)$$

If the adjustments x, y, z, u are applied independently and then combined the total effect will be the same as that of the single adjustment X, Y, Z, U taken simultaneously. By use of the quantities x, y, z, u in place of the related quantities X, Y, Z, U it is accordingly possible to treat each closure entirely independently of the remaining three.

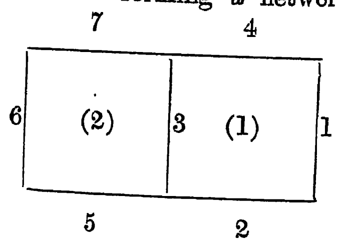
† This theorem was proved analytically in "Account of the Operations of the G.T. Survey of India, Vol. II," Appendix No. 8, pp. 151-158.

If in addition to the 4 ordinary closing quantities additional conditions such as extra bases, or fixings of latitude, longitude or azimuth are introduced, this only makes the relation more involved: it is still possible to express x, y, z, u in terms of the closing errors and proceed as though each of these quantities was independent of the other.

The principle is perfectly general and is applicable to a whole network of triangulation. As this becomes more complex the determination of the coefficients in the equations which correspond to (17) would become more difficult: but this is not necessary at least in some applications of the theorem. It is an important fact that an undetermined portion of each of the closures may be regarded quite independently of the others. The theorem may be stated as follows:—*It is possible to find quantities related to the several closing errors such that each type may be adjusted separately and independently of the others, and such that the combined effects of these several adjustments will be the same as the most probable simultaneous adjustment.*

These related quantities may be called the "independent errors" of each type. If each is adjusted independently of errors of another type, the other type adjustments being allowed to come in just as they will naturally do while the adjustment of the first type is made independently in the most probable manner, then the combined result of the adjustment of the four types will be the most probable adjustment of the closing errors which can be made.

14. Consider now in more detail the case of more than one circuit forming a network which has to be simultaneously adjusted. The case of two circuits which have a common portion is illustrative of this and will be seen to lead to a result generally applicable. Further these circuits may be considered as formed each for four series and no loss of generality occurs in taking these circuits to be of the same strength. The several series may be characterised by the numbers 1—7 and the circuits by (1) and (2).



The adjustments along any side may be made by the introduction of ϵ and η changes: and any one of the four types of closures the ϵ^s and η^s along a triangulation line will be in arithmetical progression (*vide* § 16). For the r^{th} side of the k^{th} line their values may be represented by

$$\epsilon_k + \overline{r-1}|\,\epsilon'_k \quad , \quad \eta_k + \overline{r-1}|\,\eta'_k$$

Consider first the X closure and suppose that ${}_1x$ ${}_2x$ are the "independent errors" of the two circuits: these quantities have to be determined. Then equations of the following form may be formed:

$$\left.\begin{aligned} \sum_1^4\Big(A_k\,\epsilon_k + B_k\,\epsilon'_k + C_k\,\eta_k + D_k\,\eta'_k\Big) &= {}_1x \\ \sum^{3,5,6,7}\Big(A_k\,\epsilon_k + B_k\,\epsilon_k + C_k\,\eta_k + D_k\,\eta'_k\Big) &= {}_2x \end{aligned}\right\} \quad \cdots\cdots \quad (18)$$

in which the coefficients A B C D are independent of the closing errors. The prefixes to x are the circuit numbers.

To obtain the most probable solution the quantities ϵ, η must be chosen so as to make

$$\sum_{k=1}^{k=7}\left\{\sum_{r=1}^{r=n}\Big(\epsilon_k+\overline{r-1}|\epsilon'_k\Big)^2 + \sum_{r=1}^{r=n}\Big(\eta_k+\overline{r-1}|\eta'_k\Big)^2\right\} = \text{minimum.}$$

The solution of this accordingly gives definite value for all the ϵ^s and η^s as linear functions of ${}_1x$ and ${}_2x$. The associated quantities are given by equations of form

$$
\begin{aligned}
{}_1y_x &= {}_1\beta_x\,{}_1x + {}_1\beta'_x\,{}_2x \\
{}_1z_x &= {}_1\gamma_x\,{}_1x + {}_1\gamma'_x\,{}_2x \\
{}_1u_x &= {}_1\delta_x\,{}_1x + {}_1\delta'_x\,{}_2x
\end{aligned}
$$

$$ {}_2y_x = {}_2\beta_x\,{}_1x + {}_2\beta'_x\,{}_2x \quad \text{etc.} $$

Similarly for the other closure

$$ {}_1x_y = {}_1a_y\,{}_1y + {}_1a'_y\,{}_2y \quad \text{etc.} $$

Equations similar to (14) can now be formed for each circuit

$$ {}_1x + ({}_1a_y\,{}_1y + {}_1a'_y\,{}_2y) + ({}_1a_z\,{}_1z + {}_1a'_z\,{}_2z) + ({}_1a_u\,{}_1u + {}_1a'_u\,{}_2u) = {}_1X \quad \text{etc.} \qquad (19) $$

From (19) it is clear that the determination of the "independent errors" ${}_1x$ ${}_2x$ etc. from the known closing conditions can be effected by the solution of simultaneous linear equations of the same number as there are conditions. These equations are soluble without the introduction of the principles of least squares. Their solution however will entail much the same labour as the solution of the normal equations which would arise in the simultaneous adjustment of all the four types of closure. It appears at present to be only of theoretical interest that these closures or rather the adjustment of the related independent errors may be effected, each type independently of the others.: for no material reduction in computation would be effected. However it is believed that the mechanical analogy throws some light on the question, and that developments are likely to result from its consideration.

15. The idea of a triangulation line has been introduced in § 4. It is the flank of a nearly straight series of triangulation. For the present, disregard the ϵ changes and fix attention on the η changes. Suppose a set of rigid rods are placed similarly to the several rays of the flank and that these are freely jointed at their ends, which accordingly correspond to the stations on the triangulation line. Introduce constraints at each junction which tend to maintain the angles between successive rods equal to the observed values and such that the force necessary to alter any one of these angles by a stated amount is inversely proportional to the probable error of the angle or directly proportional to the *strength* of the angle. Then it is clear that the work done in varying the angles in any way is proportional to the sum of the weighted squares of the changes of these angles. It is clear then that to bring the system into any given displaced formation the angular changes introduced are such as to make either the total work done, or the sum of the weighted squares of the adjustment in the angles, a minimum. That is to say the mechanical deformation is the same as the most probable adjustment. It is clear from this why when the angular strength is given the strength of the triangulation to resist either angular deflection at the end, or linear deflection, or a combination of the two, is the greater according as the number of stations in the line is less, or, in other words, according as the length of side is greater. The quantity M already introduced takes account of this (see Chapter VII § 1) and for certain considerations of probable error makes it unnecessary to consider the triangulation line in any detail.

The ϵ changes in a triangulation line (due to the extension of its sides) are precisely analogous to the η changes, if for angular deflection, change in ratio of final to initial side is taken, and for linear deflection at right angles to the line, linear deflection in the direction of the line is substituted. In the case of a single triangulation line (which is a straight line) the η changes are

independent of the ϵ changes and the corresponding adjustments can be performed independently: so both could be separately considered by means of this partial mechanical analogy. When however there is a set of triangulation lines in various directions the η and ϵ changes (northing and easting) of the several lines get intermixed and for this a complete analogy is required.

It is clear that for the case of trilateration it would only be necessary to arrange a set of *extensible* rods, jointed as described above, and representing a flank of the trilateration to obtain a complete analogy. But in the case of triangulation it is necessary to arrange that if any element changes length the successive elements change length in the same proportion without any force being involved. A simple mechanical analogy completely representing a triangulation line has not yet been discovered: and for this case it at present appears necessary to consider the analogy of the complete series instead of only one of its flanks (vide § 10). For purposes of probable errors it would be possible to replace the series by a simple series composed of equilateral triangles, in the same way that in the triangulation line it may be a convenient simplification to take all the sides of equal length and persisting in the same direction: and it appears from the mechanical analogy that little accuracy would be lost by so doing.

16. It is clear from the partial analogy given in the preceding section that the best adjustment to make in a triangulation line to obtain a given azimuth change is to change all the angles by amounts proportional to their probable errors: for this corresponds to the mechanical set of rods of which the first is held fast and to the last of which a suitable couple is applied. To obtain a given deflection the most probable adjustment is that the angular changes divided by their probable errors should be in arithmetical progression; as would be the case in the mechanical system, when held at one end and subjected to a simple force at right angles to the line at the other end. The exact analogy between the ϵ^s and η^s shows that similar conditions hold for the ϵ^s. The above statements can be simplified for the case of a triangulation line if the weights of the several angles are considered equal. It appears that a determination of the weights of the angles may best be obtained by a consideration of all the triangular errors, which leads to one value for the probable value of any angle of the series. Any determination of probable error of each angle separately is very much vitiated by the graduation error of the instrument, which is unknown. The simplified statements for a triangulation line with all sides of the same length are:—

(*a*) for a given angular deflection or a given change of ratio of final side to initial side, that the η^s or ϵ^s are all equal.

(*b*) for a given linear deflection at right angles to or along the line, that the η^s or ϵ^s are in arithmetical progression.

On this account the cases of the η^s and ϵ^s changing in arithmetical progression (which includes (*a*) as a special case, the constant difference then being zero) have been considered in equations (8) to (10). Some departure from rigid accuracy occurs when the several sides of the triangulation line are of unequal lengths.

CHAPTER VII.

Probable errors of triangulation before and after adjustment.

1. Expressions will now be formed for the probable errors of points and sides generated in one or more series of triangulation, in which only figural conditions have been adjusted.

(1) *Probable errors in logarithm and azimuth of terminal side.*

Equation (12) of chapter VI gives a the probable value of either η or ϵ. The probable error in the logarithm of the terminal side after n sides of a triangulation line is clearly $\sqrt{n}\ \log_{10}(1+\epsilon) = \cdot 4343 a\sqrt{n}$; and in azimuth is $a\sqrt{n}$. Considering the triangulation line as practically straight and the distances between stations as equal then, if l is the average length of side and s the length of the line, so that $nl = s$,

$$a\sqrt{n} = \cdot 668 m\,(1+f)\sqrt{n} = m(1+f)\sqrt{\frac{18}{l}}\cdot\sqrt{\frac{nl}{18}}\times\cdot 668$$
$$= \cdot 1575\ \mathrm{M}\sqrt{s}\quad \ldots\ldots\ldots\ldots \quad (1)$$

Hence if M is expressed (as is always done) in seconds of arc

Probable error in azimuth at end of a triangulation line $= 0''\cdot 1575\ \mathrm{M}\sqrt{s}$

Probable error in log. side at end of a triangulation line $= \cdot 4343 \sin 1''\times 0\cdot 1575\ \mathrm{M}\sqrt{s}$
$= 3\cdot 32\times 10^{7}\ \mathrm{M}\sqrt{s}$

For the case of a number of triangulation lines it is necessary to substitute $\sqrt{\Sigma \mathrm{M}^2 s}$ for $\mathrm{M}\sqrt{s}$. It is convenient to measure lengths of triangulation lines in units of 100 miles: so replace s by $100\,S$ where the unit of S is 100 miles and then

$$\left.\begin{array}{ll} \text{Probable error in azimuth of the terminal side of a series of triangulation lines} & = 1''\cdot 575\ \sqrt{\Sigma \mathrm{M}^2 S} \\ \text{Probable error in seventh place of logarithm of the terminal side of a series of triangulation lines} \ldots \ldots & = 33\cdot 2\ \sqrt{\Sigma \mathrm{M}^2 S} \end{array}\right\} \quad \ldots\ldots \quad (2)$$

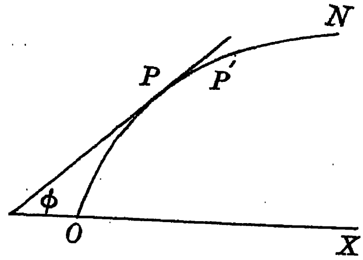
In the above S is measured along the triangulation, and it is immaterial whether this is straight or not: but if the elements of the summation indicated by Σ are straight, then S may be replaced by L the length of any triangulation line, bringing formulæ (2) into similar terms to those of (5) below.

(2) *Probable errors in easting and northing of terminal points.*

Consider any curve defined by s the distance measured along it and ϕ the angle the tangent makes with OX. Let this curve be divided into elements of length l. Suppose that for purposes of adjustment, or on account of errors, the ratio of the $\overline{m+1}|^{\text{th}}$ element to the m^{th} is changed by factor $1+\epsilon_m$ and the angle at the junction of these two elements by η_m. Then if $P\,P'$ is the $\overline{m+1}|^{\text{th}}$ element the relative shift of P' to P is

$$l\cos\phi_m \sum_1^{m+1} \epsilon - l\sin\phi_m \sum_1^{m+1} \eta \qquad \text{in easting}$$

and $$l\sin\phi_m \sum_1^{m+1} \epsilon + l\cos\phi_m \sum_1^{m+1} \eta \qquad \text{in northing}$$



and the total change relative to O of N is given by

$$\Delta x = l\sum_1^n \left(\cos\phi_m \sum_1^{m+1} \epsilon\right) - l\sum_1^n \left(\sin\phi_m \sum_1^{m+1} \eta\right)$$

$$\Delta y = l\sum_1^n \left(\sin\phi_m \sum_1^{m+1} \epsilon\right) + l\sum_1^n \left(\cos\phi_m \sum_1^{m+1} \eta\right)$$

Hence

$$\Delta x = l\epsilon_1 \sum_1^n \cos\phi + l\epsilon_2 \sum_2^n \cos\phi + l\epsilon_3 \sum_3^n \cos\phi + \quad \cdots\cdots - l\eta_1 \sum_1^n \sin\phi - \cdots\cdots$$

$$= \epsilon_1 (x_n - x_0) + \epsilon_2 (x_n - x_1) + \epsilon_3 (x_n - x_2) \quad \cdots\cdots - \eta_1 (y_n - y_0) - \cdots\cdots$$

since $$\cos\phi = \frac{x'-x}{l}, \quad \text{and } \sin\phi = \frac{y'-y}{l}$$

The most probable value of Δx is accordingly

$$\left[\epsilon^2 \left\{(x_n-x_0)^2 + (x_n-x_1)^2 + \quad \cdots\cdots \right\} + \eta^2 \left\{(y_n-y_0)^2 + (y_n-y_1)^2 + \quad \cdots\cdots \right\}\right]^{\frac{1}{2}}$$

and since the probable values of ϵ and η are both equal to a this reduces to $a\sqrt{\Sigma r^2}$ where r is the radius vector measured from the point N. The probable value of Δy is the same. When the elements are increased in number Σr^2 may be replaced by $\frac{1}{l}\int r^2 ds$: also by (12) of chapter VI and (1)

$$a = \cdot 1575 \text{ M} \sin 1'' \sqrt{\frac{s}{n}} = \cdot 1575 \text{ M} \sin 1'' \sqrt{l}$$

and $\qquad a\sqrt{\Sigma r^2}$ becomes equal to $\cdot 1575 \text{ M} \sin 1'' \sqrt{\int r^2 ds}$

Hence the probable value of Δx or $\Delta y = \cdot 1575 \text{ M} \sin 1'' \sqrt{\int r^2 ds}$ . . . . (3)

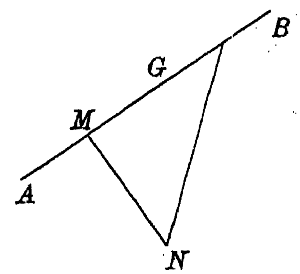
So far only a single triangulation line has been considered. If there are several it is clear that it is necessary to alter this expression to $\cdot 1575 \sin 1'' \sqrt{\Sigma \text{ M}^2 \int r^2 ds}$. This is expressed in units of a mile. To express it in feet multiply by 5280. It is convenient to measure r and s in units of 100 miles: denote their values in units of 100 miles by R and S. Finally if P (feet) is the probable error at a point N in easting or northing, and R is the radius vector measured from N, and S is the distance measured along the triangulation, both R and S being expressed in units of 100 miles, then

$$P = \cdot 1575 \times 5280 \times 10^3 \sin 1'' \sqrt{\Sigma M^2 \int R^2 \, dS}$$

$$= 4\cdot 03 \sqrt{\Sigma M^2 \int R^2 \, dS} \quad \ldots \ldots \ldots \ldots \quad (4)$$

The integral $\int R^2 \, dS$ may be taken out for each triangulation line (assumed to be a straight line). If $A\,B$ be one of these and S is measured from M the foot of the perpendicular from N on $A\,B$ and $N\,M = p$, then $R^2 = S^2 + p^2$

$$\therefore \quad \int R^2 \, dS = \Big[\tfrac{1}{3} S^3 + S p^2 \Big]_{MA}^{MB} = A\,B\,(\tfrac{1}{3} AB^2 + p^2 + MA.\,MB)$$



Now if G is the middle point of $A\,B$

$$MA.\,MB = -\left(\frac{L}{2} + MG\right)\left(\frac{L}{2} - MG\right) = -\frac{L^2}{4} + MG^2$$

Hence $\int R^2 \, dS = L\left(\tfrac{1}{3} L^2 + p^2 - \tfrac{1}{4} L^2 + MG^2\right) = L\left(R_0^2 + \frac{L^2}{12}\right)$

where R_0 is the radius vector to the middle point of $A\,B$ and L is the length $A\,B$. Therefore for a series of triangulation lines

$$P = 4\cdot 03 \sqrt{\Sigma M^2 L\left(R_0^2 + \frac{L^2}{12}\right)} \quad \ldots \ldots \ldots \ldots \quad (5)$$

in which the quantities L, R_0 may be measured off a chart in units of 100 miles. From either (4) or (5) it is clear that the probable closing error in northing or easting is different according as different points of a circuit are selected on which to close and from which to start.

2. Formulæ (2), (5) show that the probable errors in azimuth and logarithm of terminal side increase as the square root of the length of the several triangulation lines involved, while those of easting and northing increase at a much more rapid rate namely as the three halves power of the lengths, the triangulation lines remaining similar and similarly situated. On account of this latter fact it is desirable to have more frequent checks to prevent accumulation of errors than would be necessary if only length and azimuth of side were required. These checks may be obtained by measurement of bases and by forming Laplace stations, that is stations whose longitudes are observed telegraphically and at which astronomical azimuths are also observed. These two checks are of precisely equal importance; and applying only one of them does not serve a very useful purpose. In the Indian triangulation eight base lines have been measured to date (1916), excluding the short Mergui base in Burma, and these have been made use of in the adjustment of the triangulation. The longitude arcs were not available when (previous to 1879) the main adjustment was carried out. They do not all admit of the formation of Laplace equations, as the longitude stations are not coincident with the triangulation stations, nor can they be connected with satisfactory accuracy (as regards azimuth) in all cases. Only latterly* (1906) have they been applied to control the azimuth observations with a view to determining corrected plumb-line deflections in the prime vertical. This application does not improve the probable error in easting and northing of the points concerned or any other points of the triangulation.

3. It is necessary for the full consideration of the problem to find expressions for the probable errors after certain further adjustments, *viz.* closing on extra base lines, closing of circuits or (what has not been done in India) closing on Laplace stations, have been effected. Before treating

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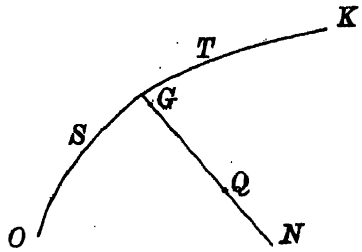
* Account of the Operations of the G. T. Survey of India, Vol. XVIII, Appendix 5.

this question quite generally it may be of interest to consider a special case. Suppose a series of triangulation lines closes between two bases. The probable error in easting or northing at any point of it after the adjustment has been performed is required. It is to be noticed in (4) that each portion of the triangulation is fully taken account of by the portion of the integral $\int M^2 R^2 \, dS$ which applies to it: that is to say, this portion of the integral represents the errors of displacement generated in the corresponding part of the triangulation, as if carried on to the closing point by perfect triangulation.

Let O,K be the two bases and suppose ST is the portion whose error as generated at N is required.



As in § 1 the expression for Δx for ST may be written

$$\Delta x = \sum_s^t \left(\epsilon_r x_r - \eta_r y_r \right)$$

The adjustment under consideration does not affect the η terms. The condition of closing gives (assuming uniform strength along OK)

$$\sum_1^k \epsilon_r = \text{a known quantity}$$

and in adjusting for this ϵ_r is replaced by $\epsilon_r - \frac{1}{k}\sum_1^k \epsilon_r$.

Denote the adjusted value of Δx by ${}_a\Delta x$. Then

$$ {}_a\Delta x = \sum_s^t \left\{ x_r \left(\epsilon_r - \frac{1}{k} \sum_1^k \epsilon_r \right) - \eta_r y_r \right\} $$

$$ = \sum_s^t \epsilon_r x_r - \frac{t-s}{k} \overset{t}{\underset{s}{X}} \sum_1^k \epsilon_r - \sum_s^t \eta_r y_r $$

where $\overset{t}{\underset{s}{X}}$ is the x-coordinate of the centre of gravity of ST. Hence

$$ {}_a\Delta x = -\frac{t-s}{k} \overset{t}{\underset{s}{X}} \left(\sum_1^{s-1} \epsilon + \sum_{t+1}^k \epsilon \right) + \sum_s^t \epsilon_r \left(x_r - \frac{t-s}{k} \overset{t}{\underset{s}{X}} \right) - \sum_s^t \eta_r y_r $$

Hence the probable error of displacement at N, after OK has been adjusted, due to the portion ST is the square root of

$$ ({}_a\Delta x)^2 + ({}_a\Delta y)^2 = (k-t+s) \left(\frac{t-s}{k} \right)^2 \epsilon^2 NG^2 + \epsilon^2 \sum_s^t R_Q^2 + \eta^2 \sum_s^t R_N^2 $$

where G is the centre of gravity of ST and Q lies on NG and $QN/GN = (t-s)/k$: also R_Q and R_N are radius vectors measured from Q and N respectively. Denoting the resultant probable displacement by D, this relation may be written, putting $OK = S_0$, $ST = S$ (measured along the curve)

$$ D^2 = k-t+s \left(\frac{S}{S_0} \right)^2 . NG^2 . \epsilon^2 + \epsilon^2 \sum^t R_Q^2 + \eta^2 \sum_s^t R_N^2 $$

$$ = \epsilon^2 \left\{ n \, NQ^2 + \sum_s^t R_Q^2 \right\} + \eta^2 \sum_s^t R_N^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6) $$

where $n = k - t + s$ is the number of sides in the line ST. Hence

$$D^2 = a^2 n \left\{ NQ^2 + \frac{1}{S}\int R^2_Q \, dS + \frac{1}{S}\int R^2_N \, dS \right\}$$

If the portion ST is a single triangulation line this becomes

$$D^2 = a^2 n \left\{ NQ^2 + \rho_0^2 + \frac{L^2}{12} + R_0^2 + \frac{L^2}{12} \right\}$$

where ρ_0 R_0 are the distances of the centre of gravity of ST from N and Q respectively and S now becomes equal to L. Putting in numerical quantities and expressing the results in feet it follows, as in (4), that

$$D = 4\cdot 03\,\mathrm{M}\sqrt{L\left(R^2_0 + \rho^2_0 + \frac{L^2}{6} + NQ^2\right)}$$

Finally if ST is composed of a number of triangulation lines, the probable displacement in any direction

$$P = \frac{D}{\sqrt{2}} = 2\cdot 85\sqrt{\Sigma \mathrm{M}^2 L\left(R_0^2 + \rho_0^2 + \frac{L^2}{6} + NQ^2\right)} \quad . \; . \; . \; . \; . \quad (7)$$

where the points Q differ for each triangulation line and are found for each as has been done above for ST alone.

If closure had also been effected on Laplace station as well as on base lines at O and K this clearly would become

$$P = 4\cdot 03\sqrt{\Sigma \mathrm{M}^2 L\left(\rho_0^2 + \frac{L^2}{12} + NQ^2\right)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Equations (7) and (8) illustrate the statement made in §2 that closure only on a base and not on a Laplace station does not improve the results nearly so much as closure of both kinds simultaneously will probably do.

Probable errors after adjustment.

4. Consider any function F of quantities x_1, x_2, x_3 . . which have been found by measurement. If the true values of these quantities are $x_1 + v_1$, $x_2 + v_2$, . . and the true value of the function is $F + dF$ then

$$\begin{aligned} dF &= F(x_1 + v_1,\ x_2 + v_2,\ \ldots) - F(x_1, x_2\ \ldots) \\ &\doteq f_1 v_1 + f_2 v_2 + \quad \ldots \ldots \ldots \end{aligned} \quad (9)$$

where $$f_1 = \frac{dF}{dx_1},\ f_2 = \frac{dF}{dx_2},\ \text{etc.}\ \ldots$$

Any conditions which may be imposed on x_1, x_2 . . . result in equations of form

$$\left.\begin{aligned} a_1 v_1 + a_2 v_2 + \ \ldots \ + a_n v_n - l_1 = 0 \\ b_1 v_1 + b_2 v_2 + \ \ldots \ + b_n v_n - l_2 = 0 \\ \text{etc.} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

Then dF may be written

$$dF = f_1 v_1 + f_2 v_2 \quad . \quad . \quad - k_1 (a_1 v_1 + a_2 v_2 + \quad . \quad . \quad - l_1) - k_2 (b_1 v_1 + b_2 v_2 + \quad . \quad . \quad - l_2) - . \quad .$$

in which any values may be assigned to k_1, k_2 . . Hence

$$dF = (f_1 - k_1 a_1 - k_2 b_1 - \quad . \quad . \quad . \quad) v_1 + (f_2 - k_1 a_2 - k_2 b_2 - \quad . \quad . \quad . \quad) v_2$$
$$. \quad + \quad . \quad . \quad . \quad + k_1 l_1 + k_2 l_2 + \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

Let the reciprocal weights of $x_1 x_2$ . . be u_1, u_2 . . and let the reciprocal weight of F be u_F. Then it follows that

$$u_F = (f_1 - k_1 a_1 - k_2 b_1 - \quad . \quad . \quad)^2 u_1 + (f_2 - k_1 a_2 - k_2 b_2 - \quad . \quad . \quad)^2 u_2 + \quad . \quad . \quad . \quad . \qquad (12)$$

If v_1, v_2 . . are the most probable values of the corrections, then u_F must be a minimum, and accordingly $k_1, k_2,$ . . are to be determined from the following equations

$$\left.\begin{array}{ll} [u a a] k_1 + [u a b] k_2 + \quad . \quad . \quad . \quad . \quad . \quad . \quad . & = [u a f] \\ [u b a] k_1 + [u b b] k_2 + \quad . \quad . \quad . \quad . \quad . \quad . \quad . & = [u b f] \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . & \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

in which $[u a b]$ is written for $u_1 a_1 b_1 + u_2 a_2 b_2 +$ . . , etc.

Reverting to (12) and developing the squares it is clear that

$$u_F = [uff] - 2[uaf] k_1 - 2[ubf] k_2 - \quad . \quad . \quad . \quad .$$
$$+ k_1 ([u a a] k_1 + [u a b] k_2 + \quad . \quad) + k_2 ([uab] k_1 + [ubb] k_2 + \quad . \quad) + \quad .$$
$$= [uff] - [uaf] k_1 - [ubf] k_2 - \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

by using the equations (13)*.

5. In considering the probable errors of triangulation it will be permissible to treat the quantities ϵ and η as errors in observed quantities, and as being independent except in so far as the several closing conditions relate them. Distinguish the several triangulation lines which make up a network of triangulation by the prefixes 1, 2, 3, etc., and the several stations on any triangulation by suffixes 1, 2, 3, etc. It will also be permissible in an investigation into the probable errors after adjustment to replace the triangulation by its projection on a plane, the projection of the general map of India naturally being selected for this purpose. This enables the closing conditions to be written in the following form, either for closure round a circuit or for closure between base lines or Laplace stations:

$$\left.\begin{array}{lll} \Sigma \Sigma \, {}_r\epsilon_s & = 0 & \text{side closure} \\ \Sigma \Sigma \, {}_r\eta_s & = 0 & \text{azimuth closure} \\ \Sigma \Sigma \, {}_r(x\epsilon - y\eta)_s & = 0 & \text{easting closure} \\ \Sigma \Sigma \, {}_r(y\epsilon + x\eta)_s & = 0 & \text{northing closure} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

the first summation for S corresponding to the number of sides in a triangulation line and the second for r corresponding to the number of triangulation lines: and x, y being the coordinates of any station referred to the closing point of the particular circuit as origin.

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* This deduction is taken from p. 229 of "*A Treatise on the Adjustment of Observations*" by T. W. Wright, New York, 1884.

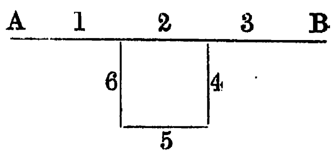
There may be any number of each type of closures, and not necessarily the same number for each type. The triangulation lines over which the summations are taken differ in part for each closure of any type. Thus in the case supposed in Chapter VI §14 the line 3 occurs in both circuits. The quantities whose probable errors are required are of the same form as the left hand sides of equations (15): but the limits are different.

It is clear that the coefficients in (13) may be written, agreeably with the notation already adopted

$$\left.\begin{array}{l}[u\,a\,a] = \Sigma_r[u\,a\,a] \\ [u\,a\,b] = \Sigma_r[u\,a\,b] \\ \quad \text{etc.}\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (16)$$

and accordingly the portion relating to each triangulation line may be separately computed. [For the side and azimuth closures the coefficients a, b are all unity: for the other two closures they are x or y. Consider the case of a triangulation line which forms part of a closed circuit, so that there are closures of each type. It may also form part of a closure between base lines and Laplace stations. If so the coefficients of the several ϵ^s and η^s remain the same as in corresponding type of closure in the circuit, and the coefficients reduce to type $[u\,a\,a]$. For the case of clearness take the specific case shown in the diagram when there are base lines and Laplace stations at A and B.

Consider the line 2. It enters into the following relations:



$$\begin{array}{ll} a & \Sigma_1\epsilon_s + \Sigma_2\epsilon_s + \Sigma_3\epsilon_s = 0 \\ b & \Sigma_2\epsilon_s + \Sigma_4\epsilon_s + \Sigma_5\epsilon_s + \Sigma_6\epsilon_s = 0 \\ c & \Sigma_1\eta_s + \Sigma_2\eta_s + \Sigma_3\eta_s = 0 \\ d & \Sigma_2\eta_s + \Sigma_4\eta_s + \Sigma_5\eta_s + \Sigma_6\eta_s = 0 \end{array}$$

$$\begin{array}{ll} e & \Sigma_2\,(x\epsilon - y\eta)_s + \Sigma_4(x\epsilon - y\eta)_s + \Sigma_5(x\epsilon - y\eta)_s + \Sigma_6(x\epsilon - y\eta)_s = 0 \\ f & \Sigma_2\,(y\epsilon + x\eta)_s + \Sigma_4(y\epsilon + x\eta)_s + \Sigma_5(y\epsilon + x\eta)_s + \Sigma_6(y\epsilon + x\eta)_s = 0 \end{array}$$

The symbolic coefficients $a\ b\ c\ d\ e\ f$ are each written opposite one of these equations.

Along any triangulation line for u write a^2, a being the probable value of ϵ or η. Suppose that ${}_2n$ is the number of sides in the triangulation line, all considered of equal length. Then omitting the prefix 2 for simplicity and considering only the line 2

$$\begin{array}{l} [u\,a\,a] = na^2 \\ [u\,a\,b] = na^2 \\ [u\,a\,c] = 0 \\ [u\,a\,d] = 0 \\ [u\,a\,e] = a^2\Sigma x = na^2X \\ [u\,a\,f] = a^2\Sigma y = na^2Y \end{array}$$

where X, Y are the coordinates of the centre of gravity of the line 2. These are typical of all the combinations of $a\ b\ c\ d$ with $a\ b\ c\ d\ e\ f$. The remaining typical coefficients are represented by

$$[u\,e\,e] = [u\,f\,f] = a^2\Sigma\,(x^2 + y^2)$$

If x_1 x_2 are the limits of x and $x_1 = X - x_0$ and $x_2 = X + x_0$

$$\Sigma x^2 \doteq \frac{n}{x_2 - x_1} \int x^2 \, dx$$

$$= \frac{n}{3}\left(x_2^2 + x_2 x_1 + x_1^2\right)$$

$$= \frac{n}{3}\left(\overline{X + x_0}|^2 + (X + x_0)(X \quad x_0) + \overline{X - x_0}|^2\right)$$

$$= n(X^2 + \tfrac{1}{3} x_0^2) \quad \ldots \quad (17)$$

Similarly $\Sigma y^2 = n(Y^2 + \tfrac{1}{3} y_0^2)$

Hence

$$\Sigma (x^2 + y^2) = n(R^2 + \tfrac{1}{3} r_0^2) = n\left(R^2 + \frac{L^2}{12}\right) \quad \ldots \quad (18)$$

Finally $[uee] = na^2 \left(R^2 + \frac{L^2}{12}\right)$

Also $[uef] = a^2 \Sigma (xy - yx) = 0$

The complete result is given in tabular form:—

Values of ${}_2[u\,a\,a] \div na^2$, *etc.*

		a	b	c	d	e	f	
Side	a	1	1	0	0	X	Y	Base
Side	b	1	1	0	0	X	Y	
Azimuth	c	0	0	1	1	$-Y$	X	Laplace
Azimuth	d	0	0	1	1	$-Y$	X	
Easting	e	X	X	$-Y$	$-Y$	$R^2 + \frac{L^2}{12}$	0	
Northing	f	Y	Y	X	X	0	$R^2 + \frac{L^2}{12}$	

It is clear from this that there are really only four types of quantities, and that a closure on an outer circuit or base gives rise to the same coefficient as the corresponding closure within the circuit. All the quantities of the form ${}_r[u\,a\,b]$ are given by the following schemes, in which the four closures, side azimuth, easting, northing are represented by S, A, E, N.

Values of ${}_r[u\,a\,b] \div na^2$ . . . . (19)

	S	A	E	N
S	1	0	X	Y
A	0	1	$-Y$	X
E	X	$-Y$	$R^2 + \frac{L^2}{12}$	0
N	Y	X	0	$R^2 + \frac{L^2}{12}$

The value of na^2 is given by (1) in terms of M and S, or M and L if straight portions are considered separately.

The above scheme deals with the cases in which the coordinates x, y which occur all refer to the same origin. It only remains to take the case where two origins occur and to form the typical expression $_r[u\,a\,f]$. The coefficients a, etc. are either 0, 1, x or $\pm\, y$ in which x, y are the coordinates of any point on the line referred to the point of closure for which the corresponding condition of closure was formed. When it is desired to find the probable error with regard to any point O (for example Kalianpur, the origin of the survey), then the coefficients f, etc. are either 0, 1, x_0 or $\pm\, y_0$ where $x_0\; y_0$ are coordinates referred to this point. If the circuit closing point is P let $x_p\; y_p$ be the coordinates of P with regard to origin O then

$$x_0 = x_p + x \qquad y_0 = y_p + y.$$

The values of $_r[u\,a\,f] \div na^2$ are now indicated in tabular form similar to (19). As only the ratios of the coefficients in (13) are required na^2 may be replaced by M^2L by means of (1).

	S_f	A_f	E_f	N_f
S	1	0	$X + x_p = X_0$	$Y + y_p = Y_0$
A	0	1	$-Y - y_p = -Y_0$	$X + x_p = X_0$
E	X	$-Y$	$R^2 + \frac{L^2}{12} + x_p X + y_p Y$	$-x_p Y + y_p X$
N	Y	X	$x_p Y - y_p X$	$R^2 + \frac{L^2}{12} + x_p X + y_p Y$

. . . . . (20)

All the quantities X, Y, R are measured from P while $x_p\; y_p$ are coordinates of P and $X_0\; Y_0$ are the coordinates of the mid point of the triangulation line relative to O.

6. The method explained in §§ 4,5 will now be applied to the determination of probable errors in the N.W. Quadrilateral of the Indian triangulation, after all adjustments have been carried out. The probable errors most generally required will be those with reference to the origin of the survey at Kalianpur (Sironj Base). In putting down the conditions of closure any closing point may be chosen: but when probable errors with regard to Kalianpur are desired, Kalianpur will naturally be selected as the closing point and origin for X and Y. Chart I shows all the triangulation of India: in charts II . . . . V it is represented diagrammatically, each series being replaced by one or more straight lines, which may be regarded as the equivalent triangulation lines. The Indian triangulation was divided for purposes of adjustment into five portions, *viz.* the N.W. Quadrilateral, S.E. Quadrilateral, N.E. Quadrilateral, Southern Trigon, S.W. Quadrilateral. These were adjusted in the order stated, so that the first two were quite independently adjusted while the third was adjusted on the first two: the fourth was adjusted on the second, and the fifth was adjusted on the first, second and fourth. The Burma quadrilateral (chart IV) has just* been adjusted by the methods of Chapter VI and is being adjusted on the eastern series (Shillong Meridional, No. 44) of the N.E. Quadrilateral.

In chart II the series which were taken account of simultaneously in each quadrilateral or trigon are shown in full lines, while some additional series afterwards adjusted on these series are shown in broken lines. The series which are common to adjacent quadrilaterals or trigon are distinguished by heavy lines. The numbers written by the side of each triangulation line in the chart are those which have been applied in Table XLIV to the several series of the triangulation. The eight base lines of the triangulation of India and the Mergui base in Burma are shown: also the points at which it has been possible to form Laplace equations. The circuits are indicated by roman numerals and the points of closure by small arcs at the closing angle, *e. g* at D. The junction points of the triangulation lines are distinguished by letters A, B . . Z, a, b . . with suffixes 1, 2, 3, 4, 5, 6 corresponding to the N.W. Quadrilateral, S.E. Quadrilateral, N.E. Quadrilateral, S. Trigon, S.W. Quadrilateral, Burma Quadrilateral.

* 1917.

The first step is to find M^2L for each triangulation line, L being the length in units of 100 miles. Chart II is on the scale of 100 miles = 1 inch, so that L is the length of each line on the chart in inches. As an example take the line between the Sironj base and the Dehra base. This is composed of two triangulation lines representing a portion of the Great Arc Series, No. 6. From table XLIV the value of M is 0·71 and by measurement on the chart the values of L for the two parts are found to be 2·15 and 2·22 inches. Hence the values of M^2L are 1·083 and 1·109 respectively.

The values of M^2L for all the triangulation lines are exhibited in table XLV together with certain related quantities to which reference will be made later. To proceed with the formation of the equations of form (13) which are necessary to determine the several probable errors, all quantities of the types indicated in (20) have to be formed. To any coefficient $[u\,g\,h]$ several component terms, each corresponding to a particular triangulation line, may contribute. Some of these are exhibited in table XLV, while the remainder are found in table XLVI. To go more into detail, the N.W. Quadrilateral (*vide* chart II) is divided into five circuits I, II, III, IV, V. In each circuit there are four types of closure—side, azimuth, easting, northing—which may be characterised by suffixes *s, a, e, n*. These give rise to twenty conditions I_s, I_a, I_e, I_n, II_s, . . . V_n. In addition there are three extra base lines giving conditions VI_s, VII_s, $VIII_s$. To investigate the additional value of having as many Laplace stations as there are base lines, the cases have also been worked out for three Laplace closures at the same points as the base line closures, giving rise to conditions VI_a, VII_a, $VIII_a$. The 26 conditions which result make it necessary to determine 26 multipliers k_1 . . . k_{26} by equations of form (13). In table XLVII the coefficients of the left hand sides of these equations are given. The method by which these coefficients are derived will now be given in detail for a few of them. The letters used correspond to those shown marginally in table XLVII.

By (20) ${}_r[u\,a\,a] + M^2L = 1$: hence $[u\,a\,a] = \Sigma M^2L$ round circuit I = 3·66 from table XLV. Also $[u\,a\,b]=0$, $[u\,a\,c]=\Sigma X\,M^2L=2 \cdot 20$ from table XLV. In the formation of any coefficients involving any pair of a, b, c or d it is clear that the summation extends around the circuit I, for the corresponding conditions relate to this complete circuit. The case is different when a coefficient involving one of the quantities, a, b, c, d and one other quantity, say e, are considered. In this case both circuits I and II are involved and it is only the part common to the two circuits that has to be considered. Moreover in this case two origins are introduced. The portion common to circuits I and II is the line D_1T_1. It will be seen by (20) that

$[u\,a\,e] = M^2L$ for $D_1T_1 = \cdot 70$ from table XLV.

$[u\,a\,f] = 0$

$[u\,a\,g] = X_0M^2L$ for D_1T_1, X_0 referring to the circuit to which g relates, *i. e.* circuit II
$= + \cdot 41$

$[u\,a\,h] = Y_0M^2L$ for D_1T_1 from circuit II $= -2 \cdot 30$

This deals with all the conditions which relate to a portion which occurs also in condition a, except the base line conditions VI_s and VII_s. The most complex case is the coefficient corresponding to northing or easting relations in two circuits which have a portion in common. An example of this is $[u\,d\,g]$. The circuits involved I, II have the portion T_1D_1 only in common. Hence from (20) as may be seen opposite the entry I, II in table XLVI, $[u\,d\,g] = M^2L\,(x_0Y - y_0X) = -1 \cdot 14$. Similarly $[u\,d\,h] = M^2L\,(R^2 + L^2/12 + x_0X + y_0Y) = 7 \cdot 35$. The quantities in tables XLV, XLVI depend on measurements of L_1, R_1, X, Y taken from a chart. These are not quite precise, and so quantities deduced from them are not quite precise. All the quantities which are known by symmetry to be equal in pairs have been separately determined by way of a check. They differ slightly as will be seen in table XLVI, the worst case being the coefficient which occurs in line II, III and also in line III, II of which the values 29·56 and 29·68 are obtained. The quantity made use of is the mean 29·62 shown in block type.

TABLE XLV.

N.W. Quadrilateral.

Base line closures	Circuit	Line	Series	M	L	A = M^2L	Closing point referred to Kalianpur	R^2	$\frac{L^2}{12}$	$R^2+\frac{L^2}{12}$	X	Y	A × $\left(R^2+\frac{L^2}{12}\right)$	AX	AY
	I	$D_1\ T_1$	33	0·37	5·10	0·608	D_1	6·45	2·17	8·62	−0·04	−2·54	6·02	− 0·03	− 1·77
4		$T_1\ A_1$	25	0·60	0·98	0·353	$x_p = -0·91$	26·50	0·08	26·58	+0·42	−5·13	9·38	+ 0·15	− 1·81
1		$A_1\ B_1$	6	0·71	2·15	1·083	$y_p = +5·18$	17·63	0·39	18·02	+0·81	−4·12	19·50	+ 0·88	− 4·46
1		$B_1\ C_1$	6	0·71	2·22	1·109		4·57	0·41	4·98	+0·88	−1·95	5·52	+ 0·98	− 2·16
3		$C_1\ D_1$	22	0·55	1·38	0·418		0·46	0·16	0·62	+0·53	−0·42	0·26	+ 0·23	− 0·18
						3·661							40·68	+ 2·20	−10·38
	II	$E_1\ U_1$	23	1·21	1·36	1·990	E_1	0·48	0·15	0·63	−0·04	−0·60	1·25	− 0·08	− 1·37
		$U_1\ X_1$	23	1·21	1·85	2·706	$x_p = -1·54$	5·16	0·29	5·45	−0·11	−2·27	14·75	− 0·30	− 6·14
		$X_1\ Y_1$	23	1·21	1·13	1·654	$y_p = +5·94$	14·29	0·11	14·40	−0·27	−3·77	23·80	− 0·45	− 6·23
		$Y_1\ S_1$	23	1·21	1·44	2·107		25·37	0·17	25·54	−0·27	−5·03	53·80	− 0·57	−10·60
4		$S_1\ T_1$	25	0·60	0·70	0·252		33·53	0·04	33·57	+0·20	−5·79	8·45	+ 0·05	− 1·46
		$T_1\ D_1$	33	0·37	5·10	0·698		11·24	2·17	13·41	+0·59	−3·30	9·36	+ 0·41	− 2·30
3		$D_1\ E_1$	22	0·55	1·01	0·306		0·25	0·08	0·33	+0·32	−0·39	0·10	+ 0·10	− 0·12
						9·713							111·51	− 0·84	−28·22
5	III	$J_1\ L_1$	32	0·43	1·99	0·368	J_1	1·00	0·33	1·33	−0·90	−0·44	0·49	− 0·33	− 0·16
5		$L_1\ N_1$	32	0·43	1·67	0·309	$x_p = -4·26$	6·71	0·23	6·94	−1·98	−1·67	2·14	− 0·61	− 0·52
4		$N_1\ P_1$	25	0·60	2·28	0·822	$y_p = +3·36$	7·81	0·43	8·24	−1·00	−2·61	6·77	− 0·82	− 2·15
4		$P_1\ Q_1$	25	0·60	1·02	0·368		8·53	0·09	8·62	+0·64	−2·85	3·17	+ 0·24	− 1·05
4		$Q_1\ S_1$	25	0·60	1·46	0·526		12·91	0·18	13·09	+1·85	−3·08	6·88	+ 0·97	− 1·62
		$S_1\ Y_1$	23	1·21	1·44	2·106		12·00	0·17	12·17	+2·45	−2·45	25·61	+ 5·16	− 5·16
		$Y_1\ X_1$	23	1·21	1·13	1·653		7·42	0·11	7·53	+2·45	−1·19	12·46	+ 4·05	− 1·97
		$X_1\ U_1$	23	1·21	1·85	2·707		6·90	0·29	7·19	+2·61	+0·30	19·46	+ 7·06	+ 0·81
		$U_1\ V_1$	45	0·53	0·98	0·275		5·90	0·08	5·98	+2·21	+1·01	1·65	+ 0·61	+ 0·28
		$V_1\ J_1$	45	0·53	1·94	0·545		0·92	0·31	1·23	+0·88	+0·39	0·67	+ 0·48	+ 0·21
						9·679							79·30	+16·81	−11·33
	IV	$F_1\ V_1$	37	0·50	2·37	0·825	F_1	1·44	0·47	1·91	+0·01	−1·20	1·58	+ 0·01	− 0·99
		$V_1\ U_1$	45	0·53	0·98	0·275	$x_p = -2·50$	4·95	0·08	5·03	+0·45	−2·18	1·38	+ 0·12	− 0·60
		$U_1\ E_1$	23	1·21	1·36	1·990	$y_p = +6·55$	2·55	0·15	2·70	+0·93	−1·30	5·37	+ 1·85	− 2·50
3		$E_1\ F_1$	22	0·55	1·11	0·336		0·30	0·10	0·40	+0·46	−0·30	0·13	+ 0·15	− 0·10
						3·426							8·46	+ 2·13	− 4·28
5	V	$G_1\ H_1$	32	0·43	1·73	0·320	G_1	0·87	0·25	1·12	−0·56	−0·75	0·36	− 0·18	− 0·24
5		$H_1\ J_1$	32	0·43	2·03	0·376	$x_p = -2·88$	7·59	0·34	7·93	−1·20	−2·48	2·98	− 0·45	− 0·93
		$J_1\ V_1$	45	0·53	1·94	0·545	$y_p = +6·80$	9·55	0·31	9·86	−0·50	−3·05	5·37	− 0·27	− 1·66
		$V_1\ F_1$	37	0·59	2·37	0·825		2·25	0·47	2·72	+0·30	−1·45	2·24	+ 0·32	− 1·20
3		$F_1\ G_1$	22	0·55	0·54	0·163		0·04	0·02	0·06	+0·13	−0·15	0·01	+ 0·02	− 0·02
						2·229							10·96	− 0·56	− 4·05

Base line closures	Circuit	Line	A = M^2L	AX	AY
VI 1	I	$A_1\ B_1$	1·083	+0·88	−4·46
		$B_1\ C_1$	1·109	+0·98	−2·16
			+2·192	+1·86	−6·62
VII 3	I	$C_1\ D_1$	0·418	+0·22	−0·18
	II	$D_1\ E_1$	0·306	+0·10	−0·12
	IV	$E_1\ F_1$	0·336	+0·15	−0·10
	V	$F_1\ G_1$	0·163	+0·02	−0·02
			+1·223	+0·49	−0·42
VIII 5	III	$J_1\ L_1$	0·368	−0·33	−0·16
		$L_1\ N_1$	0·309	−0·61	−0·52
	V	$G_1\ H_1$	0·320	−0·18	−0·24
		$H_1\ J_1$	0·376	−0·45	−0·93
			+1·373	−1·57	−1·85

TABLE XLVI.

N.W. Quadrilateral. Common portions of adjacent circuits.

Circuits		Line	First circuit referred to second		For first circuit			AXx_p	AYy_p	Sum of last 3 columns		AYx^p	$-AXy_p$	Sum of last 2 columns	
			x_p	y_p	AX	AY	$A\left(R^2+\frac{L^2}{12}\right)$								
I	II	$D_1\ T_1$	+ ·63	− ·76	− ·03	− 1·77	+ 6·02	− 0·02	+ 1·24	+ 7·34	+ 7·35	− 1·11	− 0·02	− 1·13	∓ 1·14
II	I	$T_1\ D_1$	− ·63	+ ·76	+ ·41	− 2·30	+ 9·36	− 0·26	− 1·74	+ 7·36		+ 1·45	− 0·31	+ 1·14	
II	III	$U_1\ X_1$	+ 2·72	+ 2·58	− ·30	− 6·14	+ 14·75								
		$X_1\ Y_1$			− ·45	− 6·23	+ 23·80								
		$Y_1\ S_1$			− ·57	− 10·60	+ 53·80								
					− 1·32	− 22·07	+ 92·35	− 3·59	−59·20	+ 29·56		−62·45	+ 3·40	− 59·05	
III	II	$S_1\ Y_1$	− 2·72	− 2·58	+ 5·16	− 5·16	+ 25·61				+29·62				∓59·10
		$Y_1\ X_1$			+ 4·05	− 1·97	+ 12·46								
		$X_1\ U_1$			+ 7·06	+ 0·81	+ 19·46								
					+ 16·27	− 6·32	+ 57·53	−44·15	+16·30	+ 29·68		+17·19	+41·95	+ 59·14	
II	IV	$E_1\ U_1$	+ 0·96	− 0·61	− ·08	− 1·37	+ 1·25	− 0·08	+ 0·84	+ 2·01	+ 2·20	− 1·31	− 0·05	− 1·36	∓ 1·36
IV	II	$U_1\ E_1$	− 0·96	+ 0·61	+ 1·35	− 2·59	+ 5·37	− 1·77	− 1·58	+ 2·02		+ 2·48	− 1·13	+ 1·35	
III	IV	$U_1\ V_1$	− 1·76	− 3·10	+ ·61	+ ·28	+ 1·65	− 1·07	− 0·89	− 0·31	− 0·32	− 0·40	+ 1·94	+ 1·45	± 1·44
IV	III	$V_1\ U_1$	+ 1·76	+ 3·10	+ ·12	− ·60	+ 1·38	+ 0·21	− 1·91	− 0·32		− 1·05	− 0·38	− 1·43	
III	V	$V_1\ J_1$	− 1·38	− 3·44	+ ·48	+ ·21	+ ·67	− 0·66	− 0·72	− 0·71	− 0·71	− 0·29	+ 1·65	+ 1·36	± 1·36
V	III	$J_1\ V_1$	+ 1·38	+ 3·44	− ·27	− 1·66	+ 5·37	− 0·37	− 5·71	− 0·71		− 2·29	+ 0·93	− 1·36	
IV	V	$F_1\ V_1$	+ 0·38	− 0·25	+ ·01	− ·99	+ 1·58	0·00	+ 0·25	+ 1·83	+ 1·83	− 0·38	0·00	− 0·38	∓ 0·38
V	IV	$V_1\ F_1$	− 0·38	+ 0·25	+ ·32	− 1·20	+ 2·24	− 0·12	− 0·30	+ 1·82		+ 0·46	− 0·08	+ 0·38	

The above explanation should make clear the formation of table XLVII, which gives the right hand sides of a set of equations. The left hand sides of these equations are different according as the quantity, whose probable error is sought, is of different form or relative to a different point as origin. To obtain a solution of all the cases which may arise it is desirable to keep the left hand sides of the equation in symbolical form denoting them by A, B, C . . . . Z. It will be seen that this renders possible solution in a form which afterwards admits of the solution of the same conditions together with any further conditions that may be added.

TABLE XLVII.

N. W. QUADRILATERAL.

		I_s	I_a	I_c	I_n	II_s	II_a	II_c	II_n	III_s	III_a	III_c	III_n	IV_s	IV_a	IV_c	IV_n	V_s	V_a	V_c	V_n	VI_s	VII_s	VIII_s	VI_a	VII_a	VIII_a
		k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	k_{16}	k_{17}	k_{18}	k_{19}	k_{20}	k_{21}	k_{22}	k_{23}	k_{24}	k_{25}	k_{26}
I_s	1	+ 3·66	0	+ 2·20	−10·38	+ ·70	0	+ ·41	− 2·30	0	0	0	0	0	0	0	0	0	0	0	0	2·19	·42	0	0	0	0
a	2	0	+ 3·66	+10·38	+ 2·20	0	+ ·70	+ 2·30	+ ·41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2·19	+ ·42	0
c	3	+ 2·20	10·38	+40·68	0	− ·03	+ 1·77	+ 7·35	+ 1·14	0	0	0	0	0	0	0	0	0	0	0	0	+ 1·86	+ ·22	0	+ 6·62	+ ·18	0
n	4	−10·38	+ 2·20	0	+40·68	− 1·77	− ·03	− 1·14	+ 7·35	0	0	0	0	0	0	0	0	0	0	0	0	− 6·62	− 0·18	0	+ 1·86	+ 0·22	0
II_s	5	+ ·70	0	− ·03	− 1·77	+ 9·71	0	− ·84	−28·22	+ 6·47	0	+16·27	− 6·32	+ 1·99	0	+ 1·85	− 2·59	0	0	0	0	0	+ ·31	0	0	0	0
a	6	0	+ ·70	+ 1·77	− ·03	0	+ 9·71	+28·22	− ·84	0	+ 6·47	+ 6·32	+16·27	0	+ 1·99	+ 2·59	+ 1·85	0	0	0	0	0	0	0	0	+ ·31	0
c	7	+ ·41	+ 2·30	+ 7·35	− 1·14	− ·84	+28·22	+111·51	0	− 1·32	+22·97	+29·62	+59·10	− ·08	+ 1·37	+ 2·02	+ 1·36	0	0	0	0	0	+ ·10	0	0	+ ·12	0
n	8	− 2·30	+ ·41	+ 1·14	+ 7·35	−28·22	− ·84	0	+111·51	−22·97	− 1·32	−59·10	+29·62	− 1·37	− ·08	− 1·36	+ 2·02	0	0	0	0	0	− ·12	0	0	+ ·10	0
III_s	9	0	0	0	0	+ 6·47	0	− 1·32	−22·97	+ 9·68	0	+16·81	−11·38	+ 0·28	0	+ ·12	− ·60	+ ·55	0	− ·27	− 1·66	0	0	+ ·68	0	0	0
a	10	0	0	0	0	0	+ 6·47	+22·97	− 1·32	0	+ 9·68	+11·33	+16·81	0	+ 0·28	+ ·60	+ ·12	0	+ ·55	+ 1·66	− ·27	0	0	0	0	0	+ ·68
c	11	0	0	0	0	+16·27	+ 6·32	+29·62	−59·10	+16·81	+11·33	+79·30	0	+ ·61	− ·28	− ·32	− 1·44	+ ·48	− ·21	− ·71	− 1·36	0	0	− ·94	0	0	+ ·68
n	12	0	0	0	0	− 6·32	+16·27	+59·10	+29·62	−11·33	+16·81	0	+79·30	+ ·28	+ ·61	+ 1·44	− ·32	+ ·21	+ ·48	+ 1·36	− ·71	0	0	− ·68	0	0	− ·94
IV_s	13	0	0	0	0	+ 1·99	0	− ·08	− 1·37	+ ·28	0	+ ·61	+ ·28	+ 3·43	0	+ 2·13	− 4·28	+ ·83	0	+ ·32	− 1·20	0	+ ·34	0	0	0	0
a	14	0	0	0	0	0	+ 1·99	+ 1·37	− ·08	0	+ ·28	− ·28	+ ·61	0	+ 3·43	+ 4·28	+ 2·13	0	+ ·83	+ 1·20	+ ·32	0	0	0	0	+ ·34	0
c	15	0	0	0	0	+ 1·85	+ 2·59	+ 2·02	− 1·36	+ ·12	+ ·60	− ·32	+ 1·44	+ 2·13	+ 4·28	+ 8·46	0	+ ·01	+ ·99	+ 1·33	+ 0·38	0	+ ·15	0	0	+ ·10	0
n	16	0	0	0	0	− 2·59	+ 1·85	+ 1·36	+ 2·02	− ·60	+ ·12	− 1·44	− ·32	− 4·28	+ 2·13	0	+ 8·46	− ·99	+ ·01	− 0·38	+ 1·83	0	− ·10	0	0	+ ·15	0
V_s	17	0	0	0	0	0	0	0	0	+ ·55	0	+ ·48	+ ·21	+ ·83	0	+ ·01	− ·99	+ 2·23	0	− ·56	− 4·05	0	+ ·16	+ ·70	0	0	0
a	18	0	0	0	0	0	0	0	0	0	+ ·55	− ·21	+ ·48	0	+ ·83	+ ·99	+ ·01	0	+ 2·23	+ 4·05	− ·56	0	0	0	0	+ ·16	+ ·70
c	19	0	0	0	0	0	0	0	0	− ·27	+ 1·66	− ·71	+ 1·36	+ ·32	+ 1·20	+ 1·83	− ·38	− ·56	+ 4·05	+10·96	0	0	+ ·02	− ·63	0	+ ·02	+ 1·17
n	20	0	0	0	0	0	0	0	0	− 1·66	− ·27	− 1·36	− ·71	− 1·20	+ ·32	+ ·38	+ 1·83	− 4·05	− ·56	0	+10·96	0	− ·02	− 1·17	0	+ ·02	− ·63
VI_s	21	+ 2·19	0	+ 1·86	− 6·62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+ 2·19	0	0	0	0	0
VII_s	22	+ ·42	0	+ ·22	− ·18	+ ·31	0	+ ·10	− ·12	0	0	0	0	+ ·34	0	+ ·15	− ·10	+ ·16	0	+ ·02	− ·02	0	+ 1·22	0	0	0	0
VIII_s	23	0	0	0	0	0	0	0	0	+ ·68	0	− ·94	− ·68	0	0	0	0	+ ·70	0	− ·63	− 1·17	0	0	+ 1·37	0	0	0
VI_a	24	0	+ 2·19	+ 6·62	+ 1·86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+ 2·19	0	0
VII_a	25	0	+ ·42	+ ·18	+ ·22	0	+ ·31	+ ·12	+ ·10	0	0	0	0	0	+ ·34	+ ·10	+ ·15	0	+ ·16	+ ·02	+ ·03	0	0	0	0	1·22	0
VIII_a	26	0	0	0	0	0	0	0	0	0	+ ·68	+ ·68	− ·94	0	0	0	0	0	+ ·70	+ 1·17	− ·63	0	0	0	0	0	+ 1·37

7. To find the probable errors at any point of side azimuth easting or northing it is necessary to form the right hand sides of the equations of which the left hand side coefficients are exhibited in table XLVII, *i.e.* to complete the formation of equations (13). The quantities $[u\,a\,f]$ etc. are of the same nature as those already formed: but differ in the lines for which they have to be

TABLE XLVIII.

Line	Circuit	M	L	$A = M^2L$	R^2	$\frac{L^2}{12}$	X	Y	$R^2+\frac{L^2}{12}$	$A\left(R^2+\frac{L^2}{12}\right)$	AX	AY	X_0	Y_0
$K_1 L_1$	III	0·43	1·55	0·286	1·03	0·20	−0·91	−0·45	1·23	0·35	−0·26	−0·13	−5·17	+3·91
$M_1 N_1$	III	0·43	0·78	0·144	8·70	0·05	−2·06	−2·11	8·75	1·26	−0·30	−0·30	−6·32	+1·27
$O_1 P_1$	III	0·60	0·57	0·205	7·21	0·03	−0·14	−2·68	7·24	1·48	−0·03	−0·55	−4·40	+0·68
$R_1 S_1$	III	0·60	0·53	0·191	15·27	0·02	+2·31	−3·15	15·29	2·92	+0·44	−0·60	−1·95	+0·21
$V_1 W_1$	III	0·53	0·74	0·208	2·45	0·05	+1·43	+0·64	2·50	0·52	+0·30	+0·13	−2·83	+4·00
$V_1 W_1$	V	0·53	0·74	0·208	7·85	0·05	+0·05	−2·80	7·90	1·64	+0·01	−0·58	−2·83	+4·00
$W_1 J_1$	III	0·53	1·20	0·337	0·37	0·12	+0·55	+0·25	0·49	0·16	+0·19	+0·08	−3·71	+3·61
$W_1 J_1$	V	0·53	1·20	0·337	10·87	0·12	−0·83	−3·19	10·99	3·70	−0·28	−1·08	−3·71	+3·61

TABLE XLIX.

Circuit, Base and Laplace closures.

Line	Circuit	Closing point of circuit referred to Kalianpur x_p	y_p	(1) $A = M^2L$	(2) Ax_p	(3) Ay_p	(4) AX	(5) AY	(6)=(2)+(4) $=AX_0$	(7)=(3)+(5) $=AY_0$	(8) $A\left(R^2+\frac{L^2}{12}\right)$	(9) AXx_p	(10) AYy_p	(11)=(8)+(9)+(10)	(12) AYx_p	(13) $-AXy_p$	(14)=(12)+(13)	(15) $R_0^2+\frac{L^2}{12}$	(16)= A (15)
$A_1 B_1$	I, VI	−0·91	+5·18	1·083	− 0·99	+ 5·61	+0·88	− 4·46	−0·11	+ 1·15	+19·50	− 0·80	−23·10	− 4·40	+ 4·06	− 4·56	− 0·50	1·52	1·65
$B_1 C_1$	I, VI	−0·91	+5·18	1·109	− 1·01	+ 5·74	+0·98	− 2·16	−0·03	+ 3·58	+ 5·52	− 0·89	−11·10	− 6·56	+ 1·97	− 5·08	− 3·11	10·84	12·02
$C_1 D_1$	I, VII	−0·91	+5·18	0·418	− 0·38	+ 2·17	+0·22	− 0·18	−0·16	+ 1·99	+ 0·26	− 0·20	− 0·93	− 0·87	+ 0·16	− 1·14	− 0·98	22·96	9·59
$D_1 E_1$	II, VII	−1·54	+5·94	0·306	− 0·47	+ 1·82	+0·10	− 0·12	−0·37	+ 1·70	+ 0·10	− 0·15	− 0·71	− 0·76	+ 0·18	− 0·59	− 0·41	32·38	9·91
$E_1 F_1$	IV, VII	−2·50	+6·55	0·336	− 0·84	+ 2·20	+0·15	− 0·10	−0·69	+ 2·10	+ 0·13	− 0·37	− 0·66	− 0·90	+ 0·25	− 0·98	− 0·73	43·33	14·56
$F_1 G_1$	V, VII	−2·88	+6·80	0·163	− 0·47	+ 1·11	+0·02	− 0·02	−0·45	+ 1·09	+ 0·01	− 0·06	− 0·14	− 0·19	+ 0·06	− 0·14	− 0·08	51·81	8·45
$G_1 H_1$	V, VIII	−2·88	+6·80	0·320	− 0·92	+ 2·18	−0·18	− 0·24	−1·10	+ 1·94	+ 0·36	+ 0·52	− 1·63	− 0·75	+ 0·69	+ 1·22	+ 1·91	48·69	15·58
$H_1 J_1$	V, VIII	−2·88	+6·80	0·376	− 1·08	+ 2·56	−0·45	− 0·93	−1·53	+ 1·63	+ 2·98	+ 1·30	− 6·32	− 2·04	+ 2·68	+ 3·06	+ 5·74	35·65	13·40
$K_1 L_1$	III, VIII	−4·26	+3·36	0·286	− 1·22	+ 0·96	−0·26	− 0·13	−1·48	+ 0·83	+ 0·85	+ 1·11	− 0·44	+ 1·02	+ 0·55	+ 0·87	+ 1·42	35·40	10·12
$J_1 L_1$	III, VIII	−4·26	+3·36	0·368	− 1·57	+ 1·24	−0·33	− 0·16	−1·90	+ 1·08	+ 0·49	+ 1·41	− 0·54	+ 1·36	+ 0·68	+ 1·11	+ 1·79	35·48	13·06
$M_1 N_1$	III, VIII	−4·26	+3·36	0·144	− 0·61	+ 0·48	−0·30	− 0·30	−0·91	+ 0·18	+ 1·26	+ 1·28	− 1·01	+ 1·53	+ 1·28	+ 1·01	+ 2·29	41·56	5·98
$L_1 N_1$	III, VIII	−4·26	+3·36	0·309	− 1·32	+ 1·04	−0·61	− 0·52	−1·93	+ 0·52	+ 2·14	+ 2·60	− 1·75	+ 2·99	+ 2·22	+ 2·05	+ 4·27	42·03	12·99
$O_1 P_1$	III	−4·26	+3·36	0·205	− 0·87	+ 0·69	−0·03	− 0·55	−0·90	+ 0·14	+ 1·48	+ 0·13	− 1·85	− 0·24	+ 2·34	+ 0·10	+ 2·44	19·85	4·07
$N_1 P_1$	III	−4·26	+3·36	0·822	− 3·50	+ 2·76	−0·82	− 2·15	−4·32	+ 0·61	+ 6·77	+ 3·49	− 7·22	+ 3·04	+ 9·16	+ 2·76	+11·92	28·66	23·56
$P_1 Q_1$	III	−4·26	+3·36	0·368	− 1·57	+ 1·24	+0·24	− 1·05	−1·33	+ 0·19	+ 3·17	− 1·02	− 3·53	− 1·38	+ 4·47	− 0·81	+ 3·66	13·45	4·95
$R_1 S_1$	III	−4·26	+3·36	0·191	− 0·81	+ 0·64	+0·44	− 0·60	−0·37	+ 0·04	+ 2·92	− 1·87	− 2·02	− 0·97	+ 2·56	− 1·48	+ 1·08	3·87	0·74
$Q_1 S_1$	III	−4·26	+3·36	0·526	− 2·24	+ 1·77	+0·97	− 1·62	−1·27	+ 0·15	+ 6·88	− 4·13	− 5·44	− 2·69	+ 6·90	− 3·26	+ 3·64	6·06	3·19
$S_1 T_1$	II	−1·54	+5·94	0·252	− 0·39	+ 1·50	+0·05	− 1·40	−0·34	+ 0·04	+ 8·45	− 0·08	− 8·67	− 0·30	+ 2·25	− 0·30	+ 1·95	1·81	0·47
$T_1 A_1$	I	−0·91	+5·18	0·353	− 0·32	+ 1·83	+0·15	− 1·81	−0·17	+ 0·02	+ 9·38	− 0·14	− 9·37	− 0·13	+ 1·65	− 0·78	+ 0·87	0·32	0·11
$T_1 D_1$	I	−0·91	+5·18	0·698	− 0·64	+ 3·62	−0·03	− 1·77	−0·67	+ 1·85	+ 6·02	+ 0·08	− 9·17	− 3·12	+ 1·61	+ 0·16	+ 1·77	10·04	7·01
$T_1 D_1$	II	−1·54	+5·94	0·698	− 1·07	+ 4·15	+0·41	− 2·30	−0·66	+ 1·85	+ 9·36	− 0·63	−13·66	− 4·93	+ 3·54	− 2·44	+ 1·10		
$S_1 Y_1$	II	−1·54	+5·94	2·107	− 3·24	+12·51	−0·57	−10·60	−3·81	+ 1·91	+53·80	+ 0·88	−62·96	− 8·28	+16·32	+ 3·39	+19·71	4·28	9·02
$S_1 Y_1$	III	−4·26	+3·36	2·106	− 8·97	+ 7·08	+5·16	− 5·16	−3·81	+ 1·92	+25·61	−21·98	−17·34	−13·71	+21·98	−17·34	+ 4·64		
$Y_1 X_1$	II	−1·54	+5·94	1·654	− 2·55	+ 9·82	−0·45	− 6·24	−3·00	+ 3·58	+28·80	+ 0·69	−37·07	−12·58	+ 9·61	+ 2·67	+12·28	8·09	13·38
$Y_1 X_1$	III	−4·26	+3·36	1·653	− 7·04	+ 5·55	+4·05	− 1·97	−2·99	+ 3·58	+12·46	−17·25	− 6·62	−11·41	+ 8·39	−13·61	− 5·22		
$X_1 U_1$	II	−1·54	+5·94	2·706	− 4·17	+16·07	−0·30	− 6·14	−4·47	+ 9·93	+14·75	+ 0·46	−36·47	−21·26	+ 9·46	+ 1·78	+11·24	16·48	44·59
$X_1 U_1$	III	−4·26	+3·36	2·707	−11·53	+ 9·10	+7·07	+ 0·81	−4·46	+ 9·91	+19·46	−30·12	+ 2·72	− 7·94	− 3·45	−23·76	−27·21		
$U_1 E_1$	II	−1·54	+5·94	1·990	− 3·06	+11·82	−0·08	− 1·37	−3·14	+10·45	+ 1·25	+ 0·12	− 8·14	− 6·77	+ 2·11	+ 0·48	+ 2·59	30·21	60·12
$U_1 E_1$	IV	−2·50	+6·55	1·090	− 4·98	+13·03	+1·85	− 2·59	−3·13	+10·44	+ 5·37	− 4·63	−16·96	−16·22	+ 6·48	−12·12	− 5·64		
$U_1 V_1$	III	−4·26	+3·36	0·275	− 1·17	+ 0·92	+0·61	+ 0·28	−0·56	+ 1·20	+ 1·65	− 2·60	+ 0·94	− 0·01	− 1·19	− 2·05	− 3·24	23·38	6·43
$U_1 V_1$	IV	−2·50	+6·55	0·275	− 0·69	+ 1·80	+0·12	− 0·60	−0·57	+ 1·20	+ 1·38	− 0·30	− 3·93	− 2·85	+ 1·50	− 0·79	+ 0·71		
$V_1 W_1$	III	−4·26	+3·36	0·208	− 0·89	+ 0·70	+0·30	+ 0·13	−0·59	+ 0·83	+ 0·52	− 1·28	+ 0·44	− 0·32	− 0·55	− 1·01	− 1·56	24·05	5·00
$V_1 W_1$	V	−2·88	+6·80	0·208	− 0·60	+ 1·41	+0·01	− 0·58	−0·59	+ 0·83	+ 1·64	− 0·03	− 3·94	− 2·33	+ 1·67	− 0·07	+ 1·60		
$W_1 J_1$	III	−4·26	+3·36	0·337	− 1·44	+ 1·13	+0·19	+ 0·08	−1·25	+ 1·21	+ 0·16	− 0·81	+ 0·27	− 0·38	− 0·34	− 0·64	− 0·98	26·92	9·07
$W_1 J_1$	V	−2·88	+6·80	0·337	− 0·97	+ 2·29	−0·28	− 1·08	−1·25	+ 1·21	+ 3·70	+ 0·81	− 7·32	− 2·81	+ 3·11	+ 1·90	+ 5·01		
$V_1 F_1$	IV	−2·50	+6·55	0·825	− 2·06	+ 5·40	+0·01	− 0·99	−2·05	+ 4·41	+ 1·58	− 0·03	− 6·48	− 4·93	+ 2·48	− 0·07	+ 2·41	35·29	29·11
$V_1 F_1$	V	−2·88	+6·80	0·825	− 2·38	+ 5·61	+0·32	− 1·20	−2·06	+ 4·41	+ 2·24	− 0·92	− 8·16	− 6·84	+ 3·46	− 2·18	+ 1·28		

computed. The first step is to compute the necessary quantities for each line; it will only remain then to combine by simple addition the several lines which go to form the route between the point of reference and the point whose relative probable errors are sought. The reference point will be in general A_1 (Kalianpur), though any other point could also be used. The quantities of scheme (20) have to be formed as was done in tables XLV and XLVI. This has to be done for each section. Typical cases are S_1T_1, T_1D_1, D_1E_1. The first S_1T_1 enters only in closing condition of circuit II; T_1D_1 enters in closing conditions of circuits I and II: and D_1E_1 enters into closing condition of circuit II as well as base and Laplace closure between Dehra and Chach, VI. For most of the lines values of A, AX, AY, $A\,(R^2+L^2/12)$ are given in table XLV. For the others which are required the values are now exhibited in table XLVIII. In table XLIX all the quantities necessary for forming $[naf]\ldots$ $[uff]\ldots$ are given. In the column headed "circuit" the base line closures are indicated, and these require only A, AX, AY, AX_0, AY_0, which are the same quantities as for the circuits. To form $[uff]$, $A(R^2_0+L^2/12)$ is required, where R_0 refers to Kalianpur. This quantity is also given for all lines. It is deduced from values of X_0, Y_0.

As an example consider the probable errors at U_1, selecting the route $A_1\,B_1\,C_1\,D_1\,E_1\,U_1$. This gives a good example of the method of forming the right hand sides of the equations.

The sections A_1B_1, B_1C_1 enter into circuits	I and VI	
" C_1D_1	I "	VII
" D_1E_1	II "	VII
" E_1U_1	II "	IV

TABLE L. (formed by (21))

Equations	Side closure A_1B_1	B_1C_1	C_1D_1	D_1E_1	E_1U_1	Total	Azimuth closure	Easting closure A_1B_1	B_1C_1	C_1D_1	D_1E_1	E_1U_1	Total	Northing closure.
I 1	+1·08	+1·11	+0·42			+2·61	0	−0·11	−0·03	−0·16			− 0·30	+ 6·72
2	0	0	0			0	+2·61	−1·15	−3·58	−1·99			− 6·72	− 0·30
3	+0·88	+0·98	+0·22			+2·08	+6·80	−4·40	−6·56	−0·87			−11·83	+ 4·59
4	−4·46	−2·16	−0·18			−6·80	+2·08	−0·50	−3·11	−0·98			− 4·59	−11·83
II 5				+0·31	+ 1·99	+2·30	0				−0·37	− 3·14	− 3·51	+12·15
6				0	0	0	+2·30				−1·70	−10·45	−12·15	− 3·51
7				+0·10	−0·08	+0·02	+1·49				−0·76	− 6·77	− 7·53	− 2·18
8				−0·12	−1·37	−1·49	+0·02				−0·41	+ 2·59	+ 2·18	− 7·53
III 9						0	0						0	0
10						0	0						0	0
11						0	0						0	0
12						0	0						0	0
IV 13					+1·99	+1·99	0					− 3·13	− 3·13	+10·44
14					0	0	+1·99					−10·44	−10·44	− 3·13
15					+1·85	+1·85	+2·59					−16·22	−16·22	+ 5·64
16					−2·59	−2·59	+1·85					− 5·64	− 5·64	−16·22
V 17						0	0						0	0
18						0	0						0	0
19						0	0						0	0
20						0	0						0	0
VI 21	+1·08	+1·11				+2·19	0	−0·11	−0·03				− 0·14	+ 4·73
VII 22			+0·42	+0·31		+0·73	0			−0·16	−0·37		− 0·53	+ 3·69
VIII 23						0	0						0	0
VI 24						0	+2·19	−1·15	−3·58				− 4·73	− 0·14
VII 25						0	+0·73			−1·99	−1·70		− 3·69	− 0·53
VIII 26						0	0						0	0

There are four cases according as probable errors of side, azimuth, easting or northing are sought. The scheme (20) may be rewritten, entering the numbers of the column in table XLIX in place of the actual symbolical quantities.

	S_f	A_f	E_f	N_f
S	(1)	zero	(6)	(7)
A	zero	(1)	− (7)	(6)
E	(4)	− (5)	(11)	− (14)
N	(5)	(4)	(14)	(11)

. . . . (21).

For side error $A_1\, B_1$ contributes (1), 0, (4) and (5) to equations 1, 2, 3, 4, of circuit I and (1) to equation 21 of VI: to all other equations nothing. The numerical quantities taken from table XLIX are (1) = $+1 \cdot 08$, (4) = $+0 \cdot 88$, (5) = $-4 \cdot 46$. The complete process is shown in table L.

The azimuth closure can at once be written down from the side closure by rearrangement of terms and changing of certain signs in accordance with (21).

8. In a similar way all the quantities occurring on the right hand sides of equations (13) can be formed as required, the necessary data being taken from Table XLIX. The solution of the equations which arise for the N. W. Quadrilateral is effected in the latter portion of the next chapter (VIII). It remains only to refer to the quantities $[uff]$ occurring in equation (14). From (19) it is clear that the necessary quantities for determining these are $A = M^2L$, AX_0, AY_0, $A\,(R_0^2 + L^2/12)$, the suffix zero indicating A_1, or Kalianpur as origin. All these quantities are given in table XLIX in columns (1), (6), (7), (15) for each section of line: and the corresponding quantities for a set of lines are obtained by summation of the sectional quantities.

From (2) and (5) the probable errors before adjustment are proportional to $\sqrt{[uff]}$ the multiplying factors being $1'' \cdot 575$ for azimuth, $33 \cdot 2$ for 7th place of logarithm of side and $4 \cdot 03$ for easting and northing in feet: for it is clear that $[uff] = \Sigma M^2L$ in the cases of azimuth and side and $[uff] = M^2L\,(R_0^2 + L^2/12)$ in the case of easting or northing. By (14) the probable errors after adjustment are proportional to $[uff] - [uaf]\,k_1 - [ubf]\,k_2$ . . . .
the multiplying factors being as just given for the several cases. The ratio of probable error after to adjustment to probable error before adjustment is K where

$$K = \sqrt{1 - \frac{[uaf]}{[uff]}k_1 - \frac{[ubf]}{[uff]}k_2 -} \qquad \ldots\ldots\ldots\ldots \quad (22)$$

Numerical values of K will be given in the next chapter.

CHAPTER VIII.

Numerical values of the probable and actual errors in the Indian triangulation. Note on the solution of linear equations.

1. The formulæ (2) and (5) of chapter VII will now be applied to the actual circuits and closures of the Indian triangulation and the numerical results compared with the actual closing errors which have been found in the several circuits. The question is a little complicated by the fact that the triangulation has been adjusted in six portions and so, in the case of some circuits, the closing errors are those due to several series some of which have been adjusted in a neighbouring quadrilateral. In practice this is of little account, as the best quadrilaterals were first adjusted and those adjusted later are of considerably lower quality, so that the probable errors brought in by the adjusted quadrilaterals form only a small part of the total probable error and it is of little account whether the probable errors before or after adjustment are employed. In consideration of this question the fact that the flanking quadrilaterals were previously adjusted will be ignored. What has been said regarding the relative excellence of the several quadrilaterals does not apply to the Burma quadrilateral, which had only been begun when the Indian quadrilaterals were adjusted.

It will be seen from the equations that the quantities required for each line are M^2L and $M^2L\left(R^2 + \frac{L^2}{12}\right)$ whence L is the length of line and R the distance of its mid point from the closing point of the circuit concerned, both expressed in 100 miles. These quantities have already been taken out (table XLV) for the N.W. quadrilateral. It remains to obtain these for the rest of the triangulation. For this purpose charts III, IV, V are given. These as well as chart II are on the scale 100 miles to an inch: so that L and R are the lengths on the charts in inches. See also § 6, chapter VII. The actual measurements and necessary deductions are now shown in table LI. For the Base-line and Laplace closures it is only necessary to compute ΣM^2L along the route. This has already been done for each element of the route which enters into one of the circuits: and only a few remain to be formed for the Laplace closures. The values for the elements are combined to form the necessary values of ΣM^2L for the complete routes. The results are shown in table LII.

Having thus found the probable values of closing errors in all forms of closure, the next step is to compare the results with the closing errors which have actually been found. This is done for the Base-line and Laplace closure in table LII and for the circuit closures in table LIII. In each case the actual error is given and then this is divided by the theoretical error, giving a quantity f, the ratio of the actual error to the probable error. The actual errors of 7th place of logarithm of side, of azimuth in seconds, of easting and northing in feet are denoted by ΔS, ΔA, ΔE and ΔN respectively.

TABLE LI.

	Circuit and Closing point	Line	Series	M	L	$A = M^2L$	R	R^2	$\frac{L^2}{12}$	$R^2 + \frac{L^2}{12} = C$	AC
S. E. Quadrilateral.	I S_2	$S_2 W_2$	53	0·31	2·10	0·202	1·05	1·10	0·368	1·47	0·30
		$W_2 D_2$	53	0·31	2·10	0·202	3·22	10·37	0·368	10·74	2·17
		$D_2 A_1$	5	0·32	1·26	0·129	4·24	17·98	0·132	18·11	2·34
		$A_1 T_2$	8	0·59	4·26	1·483	3·40	11·56	1·510	13·07	19·38
		$T_2 S_2$	43	0·30	1·16	0·104	0·58	0·34	0·112	0·45	0·04
						2·120					24·23
	II R_2	$R_2 X_2$	58	0·33	2·85	0·310	1·43	2·04	0·675	2·72	0·84
		$X_2 E_2$	58	0·33	1·38	0·150	3·49	12·18	0·158	12·34	1·85
		$E_2 D_2$	5	0·32	1·00	0·102	4·25	18·06	0·083	18·14	1·85
		$D_2 W_2$	53	0·31	2·10	0·202	3·40	11·56	0·368	11·93	2·41
		$W_2 S_2$	53	0·31	2·10	0·202	1·75	3·06	0·368	3·43	0·70
		$S_2 R_2$	43	0·30	1·02	0·146	0·81	0·66	0·218	0·88	0·13
						1·112					7·78
	III Q_2	$Q_2 O_2$	24	0·70	3·22	1·578	1·61	2·59	0·864	3·45	5·44
		$O_2 M_2$	24	0·70	0·96	0·470	3·63	13·18	0·077	13·26	5·23
		$M_2 L_2$	24	0·70	0·57	0·279	4·31	18·58	0·027	18·61	5·19
		$L_2 K_2$	5	0·32	1·05	0·108	4·53	20·52	0·092	20·61	2·23
		$K_2 E_2$	5	0·32	3·40	0·348	4·10	16·81	0·963	17·77	6·18
		$E_2 X_2$	58	0·33	1·38	0·150	3·60	12·96	0·158	13·12	1·97
		$X_2 R_2$	58	0·33	2·85	0·310	1·57	2·46	0·675	2·54	0·79
		$R_2 Q_2$	43	0·30	0·70	0·063	0·35	0·12	0·041	0·16	0·01
						3·306					27·04
S. W. Quadrilateral.	I S_4	$S_4 T_2$	7	0·74	1·56	0·854	0·78	0·61	0·203	0·81	0·69
		$T_2 A_1$	8	0·59	4·26	1·483	2·35	5·52	1·512	7·03	10·43
		$A_1 S_1$	25	0·60	1·70	0·612	4·09	16·73	0·241	16·97	10·39
		$S_1 I_5$	18	1·07	1·10	1·259	3·59	12·89	0·101	12·99	16·35
		$I_5 S_4$	18	1·07	3·08	3·469	1·51	2·28	0·765	3·05	10·58
						7·677					48·44
	II H_5	$H_5 I_5$	29	1·12	0·96	1·204	0·48	0·23	0·077	0·31	0·37
		$I_5 S_1$	18	1·07	1·10	1·259	0·95	0·90	0·101	1·00	1·26
		$S_1 R_1$	25	0·60	0·52	0·187	1·10	1·21	0·225	1·44	0·27
		$R_1 H_5$	10	1·14	1·03	1·339	0·52	0·27	0·089	0·36	0·48
						3·989					2·38
	III Q_4	$Q_4 S_4$	7	0·74	1·41	0·772	0·70	0·49	0·166	0·66	0·51
		$S_4 I_5$	18	1·07	3·03	3·469	1·86	3·46	0·765	4·23	14·67
		$I_5 H_5$	29	1·12	0·96	1·204	2·95	8·70	0·077	8·78	10·57
		$H_5 Q_4$	10	1·14	2·86	3·717	1·43	2·04	0·684	2·72	10·11
						9·162					35·86
	IV G_5	$G_5 H_5$	29	1·12	0·61	0·765	0·30	0·09	0·039	0·12	0·09
		$H_5 R_1$	10	1·14	1·03	1·339	0·83	0·69	0·089	0·78	1·04
		$R_1 Q_1$	25	0·60	0·92	0·331	1·09	1·19	0·071	1·26	0·42
		$Q_1 G_5$	26	0·68	1·08	0·499	0·54	0·29	0·098	0·39	0·19
						2·934					1·74
	V C_5	$C_5 G_5$	29	1·12	1·18	1·480	0·59	0·35	0·116	0·47	0·70
		$G_5 Q_1$	26	0·68	1·08	0·499	1·25	1·56	0·098	1·66	0·83
		$Q_1 P_1$	25	0·60	1·00	0·360	1·38	1·90	0·094	1·90	0·72
		$P_1 C_5$	28	1·11	1·37	1·688	0·68	0·46	0·137	0·62	1·05
						4·027					3·30
	VI N_1	$N_1 A_5$	35	1·27	1·64	2·645	0·82	0·67	0·225	0·90	2·38
		$A_5 B_5$	35	1·27	1·12	1·806	2·07	4·28	0·105	4·39	7·93
		$B_5 P_1$	28	1·11	1·02	1·257	2·37	5·62	0·087	5·71	7·18
		$P_1 N_1$	25	0·60	2·30	0·828	1·15	1·32	0·420	1·74	1·44
						6·536					18·93

	Circuit and Closing point	Line	Series	M	L	$A = M^2L$	R	R^2	$\frac{L^2}{12}$	$R^2 + \frac{L^2}{12} = C$	AC
N. E. Quadrilateral.	I C_1	$C_1 B_1$	6	0·71	2·22	1·119	1·11	1·23	0·412	1·64	13·03
		$B_1 A_1$	6	0·71	2·15	1·084	3·28	10·76	0·383	11·14	12·08
		$A_1 C_2$	5	0·32	1·02	0·104	4·40	19·36	0·083	19·44	2·02
		$C_2 C_3$	4	1·79	3·20	10·253	2·92	8·53	0·854	9·38	96·17
		$C_3 B_3$	20	0·65	0·89	0·376	1·07	1·14	0·066	1·21	0·45
		$B_3 C_1$	20	0·65	0·63	0·266	0·32	0·10	0·033	0·13	0·03
						13·202					123·78
	II C_3	$C_3 C_2$	4	1·79	3·20	10·253	1·60	2·56	0·855	3·42	35·07
		$C_2 E_2$	5	0·32	1·22	0·125	3·29	10·82	0·124	10·94	1·37
		$E_2 W_3$	3	1·88	0·97	3·428	3·00	9·00	0·078	9·08	31·13
		$W_3 D_3$	3	1·88	2·13	7·528	1·53	2·34	0·377	2·72	20·48
		$D_3 C_3$	20	0·65	0·70	0·296	0·35	0·12	0·041	0·16	0·05
						21·630					88·10
	III D_3	$D_3 W_3$	3	1·88	2·13	7·528	1·07	1·14	0·378	1·52	11·44
		$W_3 E_2$	3	1·88	0·97	3·428	2·54	6·45	0·078	6·53	22·38
		$E_2 E_3$	12	1·81	2·60	8·518	1·68	2·82	0·564	3·38	28·79
		$E_3 D_3$	20	0·65	0·48	0·203	0·24	0·06	0·019	0·08	0·02
						19·677					62·63
	IV E_3	$E_3 E_2$	12	1·81	2·60	8·518	1·30	1·69	0·564	2·25	19·17
		$E_2 F_2$	5	0·32	0·57	0·058	2·65	7·02	0·027	7·05	0·41
		$F_2 F_3$	19	1·55	2·44	5·851	1·55	2·40	0·490	2·89	16·91
		$F_3 E_3$	20	0·65	0·60	0·254	0·30	0·09	0·030	0·12	0·03
						14·681					36·52
	V F_3	$F_3 F_2$	19	1·55	2·44	5·851	1·22	1·49	0·490	1·98	11·58
		$F_2 G_2$	5	0·32	0·56	0·058	2·50	6·25	0·028	6·28	0·36
		$G_2 G_3$	15	1·21	2·34	3·426	1·48	2·19	0·455	2·65	9·08
		$G_3 F_3$	20	0·65	0·63	0·266	0·32	0·10	0·033	0·13	0·03
						9·601					21·05
	VI G_3	$G_3 G_2$	15	1·21	2·34	3·426	1·17	1·37	0·455	1·88	0·63
		$G_2 H_2$	5	0·32	0·60	0·615	2·40	5·75	0·080	5·78	3·55
		$H_2 H_3$	21	1·92	2·26	8·349	1·43	2·04	0·425	2·47	20·62
		$H_3 G_3$	20	0·65	0·66	0·279	0·33	0·11	0·036	0·15	0·04
						12·669					24·84
	VII H_3	$H_3 H_2$	21	1·92	2·26	8·349	1·13	1·28	0·425	1·71	14·28
		$H_2 I_2$	5	1·32	9·60	0·615	2·30	5·29	0·030	5·32	3·27
		$I_2 I_3$	14	1·06	2·02	2·263	1·45	2·10	0·340	2·44	5·52
		$I_3 H_3$	20	0·65	0·74	0·312	0·37	0·14	0·046	0·19	0·06
						11·539					23·13
	VIII I_3	$I_3 I_2$	14	1·06	2·02	2·268	1·01	1·02	0·340	1·36	3·08
		$I_2 J_2$	5	0·32	0·52	0·532	2·04	4·15	0·023	4·17	2·22
		$J_2 J_3$	27	1·25	1·90	2·967	2·19	4·80	0·300	5·10	15·18
		$J_2 I_3$	20	0·65	0·40	0·169	0·20	0·04	0·013	0·05	0·01
						5·931					20·44
	IX J_3	$J_3 J_2$	27	1·25	1·90	2·967	0·95	0·90	0·300	1·20	3·56
		$J_2 K_2$	5	0·32	0·52	0·532	1·97	3·88	0·023	3·90	2·07
		$K_2 K_3$	18	1·42	1·86	3·751	1·18	1·39	0·288	1·68	6·30
		$K_3 J_3$	20	0·65	0·58	0·245	0·29	0·08	0·028	0·11	0·03
						7·495					11·96
	X K_3	$K_3 K_2$	18	1·42	1·86	3·751	0·93	0·86	0·288	1·15	4·31
		$K_2 L_2$	5	9·32	1·07	1·002	2·20	4·84	0·096	4·94	5·39
		$L_2 L_3$	16	1·99	2·56	10·138	1·47	2·16	0·545	2·71	27·48
		$L_3 K_3$	20	0·65	0·81	0·342	0·40	0·16	0·054	0·21	0·72
						15·323					37·90

TABLE LI.

	Circuit and Closing point	Line	Series	M	L	A = M²L	R	R²	L²/12	R² + L²/12 = C	AC
N.E. Quad.	XI M_3	M_3L_3	34	0·71	0·87	0·437	0·48	0·19	0·063	0·25	0·11
		L_3L_2	16	1·99	2·56	10·138	1·44	2·07	0·545	2·62	26·57
		L_2T_3	48	0·57	0·67	0·218	2·32	5·83	0·037	5·42	1·18
		T_3S_3	48	0·57	0·40	0·130	2·07	4·28	0·013	4·29	0·56
		S_3M_3	56	0·70	2·04	1·000	1·02	1·04	0·347	1·39	1·39
						11·923					29·81
S. Trigon	I O_4	O_4R_4	49	0·45	2·04	0·418	1·02	1·04	0·347	1·39	0·57
		R_4Q_4	7	0·74	0·90	0·495	2·20	4·84	0·068	4·91	2·43
		Q_4P_4	11	1·93	2·37	8·828	1·30	1·69	0·467	1·66	14·83
		P_4O_4	11	1·93	0·44	0·164	0·22	0·05	0·016	0·07	0·17
						9·900					18·00
	II B_4	B_4T_2	9	0·36	3·37	0·436	1·68	2·82	0·945	3·77	1·64
		T_2R_4	7	0·74	2·10	1·150	3·73	13·91	0·368	14·28	16·42
		R_4O_4	49	0·45	2·04	0·418	3·46	11·97	0·346	12·32	5·09
		O_4N_4	49	0·45	1·92	0·388	1·93	3·72	0·307	4·03	1·56
		N_4B_4	54	0·37	1·49	0·204	0·75	0·56	0·189	0·85	0·17
						2·591					24·88
	III H_4	H_4L_4	46	0·40	2·22	0·355	1·11	1·23	0·412	1·64	0·58
		L_4S_2	46	0·46	1·11	0·235	2·78	7·73	0·103	7·83	1·84
		S_2T_2	43	0·30	1·17	0·105	3·42	11·70	0·114	11·81	1·24
		T_2B_4	9	0·36	3·37	0·436	2·17	4·71	0·945	5·66	2·47
		B_4H_4	54	0·37	1·47	0·201	0·73	0·53	0·160	0·69	0·14
						1·332					6·27
	IV D_4	D_4F_4	63	0·65	0·66	0·279	0·33	0·11	0·036	0·15	0·04
		F_4G_4	58	0·31	1·88	0·182	1·32	1·74	0·284	2·02	0·37
		G_4H_4	53	0·31	0·90	0·086	2·62	6·86	0·068	6·93	0·60
		H_4B_4	54	0·37	1·47	0·201	2·77	7·67	0·160	7·83	1·57
		B_4D_4	9	0·36	2·64	0·343	1·32	1·74	0·581	2·32	0·80
						1·091					3·38
	V I_4	I_4Q_2	46	0·40	2·40	0·384	1·20	1·44	0·480	1·92	0·74
		Q_2S_2	43	0·30	2·30	0·207	1·48	2·19	0·441	2·63	0·54
		S_2I_4	46	0·40	1·11	0·178	0·56	0·31	0·103	0·41	0·07
						0·769					1·35

	Circuit and Closing point	Line	Series	M	L	A = M²L	R	R²	L²/12	R² + L²/12 = C	AC
N.E. Quad.	XII N_3	N_3M_3	34	0·71	0·98	0·493	0·49	0·24	0·080	0·32	0·16
		M_3S_3	56	0·70	2·04	1·000	1·40	1·96	0·346	2·31	2·31
		S_3R_3	48	6·57	1·10	0·357	2·07	4·28	0·100	4·38	1·56
		R_3Q_3	44	0·49	0·47	0·116	1·78	3·17	0·018	3·19	0·37
		Q_3P_3	44	0·49	0·79	0·187	1·18	1·39	0·052	1·44	0·27
		P_3N_3	44	0·49	0·87	0·209	0·44	0·19	0·063	0·25	0·05
						2·362					4·72
Burma Quadrilateral.	I P_3	P_3Q_3	44	0·49	0·80	0·192	0·40	0·16	0·053	0·21	0·04
		Q_3R_3	44	0·49	0·47	0·113	1·00	1·00	0·018	1·02	0·12
		R_3K_6	52	0·39	1·45	0·221	1·89	3·57	0·175	3·75	0·83
		K_6I_6	52	0·39	2·26	0·344	3·57	12·74	0·426	13·17	4·53
		I_6H_6	52	0·39	1·33	0·202	5·02	25·20	0·147	25·35	5·12
		H_6G_6	66	0·35	1·08	0·132	5·04	25·40	0·117	25·52	3·37
		G_6F_6	66	0·35	0·87	0·107	4·22	17·81	0·063	17·87	1·91
		F_6C_6	66	0·35	2·78	0·341	2·90	8·41	0·645	9·06	3·09
		C_6A_6	68	0·36	2·01	0·260	1·50	2·25	0·337	2·59	0·67
		A_6P_3	44	0·49	0·50	0·120	0·25	0·06	0·021	0·08	0·01
						2·032					19·69
	II J_6	J_6I_6	52	0·39	1·23	0·187	0·62	0·38	0·126	0·51	0·10
		I_6H_6	52	0·39	1·33	0·202	1·82	3·31	0·147	3·46	0·70
		H_6G_6	66	0·35	1·08	0·132	2·22	4·93	0·117	5·05	0·67
		G_6F_6	66	0·35	0·87	0·107	1·90	3·61	0·063	3·67	0·39
		F_6C_6	66	0·35	2·78	0·341	2·52	6·35	0·645	7·00	2·39
		C_6B_6	68	0·36	1·00	0·130	3·50	12·25	0·083	12·83	1·60
		B_6M_6	71	0·81	1·74	1·142	2·50	6·25	0·252	6·50	7·42
		M_6L_6	71	0·81	1·24	0·816	1·03	1·06	0·128	1·19	0·97
		L_6J_6	71	0·81	0·53	0·348	0·27	0·07	0·023	0·09	0·03
						3·405					14·27
	III M_6	M_6E_6	70	1·96	1·20	4·610	0·60	0·36	0·120	0·48	2·21
		E_6C_6	66	0·35	1·80	0·221	1·44	2·07	0·270	2·34	0·52
		C_6B_6	68	0·36	1·00	0·130	1·88	3·53	0·083	3·61	0·47
		B_6M_6	71	0·81	1·74	1·142	0·87	0·76	0·252	0·91	1·04
						6·103					4·24

TABLE LII.

Base line closure	Laplace closure	Line	A = M² L	B² = ΣA	33·2 *B*	1·575*B*	Δ *S*	Δ *A*	f_s Δ*S* / 33·2 *B*	f_a Δ*A* / 1·575 *B*	Reference number
Sironj-Dehra Dun ...	Kalianpur-Dehra Dun ...	$A_1B_1C_1$		2·202	49·27	2·338	+ 44·0	+0·5	0·893	0·214	35
Dehra Dun-Chach ...		$C_1D_1E_1F_1G_1$		1·221	36·69		+ 71·9		1·960		36
Sironj-Karachi ...		$A_1T_1S_1Q_1P_1N_1$		2·318	50·56		− 79·6		1·574		37
	Kalianpur-Deesa ...	$A_1T_1S_1Q_1$		1·130		1·675		−4·4		2·629	38
Karachi-Chach ...	Deesa-Karachi ...	$Q_1P_1N_1$		1·188		1·716		+1·9		1·106	39
Sironj-Calcutta ...		$N_1L_1J_1H_1G_1$		1·374	38·91		+163·8		4·210		40
	Kalianpur-Calcutta ...	$A_1C_2K_2L_2$		1·808	44·59	2·115	+ 42·5	−5·9	0·953	2·790	41
Calcutta-Vizagapatam ...	Kalianpur-Jubbulpur ...	A_1D_2 Jub.		0·158		0·626		−1·2		1·917	42
Sironj-Bider ...	Calcutta-Waltair ...	$L_2M_2O_2Q_2$		2·327	50·53	2·403	− 6·9	+4·5	0·137	1·872	43
Bider-Vizagapatam ...		A_2T_2		1·484	40·44		− 16·4		0·406		44
Calcutta-Sonakhoda ...		$T_2S_1R_2Q_2$		0·313	18·59		− 21·3		1·146		45
Sonakhoda-Dehra Dun ...	Calcutta-Jalpaiguri ...	L_2L_3		10·14	105·71	5·02	+ 36·7	+8·9	0·347	1·774	46
	Kalianpur-Fyzabad ...	$L_3K_3J_3H_3E_3C_3C_1$		3·006	57·34		− 35·2		0·614		47
Bider-Bangalore ...		A_1F_2 F_2 Fyz.	0·760 4·65	5·41		3·662		−5·3		1·447	48
		$T_2A_4B_4$		0·435	21·91		+ 20·0		0·913		49
	Waltair-Madras ...	$Q_2I_4H_4$		0·739		1·354		−1·7		1·256	50
	Madras-Bangalore ...	H_4B_4		0·201		0·706		−2·3		3·259	51
Vizagapatam-Bangalore ...		$Q_2H_4B_4$		0·940			+ 41·3		1·283		52
	Bangalore-Mangalore ...	B_4N_4		0·204		0·711		+1·4		1·970	53
Bangalore-Cape Comorin ...	Bangalore-Nagarkoil ...	$B_4L_4N_4$		0·425	21·65	1·026	+ 1·1	−3·0	0·051	2·923	54
	Kalianpur-Bombay ...	A_1T_2 T_2S_4 S_4Q_4	1·484 0·854 0·772	3·110		2·778		−2·2		0·792	55
Calcutta-Mergui ...	Calcutta-Chittagong ...	$L_2R_3I_5X_5$ $L_5T_5R_5$			45·85		+ 0·7		0·017		56
	Chittagong-Akyab ...	R_5 Chit. Chit. K_5J_5	0·702 0·050	0·752		1·365		−1·0		0·733	57
	Akyab-Prome ...	J_6I_6 Prome		0·324		0·897		−0·2		0·223	58
	Prome-Moulmein ...	Prome H_6V_6		0·269		0·817		−5·0		6·12	59
				0·375		0·964		+2·1		2·179	60

The values of ΔA in table LII are taken from the second table of chapter IX in which the accumulated errors of azimuth at Laplace stations are found. These are the errors *after* the adjustment of the triangulation has been performed.

TABLE LIII.

	Circuit	$B^2=\Sigma A$	$33\cdot2B$	ΔS	$1\cdot575B$	ΔA	$D^2=\Sigma AC$	$4\cdot03D$	ΔE	ΔN	f_s $\frac{\Delta S}{33\cdot2B}$	f_a $\frac{\Delta S}{1\cdot575B}$	f_e $\frac{\Delta E}{4\cdot03D}$	f_n $\frac{\Delta N}{4\cdot03D}$	Reference number
N.W. Quad.	I	3·661	63·512	+ 68·2	3·013	+ 5·906	40·68	25·703	+ 14·613	+ 57·503	1·074	1·960	0·569	2·237	1
	II	9·718	103·484	−124·6	4·909	+ 1·550	111·51	42·557	+ 18·298	− 39·413	1·205	0·315	0·430	0·926	2
	III	9·679	103·285	− 79·6	4·900	− 3·254	79·30	35·887	+ 26·065	+ 89·087	0·761	0·664	0·726	1·089	3
	IV	3·426	61·453	+150·9	2·915	− 4·232	8·46	11·723	− 24·477	+ 3·638	2·456	1·452	2·088	0·310	4
	V	2·229	49·568	− 5·3	2·351	− 3·000	10·96	13·343	− 24·588	− 0·505	0·107	1·276	1·843	0·038	5
S.E. Quad.	I	2·120	48·339	−54·9	2·293	+0·212	24·28	19·836	− 20·676	+ 5·048	1·654	0·092	1·042	0·254	6
	II	1·112	34·993	+31·9	1·660	−4·968	7·78	11·256	+ 19·622	− 21·785	0·961	2·993	1·726	1·936	7
	III	3·306	60·358	−17·5	2·863	−3·888	27·04	20·956	+ 23·073	− 14·322	0·527	1.393	1·101	0·683	8
N.E. Quad.	I	13·202	120·640	+287·1	5·724	+11·598	128·78	44·834	+ 81·933	+ 94·856	2·380	2·026	1·828	2·118	9
	II	21·630	154·413	− 7·3	7·325	−14·317	88·10	11·961	− 96·904	− 35·551	0·047	1·956	8·102	2·972	10
	III	19·677	147·275	+482·3	6·987	− 6·798	62·63	31·893	− 16·360	+ 93·415	3·275	0·973	0·513	2·929	11
	IV	14·681	127·222	−641·0	6·035	+10·177	36·52	24·353	− 5·388	−122·087	5·038	1·686	0·221	5·013	12
	V	9·601	102·854	+169·4	4·879	− 0·791	21·05	18·490	− 21·329	+ 38·675	1·647	0·162	1·164	2·092	13
	VI	12·669	118·192	+ 82·1	5·607	+ 4·843	24·84	20·086	+ 9·377	+ 21·806	0·695	0·864	0·467	1·061	14
	VII	11·530	112·780	−132·9	5·350	− 5·739	23·13	19·380	− 2·710	− 36·047	1·178	1·073	0·139	1·860	15
	VIII	5·931	80·875	+106·7	3·837	+ 4·441	20·44	18·220	− 2·449	+ 30·593	2·432	1·157	0·134	1·679	16
	IX	7·495	90·902	−284·6	4·312	− 2·279	11·96	13·936	+ 3·269	+ 31·703	3·130	0·528	0·234	2·275	17
	X	15·323	120·978	+190·9	6·166	+ 0·648	37·90	24·809	− 10·284	− 21·606	1·468	0·105	0·414	0·871	18
	XI	11·923	114·640	−193·7	5·438	+ 2·405	29·81	22·004	− 13·044	+ 13·023	1·690	0·442	0·592	0·592	19
	XII	2·362	51·028	+102·0	2·421	−13·140	4·72	8·757	− 57·230	+ 17·465	2·011	5·428	6·542	1·905	20
S. Trigon.	I	9·900	104·447	+136·6	4·955	−0·251	18·00	17·099	+ 7·624	+ 2·218	1·308	0·051	0·446	0·130	21
	II	2·591	53·452	− 22·7	2·536	−3·680	24·88	20·102	+ 28·479	− 0·101	0·425	1·451	1·416	0·005	22
	III	1·332	38·313	+ 39·9	1·818	+4·303	6·27	10·091	− 17·218	0·000	1·041	2·367	1·706	0·000	23
	IV	1·091	34·661	− 0·4	1·644	−9·040	3·39	7·411	+ 42·500	− 27·515	0·012	5·500	5·734	3·712	24
	V	0·760	28·917	+ 11·7	1·372	−0·319	1·35	4·683	− 0·973	− 4·235	0·404	0·233	0·208	0·904	25
S.W. Quad.	I	7·677	91·907	+189·8	4·364	−7·806	48·44	28·049	− 54·726	− 64·255	2·063	1·789	1·952	2·291	26
	II	3·989	66·300	−212·5	3·145	−7·113	2·38	6·218	+ 38·058	+ 6·862	3·207	2·260	6·121	1·104	27
	III	9·162	100·496	+ 27·4	4·768	−6·719	35·86	24·132	+ 6·238	− 43·878	0·261	1·408	0·258	1·818	28
	IV	2·934	56·872	−185·9	2·608	+2·057	1·74	5·320	− 1·215	+ 15·238	3·269	1·096	0·228	2·866	29
	V	4·027	66·632	− 62·2	3·161	+3·046	3·30	7·323	− 6·712	+ 14·028	6·934	0·964	0·916	1·916	30
	VI	6·536	84·802	+257·1	4·027	−3·912	18·93	17·535	− 23·569	+ 6·057	3·030	0·972	1·344	0·346	31
Burma Quad.	I	2·032	47·343	+ 45·0	2·246	−2·267	19·69	17·881	− 47·521	+ 41·790	0·951	1·009	2·658	2·337	32
	II	3·405	61·254	+ 47·0	2·906	−6·002	14·27	12·543	− 17·550	− 17·553	0·767	2·065	1·399	1·399	33
	III	6·103	82·037	−189·0	3·592	+3·606	4·24	6·836	+ 2·914	+ 1·427	2·304	0·927	0·426	0·209	34

2. It is not to be expected that the actual errors will be the same as the probable errors: but in a considerable number of cases, values of the ratio of actual to probable errors, that is f, should be distributed according to the laws of probability. In a given number of cases the probability is that those values of f which are comprised within certain limits will form a certain percentage of the total cases. The probability integral, between the proper ordinates, represents this distribution of errors. It is tabulated in most books on minimum squares*. By means of this it is seen that 10 per cent of the errors will most probably fall in each of the regions $A, B, \ldots \ldots J$ of table LIV defined by limiting values of f.

The actual values of f found in 167 cases of closures are classified in these columns. Each value of f is followed by a number in brackets which refers to the corresponding closing condition in tables LII or LIII.

* *Vide* Wright's "Adjustment of Observations", § 213.

TABLE LIV.

Values of f from to	A 0 ·185	B ·185 ·375	C ·375 ·572	D ·572 ·777	E ·777 1·000	F 1·000 1·240	G 1·240 1·539	H 1·539 1·900	I 1·900 2·438	J 2·438 ∞
Side (i) Circuit ...	·012 (24) ·047 (10) ·107 (5)	·261 (28)	·425 (22) ·404 (25) ·527 (8)	·605 (14) ·761 (3) ·767 (33)	·934 (30) ·951 (32) ·961 (7)	1·011 (23) 1·074 (1) 1·178 (15) 1·205 (2)	1·308 (21) 1·468 (18)	1·647 (13) 1·654 (6) 1·690 (19)	2·011 (20) 2·063 (26) 2·304 (34) 2·380 (9) 2·432 (16)	2·456 (4) 3·030 (31) 3·130 (17) 3·207 (27) 3·269 (29) 3·275 (11) 5·038 (12)
	3	1	3	3	3	4	2	3	5	7
(ii) Base ...	·017 (56) ·051 (54) ·137 (43)	·347 (46)	·406 (44)	·614 (47)	·803 (35) ·913 (40) ·953 (41)	1·146 (5)	1·283 (52)	1·574 (37)	1·960 (36)	4·210 (40)
	3	1	1	1	3	1	1	1	1	1
Azimuth (i) Circuit ...	·051 (21) ·092 (6) ·104 (18) ·162 (13)	·233 (25) ·315 (2)	·412 (19) ·528 (17)	·664 (3)	·864 (14) ·927 (34) ·964 (30) ·972 (31) ·973 (11)	1·009 (32) 1·073 (15) 1·093 (20) 1·157 (16)	1·276 (5) 1·303 (8) 1·408 (28) 1·451 (22) 1·452 (4)	1·686 (12) 1·789 (26)	1·956 (10) 1·960 (1) 2·026 (9) 2·065 (33) 2·260 (27) 2·367 (23)	2·993 (7) 5·428 (20) 5·500 (24)
	4	2	2	1	5	4	5	2	6	3
(ii) Laplace ...		·214 (35) ·223 (58)		·733 (57)	·702 (55)	1·106 (30)	1·256 (50) 1·447 (48)	1·774 (46) 1·872 (43)	1·917 (42) 1·970 (53) 2·179 (60)	2·629 (38) 2·790 (41) 2·923 (54) 3·259 (51) 6·12 (59)
	0	2	0	1	1	1	2	2	3	5
Easting ...	·134 (16) ·139 (15)	·208 (25) ·221 (12) ·228 (29) ·234 (17) ·258 (28)	·414 (18) ·420 (34) ·430 (2) ·446 (21) ·467 (14) ·513 (11) ·560 (1)	·502 (10) ·726 (3)	·910 (30)	1·042 (6) 1·101 (8) 1·154 (13)	1·314 (31) 1·399 (33) 1·416 (22)	1·706 (23) 1·726 (7) 1·828 (9) 1·843 (5)	1·952 (26) 2·088 (4)	2·653 (32) 5·734 (24) 6·121 (27) 6·542 (20) 8·102 (10)
	2	5	7	2	1	3	3	4	2	5
Northing ...	·000 (23) ·005 (22) ·038 (5) ·130 (21)	·209 (34) ·254 (6) ·310 (4) ·346 (31)		·592 (19) ·683 (8)	·871 (18) ·904 (25) ·926 (2)	1·061 (14) 1·080 (3) 1·101 (27)	1·390 (33)	1·679 (16) 1·818 (28) 1·860 (15)	1·916 (30) 1·936 (7) 1·995 (20) 2·092 (13) 2·118 (9) 2·287 (1) 2·275 (17) 2·291 (26) 2·337 (32)	2·866 (29) 2·929 (11) 2·972 (10) 3·712 (24) 5·013 (12)
	4	4	0	2	3	3	1	3	9	5
Side and Azimuth excluding Laplace	10	4	6	5	11	9	8	6	12	11
			36					46		
Northing and Easting ...	6	9	7	4	4	6	4	7	11	10
			30					38		
All except Laplace ...	16	13	13	9	15	15	12	13	23	21
			66					84		

3. On examination of table LIV it is immediately noticeable that the cases in which the actual error is greater than the computed probable error are more numerous than the cases when it is less. Considering all the 167 cases, f is less than unity for 70 cases and greater than unity for the remaining 97 cases. This unequality is largely attributable to the Laplace closures,

although in this case values of azimuth, adjusted for all circuit conditions, have been used. In the Laplace closures there are 13 cases of actual error greater than computed probable error and only 4 cases of actual error less than computed error. It is believed that the explanation of this is that given in § 6, Chapter V. The error due to acceptance of geoidal angles uncorrected, instead of spheroidal angles, to some extent magnifies the triangular error and so increases the value of M, and as a result the closures of azimuth in circuits and of the deduced quantities side, northing and easting are in better agreement with the formulæ than the closures on Laplace points; since the former do not depend on absolute errors while the latter do. If the Laplace closures are ignored the number of cases, less than and greater than the formulæ give, are 66 and 84 respectively. If the formulæ values were increased in the ratio $1 \cdot 1$ to $1 \cdot 0$* the figures would become 74 and 76 respectively. It appears then that the formulæ give values of the probable error which are some ten per cent below what the 150 cases would lead to. This is not a serious deviation from the facts and, apart from mere chance, may be attributed partly to

(1) the use of geoidal instead of spheroidal angles.
(2) the fact that M is based on certain simplifying assumptions regarding the regularity of the triangles and polygons in the series of triangulation*.

The total number of cases falling in each class A, ... J is shown at the bottom of the table and from this it is seen that the errors are fairly distributed in the various classes, except that in classes I, J a considerable excess of cases occur. The excess of large errors over the number which is given by the formulæ, viz. $45-30=15$ or 50%, in classes I, J is to be attributed to the neglect of certain sources of error. One such source of error is that, already mentioned, of treating geoidal and spheroidal angles as identical: and it may be that other undetected sources also exist. However the formulæ give practically a satisfactory indication of the probable accuracy of side, azimuth, easting or northing. They should be a useful guide to the care which ought to be expended on observing and selecting a series in order that a result of any stated precision may be arrived at. As work on such a series progresses, the value of M may be taken out and observations increased in number, or rays increased in length, until the value of M is reduced to a quantity sufficiently small to give the proper precision.

4. It has just (June 1917) been noticed that the question of probable errors of side, azimuth, easting and northing generated in a chain of triangles were considered by General Walker and Mr. W. H. Cole in 1882†. The deduction is based on the equations by which the simultaneous reduction of the triangulation of India had been effected, and the equations obtained—*vide* xxviii, xxix, xxx *ibid*—are somewhat complex. These equations are comparable with (2) and (5) of Chapter VII of this work. Dealing with the case of a simple chain of equilateral triangles (on p. 104) with sides of 15 miles and chain of length 8° of arc, it is found in the Appendix that the

$$\text{e. m. s.}\left(i.\ e.\ \frac{\text{probable error}}{\cdot 6745}\right) \text{ in azimuth} = 6'' \cdot 93\epsilon, \text{ average value}$$

$$\text{latitude} \quad 0'' \cdot 55\epsilon$$

$$\text{longitude} \quad 0'' \cdot 59\epsilon$$

the first quantity being somewhat dependent on the direction of the chain. It appears that ϵ is the quantity now denoted by m. To obtain results by the method of present work put $M = \frac{7}{8} m \sqrt{\frac{18}{15}} = 1 \cdot 278m$. Then taking 8° as equivalent to 550 miles,

$$\text{Probable error in azimuth} = 1 \cdot 575 \times 1 \cdot 278\epsilon \sqrt{5 \cdot 5} = 4'' \cdot 72\epsilon$$

$$\text{Mean error in azimuth} = \frac{4 \cdot 72}{\cdot 6745}\epsilon = 6'' \cdot 99\epsilon$$

$$\text{Probable error in easting or northing} = 4 \cdot 03 \times 1 \cdot 278\epsilon \sqrt{5 \cdot 5} \times \frac{5 \cdot 5}{\sqrt{3}} \text{ feet} = 38 \cdot 3\epsilon \text{ feet.}$$

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* *See* also § 4 below.

† *Vide* G.T.S. Vol. VII, Appendix No. 3.

$$\text{Mean error in easting or northing} = \frac{38 \cdot 3\epsilon}{\cdot 6745} = 56 \cdot 8\epsilon \text{ feet.}$$

These results are in accord with those found by the old formulæ. On pp. 105, 106 of the Appendix* additional quantities were introduced to take account of geometrical irregularity, double instead of single chains, length of side. The latter two considerations have been dealt with in the present work by use of the quantity M. In the appendix under reference geometrical irregularity of the magnitude which might occur in Survey of India work is represented by an augmenting factor κ and it is stated that "we may as a rule put $\kappa = 1 \cdot 4$ in hilly country and $\kappa = 1 \cdot 1$ in the plains". The introduction of this factor would increase most of the probable errors computed in the present work in ratio $1 \cdot 1$, an amount which would practically equalise the number of cases of errors exceeding and falling short of the probable error as already deduced in § 3. The independent opinion of the author before seeing this appendix was that it was better to leave this out of account: but it is a question as to whether the factor κ might not with advantage be incorporated in M in some cases.

The formulæ developed in the Appendix* do not appear to have been put to much use: and as far as can be seen were lost sight of. They are only applicable to straight (or approximately straight) chains of triangles, and not to circuits of all forms.

Note on the solution of equations.

5. In the adjustment of triangulation, and the calculation of its probable errors, groups of linear equations involving a large number of unknowns frequently arise. Although the solution is not necessarily required to a high order of accuracy, yet the work of elimination has generally of necessity been performed using a large number of significant figures to safeguard the solution against accumulation of computation inaccuracy. Some of the multipliers in the process of elimination become very large owing to the fact that the denominators consist of terms of which the positive and negative portions are not very different in amount. Taking the denominator to be of the form $\Sigma\alpha - \Sigma\beta$ where α, β represent the positive and negative terms respectively, while $\Sigma\alpha$ and $\Sigma\beta$ may each be formed to a fairly high percentage accuracy, the quantity $\Sigma\alpha - \Sigma\beta$ may be inaccurate by a considerable percentage.

If the ordinary Gaussian method of arranging the elimination is followed, it is to be noted that most of the solution is independent of the R. H. S.† of the equations, and is in fact just a process of elimination of the several unknowns. In some cases, *e. g.* that occurring in Chapter VII, solutions are required for a number of sets of values of the R. H. S.† It is accordingly in this case desirable to retain the R. H. S.† in symbolical form. But this has an advantage of an entirely different kind, as it permits of any number of successive approximations in the solution being made, without repeating the eliminating process, which accordingly need not be performed with such exactness as would otherwise be necessary.

First consider the L. H. S.‡ of the equations. Following Gauss's method of arrangement denote the equations by

$$\left.\begin{array}{l} a_1 x_1 + a_2 x_2 + \quad \cdots \quad + a_n x_n = A \\ b_1 x_1 + b_2 x_2 + \quad \cdots \quad = B \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ n_1 x_1 + n_2 x_2 + \quad \cdots \quad n_n x_n = N \end{array}\right\} \quad \cdots\cdots \quad (1)$$

If the first equation is multiplied by $-\dfrac{b_1}{a_1}$ and added to the second, x_1 is eliminated. Similarly

* *Vide* G.T.S. Vol. VII, Appendix No. 3. † R.H.S. = right hand side. ‡ L.H.S. = left hand side.

if it is multiplied by $-\dfrac{r_1}{a_1}$ and added to the rth equation x_1 is eliminated. The following equations are formed

$$\left.\begin{array}{ll}\left(b_2-\dfrac{b_1}{a_1}a_2\right)x_2+\left(b_3-\dfrac{b_1}{a_1}a_3\right)x_3\ \ldots\ldots & =B-\dfrac{b_1}{a_1}A \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ \left(r_2-\dfrac{r_1}{a_1}a_2\right)x_2+\left(r_3-\dfrac{r_1}{a_1}a_3\right)x_3+\ \ldots\ldots & =R-\dfrac{r_1}{a_1}A\end{array}\right\}\quad\ldots\quad(2)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

To eliminate x_2 the same process is applied, the multipliers in this case all having the denominator $b_2-\dfrac{b_1}{a_1}a_2$. It is to be observed that the denominator of the multiplying factors is always the first coefficient of the first equation of the set being operated on. The successive denominators are a_1, $b_2-\dfrac{b_1}{a_1}a_2$, $c_3-\dfrac{c_1}{a_1}a_3-\dfrac{c_2-\dfrac{c_1}{a_1}a_2}{b_2-\dfrac{b_1}{a_1}a_2}\left(b_3-\dfrac{b_1}{a_1}a_3\right)$ etc.

In the solution of normal equations the diagonal coefficients are generally larger than the others. Being of the forms Σua^2 and Σuab respectively the component parts of the first form are all positive while those of the second form are equally likely to be positive and negative, and accordingly tend to cancel. Accordingly in more cases than not $a_1 > a_r$ $(r \neq 1)$, $b_2 > b_r$ $(r \neq 2)$ . . . so that $b_2-\dfrac{b_1}{a_1}a_2$ is not likely to be small compared with b_2. But as the denominators become more complex there is more possibility of their becoming small. To avoid this, as well as may be foreseen before the actual computations are carried out, it is accordingly convenient to rearrange the equations in such order that the diagonal coefficients are of increasing magnitude. Before doing this however it is desirable that all these diagonal coefficients should be brought up to as near as may be the same order of magnitude. It is inconvenient to have quantities entering the computation, some with many figures preceding the decimal point while in others there are no figures before the decimal point and a number of zeros following the decimal point.

Any coefficient say f_r can be changed to $10f_r$ or $\frac{1}{10}f_r$ if at the same time for x_r is written $\frac{1}{10}x_r$ or $10x_r$, this being done in all the equations; and solution subsequently being performed for $\frac{1}{10}x_r$ or $10x_r$ as the case may be. It is convenient to use as multiplier a power of ten, as this involves no loss of precision in the coefficient and no labour in transforming it. If the process thus suggested is carried out in the case of symmetrical equations—such as normal equations—the symmetry is destroyed. As the symmetry is advantageous it is desirable to avoid destroying it, and this may be arranged for as follows. When any column is multiplied by a power of ten the corresponding row should be multiplied by the same quantity. Then the equations (1) may be written

$$\left.\begin{array}{l}a_1x_1+a_2x_2+\ \ldots\ +10^a a_r X_r+\ =A \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 10^a r_1x_1+10^a r_2x_2+\ \ldots\ +10^{2a} r_r X_r=R.10\end{array}\right\}\quad\ldots\ldots\quad(3)$$

in which $X_r=\dfrac{x_r}{10^a}$: which are quite symmetrical.

In the solution of (1) the quantities which should be brought up to about the same order are the diagonal coefficients, as these enter more than the others into the computations. It may be seen from (3) that the diagonal coefficients can only be conveniently changed by powers (positive or

negative) of 10^2. The first step then is to apply this process of dealing with the diagonal coefficients; the second is to rearrange the equations in such order that the modified diagonal coefficients occur in increasing order of magnitude. It can always be arranged that the largest diagonal coefficient is not so much as one hundred times the smallest.

6. Suppose now that (1) represents a set of equations dealt with and arranged as explained above. They are now in an order as favourable for solution as can be arranged for by mere inspection.

In proceeding with the elimination as explained in §5, notice that the R. H. S. quantities A, B, C do not in any way affect the elimination. The work on the left hand side of the equation is entirely independent of the values A, B, C . . . . . . Suppose then that a solution has been taken out, which is essentially only approximate: the degree of approximation depends on the number of figures retained in the arithmetical processes. Instead of the correct quantities x_1, x_2 . . this solution will determine slightly different quantities $x_1-\delta x_1, x_2-\delta x_2$. On substituting these in the L.H.S of (1) the values found will be $A-\delta A$, $B-\delta B$ . . . . Hence

$$\left.\begin{array}{l} a_1\,\delta x_1 + a_2\,\delta x_2 + \;.\;.\;.\;.\;. = \delta A \\ b_2\,\delta x_1 + \;.\;.\;.\;.\;.\;.\;.\;. = \delta B \\ \;.\;.\;.\;.\;.\;.\;.\;. \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

These equations have the same coefficients on the L.H.S. as those of the original equations (1). Hence the process of elimination in order to determine $\delta x_1, \delta x_2$ . . . is the same as that already performed for x_1, x_2 . . . : and it is only necessary to change the portion of the computation involving A, B, C. In this way a second approximation is easily arrived at. Clearly this result may again be treated in the same way, and so successive sets of higher approximation may be obtained, without any necessity of increasing the accuracy of the elimination process. The gain in accuracy is arrived at by means of accurate substitution, and this substitution may be made absolutely perfect by keeping all the figures resulting from the substitution. As an example, if the original solution consists of numbers of 4 significant figures, and the coefficients are given to 4 figures the products will consist of 7 or 8 significant figures. It may be pointed out here that if the coefficients are given to much higher accuracy it is not necessary to use the full amount of figures for the elimination process: but this must be done in the substitution.

The upshot of this is that the various multiplications and divisions which arise in the process of elimination may all be performed by slide rule, which greatly facilitates the work. The substitutions must be carried out with higher, or even absolute, accuracy, as can most conveniently be done by arithmometer.

7. As remarked in §5 it is sometimes convenient to have the solution in terms of symbolic values of the R.H.S. This gives rise to another general method of procedure as regards higher approximation to any desired degree, which will now be described. Other attendant advantages will be seen to arise in this method.

Suppose the solution of (1) is expressed in the form

$$\left.\begin{array}{l} x_1 = {}_{a}x_1 A + {}_{b}x_1 B + {}_{c}x_1 C + \ldots \\ \;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \\ x_r = {}_{a}x_r A + {}_{b}x_r B + {}_{c}x_r C + \ldots \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Then ${}_{a}x_1, {}_{a}x_2$ . . . . are the solutions of

$$\left.\begin{array}{l} a_1 x_1 + a_2 x_2 + \;.\;.\;.\;.\;. = 1 \\ b_1 x_1 + b_2 x_2 + \;.\;.\;.\;.\;. = 0 \\ r_1 x_1 + r_2 x_2 + \;.\;.\;.\;.\;. = 0 \\ \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \end{array}\right\}$$

and $\quad {}_rx_1, {}_rx_2 \quad \ldots \ldots$ are the solutions of

$$\left.\begin{array}{l} a_1x_1 + a_2x_2 \quad \ldots\ldots \quad = 0 \\ \ldots\ldots\ldots\ldots\ldots \\ r_1x_1 + r_2x_2 \quad \ldots\ldots \quad = 1 \\ \ldots\ldots\ldots\ldots \quad = 0 \\ \ldots\ldots\ldots\ldots\ldots \end{array}\right\} \quad \ldots\ldots\ldots \quad (6)$$

In (6) all the quantities on the R.H.S. are zero, except the rth, which is unity. The elimination processes are identical for all values of r, but the work on the R.H.S. differs for each value of r. The accurate solution of all the sets of form (6) gives values of all the quantities ${}_rx_s$: but any actual solution will give quantities slightly different, viz. ${}_rx_s - \delta_r x_s$. On substituting these, instead of getting the quantities unity and zero as values of the R.H.S. slightly different quantities are obtained, as indicated.

Values of R.H.S. *of equations* (6) *resulting from an approximate solution.*

Values of R.H.S.				Approximate solutions.			
	1	2	3	1	2	3	
1	$1+a_1$	a_2	a_3	${}_au_1$	${}_au_2$	${}_au_3$	
2	β_1	$1+\beta_2$	β_3	${}_bu_1$	${}_bu_2$	${}_bu_3$	
3	γ_1	γ_2	$1+\gamma_3$	${}_cu_1$	${}_cu_2$	${}_cu_3$	
.	. . .	. . .	. . . .	. . .	. . .	. . .	. . . .

If the original equations are symmetrical, so also are the quantities of the approximate solution.

It is clear that if the approximate calculation has been properly carried out all the quantities $a, \beta, \gamma \quad \ldots\ldots$ are small compared with unity.

8. Suppose the solutions 1, 2, 3 . . . are combined in any way, taking for x_1 the value $A_au_1 + B_au_2 + \ldots$ and similarly for the other x^s. Then it is clear that the corresponding values of the R. H. S. will be

$$\begin{array}{l} A(1 + a_1) + Ba_2 + Ca_3 \quad \ldots , \\ A\beta_1 + B(1+ \beta_2) + C\beta_3 \quad \ldots , \\ A\gamma_1 + B\gamma_2 + C(1 + \gamma_3) + \ldots , \end{array}$$

Putting $\quad A = 1 - a_1 \quad B = -\beta_1 \quad C = -\gamma_1 \ldots\ldots$ these become

$$\begin{array}{l} 1 - a_1^2 - \beta_1 a_2 - \gamma_1 a_3 \quad \ldots , \\ - a_1 \beta_1 - \beta_1 \beta_2 - \gamma_1 \beta_3 \quad \ldots , \\ \ldots\ldots\ldots \end{array}$$

Only products of the small quantities a, β . . . now occur except in the quantity unity in the first line: so that a higher approximation is readily found in this way. The process can obviously be repeated as often as is desirable.

9. The question can also be considered otherwise. Suppose the true value of any quantity x is $u-v$, u being a value obtained by solution and v a small correction. Then the solutions of equations

$$\begin{array}{l|c|c|c|} & A & B & C \\ a_1 x_1 + a_2 x_2 + \ldots = & 1 & 0 & 0 \\ b_1 x_1 + b_2 x_2 + = & 0 & 1 & 0 \\ \ldots & & & \end{array} \quad \text{are} \quad \begin{array}{|c|c|c|} A & B & C \\ {}_a x_1 & {}_b x_1 & {}_c x_1 \\ {}_a x_2 & {}_b x_2 & {}_c x_2 \end{array} \quad \ldots \quad (7)$$

and as above

$$\begin{array}{l|c|c|c|} & A & B & C \\ a_1 v_1 + a_2 v_2 + \ldots = & \alpha_1 & \alpha_2 & \alpha_3 \\ b_1 v_1 + b_2 v_2 + \ldots = & \beta_1 & \beta_2 & \beta_3 \\ \ldots & \cdot & \cdot & \cdot \end{array}$$

Whence by means of (7) the solutions are represented by

$$\left.\begin{array}{l} {}_s v_1 = \alpha_s\, {}_a x_1 + \beta_s\, {}_b x_1 + \gamma_s\, {}_c x_1 + \ldots \\ {}_s v_2 = \alpha_s\, {}_a x_2 + \beta_s\, {}_b x_2 + \gamma_s\, {}_c x_2 \ldots \\ {}_s v_3 = \alpha_s\, {}_a x_3 + \beta_s\, {}_b x_3 + \gamma_s\, {}_c x_3 \ldots \end{array}\right\} \quad \ldots \quad (8)$$

Hence

$${}_s v_r = \alpha_s\, {}_a x_r + \beta_s\, {}_b x_r + \gamma_s\, {}_c x_r + \ldots \quad (9)$$

The quantities ${}_a x_r$ etc. may as a first approximation be replaced by the determined quantities ${}_a u_r$ etc. The solution then takes the form

$${}_s u'_r = {}_s u_r - \alpha_s\, {}_a u_r - \beta_s\, {}_b u_r \ldots \quad (10)$$

and if further approximation is required this value may be substituted in (8) for ${}_a x_r$ giving the next approximation to ${}_s v_r$. Any number of successive approximations may be made in this manner. The *R. H. S.* corresponding to (9) may be written down. They are

$$\begin{array}{ll} 1+\alpha_1-\alpha_1(1+\alpha_1)-\alpha_2\beta_1-\alpha_3\gamma_1 \ldots, & \alpha_2-\alpha_2(1+\alpha_1)-\alpha_2\beta_2-\alpha_3\gamma_2 \ldots, \\ \beta_1-\beta_1(1+\alpha_1)-\beta_2\beta_1-\beta_3\gamma_1 & 1+\beta_2-\alpha_2\beta_1-\beta_2(1+\beta_2)-\gamma_2\beta_3 \ldots; \\ \ldots & \end{array}$$

or

$$\begin{array}{ll} 1-\alpha_1^2-\alpha_2\beta_1-\alpha_3\gamma_1 \ldots, & -\alpha_1\alpha_2-\alpha_2\beta_2-\alpha_3\gamma_2 \ldots, \\ -\beta_1\alpha_1-\beta_2\beta_1-\beta_3\gamma_1 \ldots, & 1-\beta_1\alpha_2-\beta_2^2-\beta_3\gamma_2 \ldots, \end{array}$$

which may be written with abbreviated form

$$\begin{array}{cccc} 1+\alpha_1' & \alpha_2' & \alpha_3' & \ldots \\ \beta_1' & 1+\beta_2' & \beta_3' & \ldots \\ \ldots & & & \end{array}$$

where $\alpha_1' = -\alpha_1^2 - \alpha_2\beta_1 - \alpha_3\gamma_1, \ldots$ etc.

These residuals α', β' etc., are composed of binary products of α, β etc. and accordingly are of a smaller order than α, β . . . . . Starting with the second approximate values u', . . and these residuals α', β' . . . another approximation may be made: and so on as far as is necessary to attain the accuracy of solution desired.

10. It may at times be convenient to split up a set of equations and apply the above process to any portion consisting of an equal number of rows and columns. This may be done with advantage when a considerable number of the coefficients are zero. It is to be remembered that the actual numerical labour of solving n equations varies as the cube of n, so that a group of say 30 equations presents a formidable piece of computation. By the method now proposed perhaps this labour may be considerably reduced: but one certain advantage is a substitution check at a comparatively early stage of the computation. This is a check against actual computation blunders, as well as against accumulation of error due to lack of absolute exactness in the calculation on account of the necessity of limiting the number of figures employed. In this way, as has been shown above, much of the work can be performed readily with a slide rule, greatly accelerating the work.

In a large class of equations, *e.g.* the normal equations which occur in the method of least squares, there is complete symmetry about a diagonal. This reduces the work of elimination to about one-half. It is important then that in dealing with equations of this class that the symmetry should be preserved. Gauss's arrangement secures this for his method of solution, and it will now be shown that symmetry is maintained when the equations are split up as just suggested.

Denote the equations by

$$\left.\begin{array}{l} (1,1)\,k_1 + (1,2)\,k_2 + \;\ldots\; + (1,n)\,k_n = \text{(i)} \\ (2,1)\,k_1 + (2,2)\,k_2 + \;\ldots\; + (2,n)\,k_n = \text{(ii)} \\ (n,1)\,k_1 + (n,2)\,k_2 + \;\ldots\; + (n,n)\,k_n = (n) \end{array}\right\} \quad \ldots\ldots (11)$$

Suppose the solution of

$$\left.\begin{array}{l} (1,1)\,k_1 + (1,2)\,k_2 \;\ldots\; + (1,r)\,k_r = [\text{i}] \\ (2,1)\,k_1 + (2,2)\,k_2 \;\ldots\; + (2,r)\,k_r = [\text{ii}] \\ (r,1)\,k_1 + (r,2)\,k_2 \;\ldots\; + (r,r)\,k_r = [r] \end{array}\right\} \quad \ldots\ldots (12)$$

where r is less than n, is

$$\left.\begin{array}{l} k_1 = {}_1k_1\,[\text{i}] + {}_2k_1\,[\text{ii}] + \;\ldots\; + {}_rk_1\,[r] \\ k_2 = {}_1k_2\,[\text{i}] + \;\ldots \\ k_r = {}_1k_r\,[\text{i}] + \;\ldots\; + {}_rk_r\,[r] \end{array}\right\}^{*} \quad \ldots\ldots (13)$$

Then from the first r equations of (11) it is seen that

$$\left.\begin{array}{l} [\text{i}] = \text{(i)} - (1,r+1)\,k_{r+1} - (1,r+2)\,k_{r+2} - \;\ldots\; - (1,n)\,k_n \\ [\text{ii}] = \text{(ii)} - (2,r+1)\,k_{r+1} - (2,r+2)\,k_{r+2} - \;\ldots\; - (2,n)\,k_n \\ [r] = (r) - (r,r+1)\,k_{r+1} - (r,r+2)\,k_{r+2} - \;\ldots\; - (r,n)\,k_n \end{array}\right\} \quad \ldots (14)$$

Substituting from (14) in (13) it follows that

* This may be a first or higher approximation as appears most suitable.

$$\left.\begin{array}{rl} k_1 = & {}_1k_1\,(\text{i}) + {}_2k_1\,(\text{ii}) + \;.\;.\;.\;.\; + {}_rk_1\,(r) \\ & - k_{r+1}\,\Sigma\,{}_sk_1\,(s, r+1) - k_{r+2}\,\Sigma\,{}_sk_1\,(s, r+2) - \;.\;.\;.\; - k_n\,\Sigma_s k_1\,(s, n) \\ \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot \\ k_t = & {}_1k_t\,(\text{i}) + {}_2k_t\,(\text{ii}) + \;.\;.\;.\;.\; + {}_rk_t\,(r) \\ & - k_{r+1}\,\Sigma\,{}_sk_t\,(s, r+1) - k_{r+2}\,\Sigma_s k_t\,(s, r+2)\;.\;.\;.\; - k_n\,\Sigma\,{}_sk_t\,(s, n) \\ \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot \end{array}\right\} \quad . \; . \; (15)$$

the summation indicated by Σ referring to s to which all values from 1 to r are to be given.

The values $k_1, k_2 \; . \; . \; . \; . \; k_r$ are now to be substituted in the latter $n-r$ equations of (11). It is clear that thereby the coefficients of $k_{r+1}\; k_{r+2} \; . \; . \; . \; k_n$ are altered. The original coefficients are clearly symmetrical, and it is only necessary to show that the change in the coefficient of k_u in the $r+1^{th}$ equation—the first of the equations dealt with—is the same as the change in the coefficient of k_{r+1} in the u^{th} equation. By (15) and (11) it is seen that the change in coefficient of k_u in the $r+1^{th}$ equation is

$$-(r+1, 1)\,\Sigma\,{}_sk_1\,(s, u) - (r+1, 2)\,\Sigma\,{}_sk_2\,(s, u) - \;.\;.\;.\; -(r+1, r)\,\Sigma\,{}_sk_r\,(s, u) \quad . \; . \; (16)$$

while the change in the coefficient of k_{r+1} in the u^{th} equation is

$$-(u, 1)\,\Sigma\,{}_sk_1\,(s, r+1) - (u, 2)\,\Sigma\,{}_sk_2\,(s, r+1) - \;.\;.\;.\; -(u, r)\,\Sigma\,{}_sk_r\,(s, r+1) \quad . \; . \; (17)$$

Remembering that ${}_sk_t = {}_tk_s$ notice that the sum of the coefficients of ${}_sk_t$ and ${}_tk_s$ in (16) is $-\;(r+1, t)\,(s, u) - (r+1, s)\,(t, u)$ and the corresponding quantity in (17) is $-(u, t)\;(s, r+1)$ $-(u, s)\;(t, r+1)$. These quantities are the same, since $(u, t) = (t, u)$ etc.

The equations resulting from this method of solution are accordingly symmetrical.

11. In some cases after the solution of a set of normal equations has been effected, additional conditions may have to be introduced. The form of the equations, *vide* (13), is only modified thereby by the addition of a number of terms at the end of the original equations and an addition of the same number of equations at the end. If then the original equations have been solved in the manner explained above, it is possible to proceed immediately to derive the solution of the larger number of equations, making use of the solution already obtained. If the ordinary method of solution of the original equations had been followed this would have been of little help in proceeding to the solution of the larger number of equations.

These methods will now be given effect to in the solution of the 26 equations of p. 116. In table XLVII the coefficients are marked off in the stages for which solution will be performed.

In cases where a highly accurate solution is not desired the values of the residuals are not required. It is however of importance to verify that the solution does not contain any blunders, as may easily occur in the numerical work. A check on this is obtained by substituting the values obtained for solution A, B, . . . in the last equation. This equation only enters into the final eliminant from which the value of the last unknown is determined, and not into any of the previous equations used for the actual solution—that is the first of each group of equations formed by successive elimination of the first, second, ... unknowns. If then this last equation is satisfied with satisfactory precision it is an indication that no blunder has been committed. It is not certain from this that the residuals of the other equations are equally small, and nothing short of substitution in each of these will make this point quite clear: but it is a sure check against any serious blunder.

12. Application of the method suggested above will now be made to the equation whose solution is necessary for the determination of the several probable errors of the N.W. Quadrilateral after adjustment. The L.H.S. of the equations are indicated in table XLVII. Conformably with §10 only a portion of the complete set is dealt with at first. It is at once clear that the first 8 equations and the next 12 equations form convenient groups. The first step is to solve the first 8 equations for the quantities k_1 . . . k_8 ignoring for the present quantities k_9 . . . . k_{16} which occur in the 5th—8th equations. The R.H.S. are taken as zero or unity, *vide* (6). As an example of (3) multiplications by powers of 10 are introduced and the order of equations arranged to make the diagonal quantities in increasing order of magnitude. In this particular case there is little gained by the former procedure, which is introduced merely to illustrate the method. The arrangement of the work is shown in tabular form in table LV, of which detailed explanation is now given.

TABLE LV.

Equation Number	Right Hand Side	Left Hand Side 1 k_3	 2 k_4	 3 k_7	 4 k_8	 5 $\frac{1}{10}k_1$	 6 $\frac{1}{10}k_2$	 7 $\frac{1}{10}k_5$	 8 $\frac{1}{10}k_6$
1 1 (1)	+1·000 +4·1335	+40·68	0 o	+7·35 −·0296	+1·14 −·0001	+22·0 −·1354	+103·8 −2·993	−0·3 −·0001	+17·7 +·0247
2 3 4 5 6 7 8	0 o 0 −·1807 0 −·0280 0 −·541 0 −2·552 0 +·0074 0 −·435		+40·68 o	−1·14 o +111·51 −1·33	+7·35 o 0 −·206 +111·51 −·03	−103·8 o +4·1 −3·975 −23·0 −·62 +366·0 −11·9	+22·0 o +23·0 −18·76 +4·1 −2·906 0 −56·16 +366·0 −264·9	−17·7 o −8·4 +·054 −282·2 +·0084 +70·0 +·162 0 +·7656 +971·0 −·002	−0·3 o +282·2 −3·2 −8·4 −·496 0 −9·576 +70·0 −45·17 0 +·131 +971·0 −7·7
9 9(1) 9(2)	0 −·0006	+1·000 +4·1333	+40·68	−1·14 +·0046 −·0001	+7·35 −·0008 −·0296	−103·8 +·6389 −2·993	+22·0 −·6344 −·1352	−17·7 −·0073 +·0247	−0·3 −·0004 −·0001
10 11 12 13 14 15	−·1807 o −·0280 o −·541 o −2·552 o +·0074 o −435 o	0 +·0280 0 −·1807 0 +2·552 0 −·541 0 +·435 0 +·0074		+110·18 −·03	−·206 +·206 +111·48 −1·33	+·125 −2·906 −23·62 +18·76 +354·1 −264·9	+4·24 +·62 +1·194 −3·975 −56·16 +56·144 +101·1 −11·9	−8·346 −·496 −282·2 +3·2 +70·16 −45·17 +·7656 +9·576 +971·0 −7·7	+279·0 −·01 −8·896 +·054 −9·576 −·7656 +24·83 +·16 +·131 −·131 +963·3 −·002
16 16(1) 16(2) 16(3)	−·1807 −·4433	+·0280 +·0116	+1·000 +3·7709	+110·15	0	−2·781 +·0171 −·0802 +·0005	+4·86 −·1401 −·0299 +·0043	−8·842 −·0036 +·0123 −·0027	+279·0 +·3892 +·1142 −2·773
17 18 19 20 21	−·0280 o −·541 −·0046 −2·552 +·008 +·0074 −·0145 −·435 +·4577	−·1807 o +2·552 +·0007 −·541 −·0012 +·435 +·0022 +·0074 −·0709	0 o 0 +·02525 0 −·04412 0 +·0803 0 −2·533		+110·15 o	−4·86 o +89·2 −·07	−2·781 o −·016 +·1227 +89·2 −·21	−279·0 o +24·99 −·223 +10·342 +·39 +963·3 −·71	−8·842 o −10·342 +7·045 +24·99 −12·31 0 +22·40 +963·3 −706·7

TABLE LV (Continued).

Equation Number	Right Hand Side				Left Hand Side				
					4 k_8	5 $\frac{1}{10}k_1$	6 $\frac{1}{10}k_2$	7 $\frac{1}{10}k_5$	8 $\frac{1}{10}k_6$
22 22(1) 22(2) 22(3) 22(4)	− ·028 − ·0114	− ·1807 − ·4436	0 + ·0002	+ 1·000 + 3·7709	+110·15	− 4·86 + ·0299 − ·1401 + ·0008 + ·0043	− 2·781 + ·0802 + ·0171 − ·0025 + ·0005	−270·0 − ·1144 + ·3895 − ·0864 − 2·773	− 8·842 − ·0132 − ·0036 + ·0879 − ·0027
23 24 25 26	− ·5456 − ·0012 − 2·544 − ·001 − ·0071 − ·0709 + ·0227 − ·0022	+ 2·5527 − ·0080 − ·5422 − ·0046 + ·4372 − ·4577 − ·0635 − ·0145	+ ·02525 0 − ·0441 0 + ·0803 0 − 2·533 0	0 + ·04412 0 + ·02525 0 + 2·533 0 + ·0803		+ 89·13 − ·21	+ ·1067 − ·1227 + 88·99 − ·07	+ 24·77 − 12·31 + 10·732 − 7·045 +962·6 −706·7	− 8·297 − ·390 + 12·68 − ·22 + 22·40 − 22·40 +256·6 − ·71
27 27(1) 27(2) 27(3) 27(4) 27(5)	− ·5468 − ·5473	+ 2·5447 + 2·5635	+ ·02525 − ·01525	+ ·04412 − ·0787	+ 1·000 + 1·0075	+ 88·92	− ·016 + ·0005 + ·0001 0 0 0	+ 12·46 + ·0051 − ·0174 + ·0039 + ·1239 − ·0069	− 8·687 − ·0051 − ·0015 + ·0366 − ·0011 − ·0006
28 29 30	− 2·545 − ·0001 − ·078 + ·0766 + ·0205 − ·0227	− ·5468 + ·0005 − ·0205 − ·3566 − ·078 + ·1055	− ·04412 0 + ·0803 − ·0035 − 2·533 + ·001	+ ·02525 + ·00001 + 2·533 − ·006 + ·0803 + ·0018	0 + ·00018 0 − ·1401 0 + ·04146		+ 88·92 0	+ 3·687 + ·002 +255·9 − 1·75	+ 12·48 − ·001 0 + ·5165 +255·9 − ·15
31 31(1) 31(2) 31(3) 31(4) 31(5) 31(6)	− 2·5451 − 2·564	− ·5463 − ·5463	− ·04412 + ·0786	+ ·02526 − ·0153	+ ·00018 + ·00018	+ 1·000 + 1·0075	+ 88·92	+ 3·689 + ·0015 − ·0051 + ·0011 + ·0367 − ·0020 − ·0006	+ 12·459 + ·0174 + ·0051 − ·1238 + ·0039 + ·0020 − ·0069
32 33	− ·0014 + ·1056 − ·0022 + ·3566	− ·3771 + ·0227 + ·0275 + ·0766	+ ·0768 + ·0018 − 2·532 + ·006	+ 2·527 − ·001 + ·0821 − ·0035	− ·1401 0 + ·04146 0	0 − ·04149 0 − ·1401		+254·15 − ·153	+ ·5165 − ·5169 +255·75 − 1·75
34 34(1) 34(2) 34(3) 34(4) 34(5) 34(6) 34(7)	+ ·1042 + ·1042	− ·3544 − ·3544	+ ·0786 + ·0786	+ 2·526 + 2·526	− ·1401 − ·1401	− ·04149 − ·04149	+ 1·000 + 1·000	+254·0	− ·0004 0 0 0 0 0 0 0
35	+ ·3544 0	+ ·1040 0	− 2·526 0	+ ·0786 0	+ ·04146 0	− ·1401 0	+ 0·000 + ·000002		+254·0 0
36	− ·3544	+ ·1040	− 2·526	+ ·0786	+ ·04146	− ·1401	+ 0·000	+ 1·000	+254·0
Case		I	II	III	IV	V	VI	VII	VIII

Solution 1st approximation.	k_3	k_4	k_7	k_8	$\frac{1}{10}k_1$	$\frac{1}{10}k_2$	$\frac{1}{10}k_5$	$\frac{1}{10}k_6$
Case I	+ ·1016	− ·00001474	− ·004025	− ·0001035	− ·006155	− ·028835	+ ·00041	+ ·001395
II		+ ·1016	+ ·0001053	− ·004027	+ ·028835	− ·006144	− ·001396	+ ·0004094
III			+ ·03423	+ ·00000182	− ·00017	+ ·000884	+ ·0008095	− ·00994
IV				+ ·03423	− ·000875	− ·000172	+ ·00994	+ ·0008095
V					+ ·01133	+ ·00000202	− ·000552	+ ·000163
VI						+ ·01133	− ·0001635	− ·000552
VII							+ ·003937	0
VIII								+ ·003937

The first, second, fifth and sixth lines and columns of table XLVII are multiplied by 10 and their rows and columns are arranged in order 3, 4, 7, 8, 1, 2, 5, 6: giving rise to equations whose coefficients are written down in column headed "Left Hand Side 1 . . . 8," and in lines whose equation numbers are 1 to 8. The Right Hand Sides of these equations are

$$\begin{array}{c|c|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ . & . & . \\ . & . & . \end{array}$$

It is however only necessary to write down the first column under heading R.H.S. To eliminate the first quantity k_3 from these 8 equations, the first equation is multiplied successively by $-\frac{0}{40 \cdot 68}=0$, $\frac{-7 \cdot 35}{40 \cdot 68}=-\cdot 1807$, $\frac{-1 \cdot 14}{40 \cdot 68}=-\cdot 280$ . . . . . and the results written below the 2nd, 3rd, 4th equations in old face type. These multipliers are also applied to the right hand side of the first equation which is unity, and therefore appear in this column. The work is performed by sliderule, making one setting representing division by $40 \cdot 68$, and reading off opposite the quantities 0, $7 \cdot 35$, $1 \cdot 14$ etc. these quantities being at once entered in R.H.S. column (shown in old face type). In completing the multiplication of the first equation by these several factors, it is noticed that the quantity to be entered is the product of the factor of the particular line by the coefficient of the particular column in equation No. 1, *e. g.* old face figures $-3 \cdot 975$ (line 3, column 5) $= -\cdot 1807 \times 22 \cdot 0$. The old face figures thus formed for all the equations Nos. $2-8$, are added to the corresponding coefficients and give rise to 7 equations (numbered 9—15) from which k_3 has been eliminated. The process as regards the right hand side has only actually been applied for case I in which the right hand side of the first equation is unity and the rest are zero (*vide* § 7): but it will be easily seen that for the other cases all the old face quantities would be zero having zero as a factor. Case II is accordingly brought in conveniently after the elimination of k_3 has been completed.

The necessary multipliers for the next elimination, that of k_4, are now duly entered of the R.H.S. for case II. They are the old face figures $+\cdot 0280$, $-\cdot 1807$, $+2\cdot 552$ . . . : and the process of elimination is proceeded with. In this way the groups of equations $16-21$, $22-26$, $27-30$, $31-33$, $34-35$, 36 are formed successively in the last of which only $\frac{1}{10} k_6$ occurs. From this values of $\frac{1}{10} k_6$ are written down at the bottom of the table for each of the eight cases. These values would be substituted in 34, except that the coefficient of k_8 is so small that they are negligible. Thus equations 34 (1) . . . 34 (7) are formed giving values of $\frac{1}{10} k_5$ for seven cases. Then the values of $\frac{1}{10} k_6$ and $\frac{1}{10} k_5$ are substituted in 31, giving 31 (1) . . 31 (6) from which six values of $\frac{1}{10} k_2$ are formed. These several substitutions are shown in old face. The results of the terms on the left hand sides are combined and with changed sign applied to the right hand side for the corresponding case, *e. g.* 31 (3) $+\cdot 0786 = -\cdot 04412 - \cdot 0011 + \cdot 1238$. All the values obtained are exhibited at the foot of the table: they are the values of ${}_r x_s$ in the notation of § 7. Since the equations are symmetrical the solution is also symmetrical and ${}_r x_s = {}_s x_r$: so that it is only necessary to take out half of the quantities. Had the equations not been symmetrical all work below the diagonal in each group of eliminants would have had to be completed and the full number of cases, eight, would have been necessary in substituting in each of equations 1, 9, 16, 22, 27, 31, 34, 36.

13. An approximate solution of the first eight equations has now been found. As it will be necessary to substitute from the solution in the remaining equations, it is desirable in any case at this stage to check the solution; but this substitution at the same time enables a higher approximation to be reached by (8). This is desirable; although a high order of accuracy of final solution is not desired, yet it is proper to avoid the introduction of computation inaccuracy at an early stage in the work. The first step is to substitute in the equations and so to find the quantities α, β . . . of § 7. In performing this substitution a sliderule cannot be used, as greater precesion is desired. If an arithmometer is used, then there is little extra labour in taking out the work to the full number of figures which occur. When this has been done (10) gives a means of an infinite number of successive approximations. As exemplifying this, full accuracy is kept in this substitution, which is reproduced for Case I only in table LVI. It is to be remarked that there is no advantage in keeping the solution of

table LV to as many figures as has been done, as the latter figures cannot be accurate: and their presence adds to the labour of multiplication. However in the present case the substitution had already been carried out by the computer before this simplification could be given effect to.

TABLE LVI.

Equation	1	2	3	4	5	6	7	8
Case I	+4·183,088,00	0	+·746,760,00	+·115,824,00	+2·235,200	+10·546,080,00	−·030,480	+1·798,320,00
	0	−·000,599,623,2	+·000,016,803,6	−·000,108,339	+0·001,530,012	− 0·000,324,28	+·000,260,898	+0·000,004,422
	−0·029,588,75	+·004,588,85	−·448,827,75	0	−0·016,502,5	− 0·092,575	+·033,810	−1·135,855
	−0·000,117,99	−·000,760,725	0	−·011,541,285	+0·002,380,5	− 0·000,424,35	+·029,207,7	+0·000,869,4
	−0·135,410	+·638,889	−·025,235,5	+·141,565	−2·252,730	0	−·430,850	0
	−2·993,073	−·634,370	−·663,205	−·118,228,5	0	−10·553,610	0	−2·018,450
	−0·000,123	−·007,257	−·003,444	−·115,702	+0·028,700	0	+·398,110	0
	+0·024,691,5	−·000,418,5	+·393,669	−·011,718	0	+ 0·097,650	0	+1·354,545
Sum	+0·999,471,76	+·000,072,001,8	−·000,266,446,4	+·000,095,876	−0·001,421,988	− 0·003,203,63	+·000,058,598	−0·000,566,178

Table LVII gives the results of the substitution for all cases, that is the quantities $\alpha\ \beta\ \gamma$ . . in notation of § 7. To avoid constant repetition of zeros, they have all been multiplied by 10^3. It is now perfectly straightforward to substitute in (10) and obtain a second approximation. Since the solution is known to be symmetrical, it is not necessary to perform the substitution on both sides of the diagonal: but as a check it may sometimes be useful to do so. When two determinations of what is known by symmetry to be one quantity differ slightly, the mean can be taken.

TABLE LVII.

r	1	2	3	4	5	6	7	8
$10^3\alpha_r$	− ·528,24	+ ·072,001,8	− ·266,446,4	+ ·095,876	−1·421,988	−3·203,63	+ ·058,598	− ·566,178
$10^3\beta_r$	− ·128,468,2	− ·285,112	− ·019,756	− ·190,733,6	+2·538,45	− ·464,812	+ ·153,302	+ ·248,962
$10^3\gamma_r$	− ·156,075,2	− ·127,369	− ·749,292	+ ·077,903,2	+ ·266,00	− ·436,938	− ·077,414	−1·443,378
$10^3\delta_r$	+ ·217,747	−1·219,784,8	+ ·334,403,2	− ·977,14	+3·993,062	− ·239,44	+2·077,762	+ ·842,254
$10^3\epsilon_r$	− ·112,024	+ ·482,29	+ ·067,01	+1·172,782	− ·915	+ ·132,82	−3·072,0	+ ·196,4
$10^3\zeta_r$	− ·329,39	− ·100,006	− ·160,968	− ·026,98	+ ·251,92	− ·474,2	− ·105,0	− ·518,7
$10^3\eta_r$	− ·012,175	− ·067,41	+ ·022,785	− ·359,55	+ ·131,75	− ·122,5	+1·105,4	+ ·075,7
$10^3\theta_r$	+ ·015,73	+ ·016,317	+ ·370,834	+ ·028,735	− ·020,22	+ ·014,75	− ·099,78	+1·087,88

The result of the second approximation, found by means of (8), is given in table LVIII which has been rearranged in the order of the original quantities.

TABLE LVIII.

	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8
1	+1·1325	+ ·000,006	−·061,44	+·288,135	−·055,14	+·016,27	−·001,702	−·008,835
2		+1·132,5	−·288,135	−·061,44	−·016,27	−·055,14	+·008,835	−·001,702
3			+·101,552	0	+·004,079	+·018,936	−·004,021	−·000,108,5
4				+·101,552	−·013,936	+·004,079	+·000,108,5	−·004,021
5					+·393,63	0	+·003,092	+·099,41
6						+·393,63	−·099,41	+·003,092
7							+·034,241	0
8								+·034,241

14. This solution corresponds to that indicated in (13). The next step is to form (15) with a view to substitution for k_1 . . . . k_8 in the equations Nos. 9—20. For this only values of k_5 . . k_8 are required since the coefficients of k_1 . . . . k_4 are zero in these equations. However values of k_1 . . k_4 are also required at a later stage, so the complete series of quantities k_1 to k_8 are expressed

according to (15). Denote the quantities given in table LVIII by ${}_s\mathrm{K}_r$ where r and s have all values from 1 to 8. It is necessary to compute all the quantities $\Sigma\ {}_s\mathrm{K}_r\ (s, t)$ where s has all values from 1 to 8 and is the quantity to which Σ refers, for each value of r from 1 to 8 and each value of t from 9 to 20 : (s, t) in the notation of (11) indicate the coefficients shown in table XLVII, while ${}_s\mathrm{K}_r$ are the quantities of table LVIII. Owing to zero coefficients it has only to be taken for values 9 to 16 and so the number of quantities $\Sigma_s\mathrm{K}_r\ (s, t)$ actually to be computed is $8 \times 8 = 64$; and among them there is a sort of skew symmetry due to equality in pairs of coefficients in table XLVII, making altogether only 32 independent quantities. Further the summation Σ although relating to 8 values of s, actually only gives rise to four terms owing to zero coefficients. The details of the computation are given in full in table LIX. The computation is taken out to full accuracy to exhibit the symmetry which afterwards occurs when substituting in equations Nos. 9—20.

TABLE LIX.

Values of ${}_s\mathrm{K}_r\ (s, t)$ and of $\Sigma^s\ {}_s\mathrm{K}_r\ (s, t)$, the latter in old face type.

r	s	t = 9	10	11	12	13	14	15	16
1	5	− ·356,755,80	0	− ·897,127,80	+ ·348,484,8	− ·109,728,60	0	− ·102,009,00	+ ·142,812,60
	6	0	+ ·105,266,90	+ ·102,826,40	+ ·264,712,9	0	+ ·032,377,30	+ ·042,139,30	+ ·030,099,50
	7	+ ·002,246,64	− ·039,094,94	− ·050,418,24	− ·100,588,2	+ ·000,136,16	− ·002,331,74	− ·003,438,04	− ·002,314,72
	8	+ ·202,939,95	+ ·011,662,20	+ ·522,148,50	− ·261,692,7	+ ·012,103,95	+ ·000,706,80	+ ·012,015,60	− ·017,846,70
Sum		− ·151,569,21	+ ·077,834,16	− ·322,566,14	+ ·250,916,8	− ·097,488,49	+ ·030,752,36	− ·051,292,14	+ ·152,750,68
2	5	− ·105,266,90	0	− ·284,712,9	+ ·102,826,40	− ·032,377,30	0	− ·030,099,50	+ ·042,139,30
	6	0	− ·356,755,80	− ·348,484,8	− ·827,127,80	0	− ·109,728,60	− ·142,812,60	− ·102,009,00
	7	− ·011,662,20	+ ·202,939,95	+ ·261,692,7	+ ·522,148,50	− ·000,706,80	+ ·012,103,95	+ ·017,846,70	+ ·012,015,60
	8	+ ·039,094,94	+ ·002,246,64	+ ·100,588,2	− ·050,413,24	+ ·002,331,74	+ ·000,136,16	+ ·002,314,72	− ·003,438,04
Sum		− ·077,834,16	− ·151,569,21	− ·250,916,8	− ·322,566,14	− ·030,752,36	− ·097,488,49	− ·152,750,68	− ·051,292,14
3	5	+ ·026,391,130	0	+ ·066,365,33	− ·025,779,28	+ ·008,117,210	0	+ ·007,546,15	− ·010,564,61
	6	0	+ ·000,165,92	+ ·088,075,52	+ ·226,738,72	0	+ ·027,732,64	+ ·036,094,24	+ ·025,781,60
	7	+ ·005,307,720	− ·092,362,37	− ·119,102,02	− ·237,641,10	+ ·000,321,680	− ·005,508,77	− ·008,122,42	− ·005,468,56
	8	+ ·002,492,245	+ ·000,143,22	+ ·006,412,35	− ·003,213,77	+ ·000,148,645	+ ·000,008,68	+ ·000,147,56	− ·000,219,17
Sum		+ ·034,191,095	− ·002,053,23	+ ·041,751,18	− ·039,895,43	+ ·008,587,535	+ ·022,232,55	+ ·035,665,53	+ ·009,529,26
4	5	− ·090,165,92	0	− ·226,738,72	+ ·088,075,52	− ·027,732,64	0	− ·025,781,60	+ ·036,094,24
	6	0	+ ·026,391,130	+ ·025,779,28	+ ·066,365,33	0	+ ·008,117,210	+ ·010,564,61	+ ·007,546,15
	7	− ·000,143,22	+ ·002,492,245	+ ·003,213,77	+ ·006,412,35	− ·000,008,68	+ ·000,148.645	+ ·000,219,17	+ ·000,147,56
	8	+ ·092,362,37	+ ·005,307,720	+ ·237,641,10	− ·119,102,02	+ ·005,508,77	+ ·000,321,680	+ ·005,468,56	− ·008,122,42
Sum		+ ·002,053,23	+ ·034,191,095	+ ·039,895,43	+ ·041,751,18	− ·022,232,55	+ ·008,587,535	− ·009,529,26	+ ·035,665,53
5	5	+2·546,786,10	0	+6·404,360,10	−2·487,741,6	+ ·783,323,70	0	+ ·728,215,50	−1·019,501,70
	6	0	0	0	0	0	0	0	0
	7	− ·004,081,44	+ ·071,023,24	+ ·001,585,04	+ ·182,737,2	− ·000,247,36	+ ·004,236,04	+ ·006,245,84	+ ·004,205,12
	8	−2·283,447,70	− ·131,221,20	−5·875,131,00	+2·044,524,2	− ·136,191,70	− ·007,952,80	− ·135,197,60	+ ·200,808,20
Sum		+ ·259,256,96	− ·060,197,96	+ ·620,814,14	+ ·639,519,8	+ ·646,884,64	− ·003,716,76	+ ·599,263,74	− ·814,488,38
6	5	0	0	0	0	0	0	0	0
	6	0	+2·546,786,10	+2·487,741,6	+6·404,360,10	0	+ ·783,323,70	+1·019,501,70	+ ·728,215,50
	7	+ ·131,221,20	−2·283,447,70	−2·044,524,2	−5·875,131,00	+ ·007,952,80	− ·136,191,70	− ·200,808,20	− ·135,197,60
	8	− ·071,023,24	− ·004,081,44	− ·182,737,2	+ ·001,585,04	− ·004,236,04	− ·000,247,36	− ·004,205,12	+ ·006,245,84
Sum		+ ·060,197,96	+ ·259,256,96	− ·639,519,8	+ ·620,814,14	+ ·003,716,76	+ ·646,884,64	+ ·814,488,38	+ ·599,263,74
7	5	+ ·020,005,24	0	+ ·050,306,84	− ·019,541,44	+ ·006,153,08	0	+ ·005,720,20	− ·008,008,28
	6	0	− ·643,182,70	− ·628,271,20	−1·617,400,70	0	− ·197,825,90	− ·257,471,90	− ·183,908,50
	7	− ·045,198,12	+ ·786,515,77	+1·014,218,42	+2·023,643,10	− ·002,739,28	+ ·046,910,17	+ ·069,166,82	+ ·046,567,76
	8	0	0	0	0	0	0	0	0
Sum		− ·025,192,88	+ ·143,333,07	+ ·436,254,06	+ ·386,700,96	+ ·003,413,80	− ·150,915,73	− ·182,584,88	− ·145,349,02
8	5	+ ·643,182,70	0	+1·617,400,70	− ·628,271,20	+ ·197,825,90	0	+ ·183,908,50	− ·257,471,90
	6	0	+ ·020,005,24	+ ·019,541,44	+ ·050,306,84	0	+ ·006,153,08	+ ·008,008,28	+ ·005,720,20
	7	0	0	0	0	0	0	0	0
	8	− ·786,515,77	− ·045,198,12	−2·023,643,10	+1·014,218,42	− ·046,910,17	− ·002,739,28	− ·046,567,76	+ ·069,166,82
Sum		− ·143,333,07	− ·025,192,88	− ·386,700,96	+ ·436,254,06	+ ·150,915,73	+ ·003,413,80	+ ·145,349,02	− ·182,584,88

15. Values of k_5, k_6, k_7, k_8 given in (15) contain terms in $k_4 \ldots k_{16}$ of which the coefficients have just been found in table LIX. These are to be substituted in equations 9—16, they do not occur in equations 17 to 20. The formation of the products and the collecting of coefficients is carried out in Table LX. In this the values of $\Sigma\ {}_s\mathrm{K}_r\ (s, t)$ are rewritten with sign changed at the top, kept to six places, while the multiplying coefficients are all shown in the first column. The previously existing

coefficients of k_9 . . k_{20} are also included. The complete coefficients of k_9 to k_{20} after including the portions due to substitution of k_5 to k_8, are shown in Table LX in old face. Thus twelve symmetrical equations relating k_9 to k_{20} have been formed, k_1 to k_8 having been eliminated. The solution of these twelve equations is performed in a manner similar to that employed for the solution of the first eight. The equations are first rearranged in increasing order of the diagonal coefficients, the whole process being given in table LXI. It does not seem likely that a second approximation is necessary in this case, so a verification, as described in § 11, is carried out in table LXII showing the degree of precision with which the last equation of the group of 12 is satisfied for each of the 12 cases.

TABLE LX.

Substitution of k_5—k_8 in equations 9 to 20.

r	t	9	10	11	12	13	14	15	16	17	18	19	20
5	$\Sigma^s_s K_r(s,t)$	−·259,257	+·060,198	−·620,814	−·639,520	−·646,885	+·003,717	−·599,264	+·814,488				
6		−·060,198	−·259,257	+·639,520	−·620,814	−·003,717	−·646,885	−·814,488	−·599,264				
7		+·025,193	−·143,333	−·436,254	−·386,701	−·003,414	+·150,916	+·182,585	+·145,349				
8		+·143,333	+·025,193	+·386,701	−·436,254	−·150,916	−·003,414	−·145,349	+·182 585				
Equation 9	Multiplier												
	+ 6·47	− 1·6774	+ ·3895	− 4·0167	− 4·1377	− 4·1853	+ ·0240	− 3·8772	+ 5·2697				
	0	0	0	0	0	0	0	0	0				
	− 1·32	− ·0333	+ ·1892	+ ·5759	+ ·5104	+ ·0045	− ·1992	− ·2410	− ·1919				
	− 22·97	− 3·2924	− ·5787	− 8·8825	+10·0208	+ 3·4665	+ ·0784	+ 3·3387	− 4·1940				
	(9,t)	+ 9·68	0	+16·81	−11·33	+ 0·28	0	+ 0·12	− 0·60	+ 0·55	0	− 0·27	− 1·66
	Sum	+ 4·6770	0	+ 4·4867	− 4·9365	− 0·4343	− 0·0968	− 0·6595	+ 0·2838				
10	0	0	0	0	0	0	0	0	0				
	+ 6·47	− ·3895	− 1·6774	+ 4·1377	− 4·0167	− ·0240	− 4·1853	− 5·2697	− 3·8772				
	+ 22·97	+ ·5787	− 3·2924	−10·0208	− 8·8825	− ·0784	+ 3·4665	+ 4·1940	+ 3·3387				
	− 1·32	− ·1892	− ·0333	− ·5104	+ ·5759	+ ·1992	+ ·0045	+ ·1919	− ·2410				
	(10,t)	0	+ 9·68	+11·33	+16·81	0	+ ·28	+ ·60	+ ·12	0	+ 0·55	+ 1·66	− 0·27
	Sum	0	+ 4·6770	+ 4·9365	+ 4·4867	+ 0·0968	− 0·4343	− 0·2838	− 0·6595				
11	+ 16·27	− 4·2181	+ ·9794	−10·1006	−10·4050	−10·5248	+ ·0605	− 9·7500	+13·2517				
	+ 6·32	− ·3804	− 1·6385	+ 4·0418	− 3·9235	− ·0235	− 4·0883	− 5·1476	− 3·7873				
	+ 29·62	+ ·7462	− 4·2455	−12·9218	−11·4541	− ·1011	+ 4·4701	+ 5·4082	+ 4·3052				
	− 59·10	− 8·4710	− 1·4889	−22·8540	+25·7826	+ 8·9191	+ ·2018	+ 8·5901	−10·7908				
	(11,t)	+16·81	+11·33	+79·30	0	+ ·61	− ·28	− ·32	− 1·44	+ 0·48	− 0·21	− 0·71	− 1·36
	Sum	+ 4·4867	+ 4·9365	+37·4654	0	− 1·1203	+ 0·3640	− 1·2193	+ 1·5388				
12	− 6·32	+ 1·6385	− ·3804	+ 3·9235	+ 4·0418	+ 4·0883	− ·0235	+ 3·7873	− 5·1476				
	+ 16·27	− ·9794	− 4·2181	+10·4050	−10·1006	− ·0605	−10·5248	−13·2517	− 9·7500				
	+ 59·10	+ 1·4889	− 8·4710	−25·7826	−22·8540	− ·2018	+ 8·9191	+10·7908	+ 8·5901				
	+ 29·62	+ 4·2455	+ ·7462	+11·4541	−12·9218	− 4·4701	− ·1011	− 4·3052	+ 5·4082				
	(12,t)	−11·33	+16·81	0	+79·30	+ ·28	+ ·61	+ 1·44	− ·32	+ 0·21	+ 0·48	+ 1·36	− 0·71
	Sum	− 4·9365	+ 4·4867	0	+37·4654	− 0·3640	− 1·1203	− 1·5388	− 1·2193				
13	+ 1·99	− ·5159	+ ·1198	− 1·2354	− 1·2726	− 1·2873	+ ·0074	− 1·1925	+ 1·6208				
	0	0	0	0	0	0	0	0	0				
	− 0·08	− ·0020	+ ·0115	+ ·0349	+ ·0309	+ ·0003	− ·0121	− ·0146	− ·0116				
	− 1·37	− ·1964	− ·0345	− ·5298	+ ·5977	+ ·2068	+ ·0047	+ ·1991	− ·2501				
	(13,t)	+ ·28	0	+ ·61	+ ·28	+ 3·43	0	+ 2·13	− 4·28	+ 0·83	0	+ 0·32	− 1·20
	Sum	− 0·4343	+ 0·0968	− 1·1203	− 0·3640	+ 2·3497	0	+ 1·1220	− 2·9209				
14	0	0	0	0	0	0	0	0	0				
	+ 1·99	− ·1198	− ·5159	+ 1·2726	− 1·2354	− ·0074	− 1·2873	− 1·6208	− 1·1925				
	+ 1·37	+ ·0345	− ·1964	− ·5977	− ·5298	− ·0047	+ 0·2068	+ ·2501	+ ·1991				
	− 0·08	− ·0115	− ·0020	− ·0300	+ ·0349	+ ·0121	+ ·0003	+ 0·116	− ·0146				
	(14,t)	0	+ ·28	− ·28	+ ·61	0	+ 3·43	+ 4·28	+ 2·13	0	+ 0·83	+ 1·20	+ 0·32
	Sum	− 0·0968	− 0·4343	+ 0·3640	− 1·1203	0	+ 2·3497	+ 2·9209	+ 1·1220				
15	+ 1·85	− ·4796	+ ·1114	− 1·1485	− 1·1831	− 1·1967	+ ·0069	− 1·1086	+ 1·5068				
	+ 2·59	− ·1559	− ·6715	+ 1·6564	− 1·6079	− ·0096	− 1·6754	− 2·1095	− 1·5521				
	+ 2·02	+ ·0509	− ·2895	− ·8812	− ·7811	− ·0069	+ ·3048	+ ·3688	+ ·2936				
	− 1·36	− ·1049	− ·0343	− ·5259	+ ·5933	+ ·2052	+ ·0046	+ ·1977	− ·2483				
	(15,t)	+ ·12	+ ·60	− ·32	+ 1·44	+ 2·13	+ 4·28	+ 8·46	0	+ 0·01	+ 0·99	+ 1·83	+ 0·38
	Sum	− 0·6595	− 0·2839	− 1·2192	− 1·5388	+ 1·1220	+ 2·9209	+ 5·8084	0				
16	− 2·59	+ ·6715	− ·1559	+ 1·6079	+ 1·6564	+ 1·6754	− ·0096	+ 1·5521	− 2·1095				
	+ 1·85	− ·1114	− ·4796	+ 1·1831	− 1·1485	− ·0069	− 1·1967	− 1·5068	− 1·1086				
	+ 1·36	+ ·0343	− ·1949	− ·5933	− ·5259	− ·0046	+ ·2052	+ ·2483	+ ·1977				
	+ 2·02	+ ·2895	+ ·0509	+ ·7811	− ·8812	− ·3049	− ·0069	− ·2036	+ ·3688				
	(16,t)	− ·60	+ ·12	− 1·44	− ·32	− 4·28	+ 2·18	0	+ 8·46	− 0·99	+ 0·01	− 0·38	+ 1·83
	Sum	+ 0·2839	− 0·6595	+ 1·5388	− 1·2192	− 2·9210	+ 1·1220	0	+ 5·8084				
17	(17,t)	+ 0·55	0	+ 0·48	+ 0·21	+ 0·83	0	+ 0·01	− 0·99	+ 2·23	0	− 0·56	− 4·05
18	(18,t)	0	+ 0·55	− 0·21	+ 0·48	0	+ 0·83	+ 0·99	+ 0·01	0	+ 2·23	+ 4·05	− 0·56
19	(19,t)	− 0·27	+ 1·66	− 0·71	+ 1·36	+ 0·32	+ 1·20	+ 1·83	− 0·38	− 0·56	+ 4·05	+10·96	0
20	(20,t)	− 1·66	− 0·27	− 1·36	− 0·71	− 1·20	+ 0·32	+ 0·38	+ 1·83	− 4·05	− 0·56	0	+10·96

TABLE LXI.

Equation Number	Right hand side	Left Hand Side.											
		k_{17}	k_{18}	k_{13}	k_{14}	k_{9}	k_{10}	k_{15}	k_{16}	k_{19}	k_{20}	k_{11}	k_{12}
1	+1·00 +3·77446	+2·23	0 o	+ ·83 − ·4318	0 o	+ ·55 − ·01716	0 o	+ ·01 + ·00188	− ·99 + ·08799	− ·56 − ·06054	− 4·05 − 2·35406	+ ·48 + ·00226	+ ·21 − ·003034
2	0		+2·23	0	+ ·83	0	+ ·55	+ ·99	+ ·01	+ 4·05	− ·56	− ·21	+ ·48
	o		o	o	o	o	o	o	o	o	o	o	o
3	0			+2·35	0	− ·43	+ ·10	+1·12	−2·92	+ ·32	− 1·20	− 1·12	− ·36
	− ·3722			− ·309	o	− ·2047	o	− ·0037	+ ·3685	+ ·2084	+ 1·5074	− ·1787	− ·0782
4	0				+2·35	− ·10	− ·43	+2·92	+1·12	+ 1·20	+ ·32	+ ·36	− 1·12
	o				o	o	o	o	o	o	o	o	o
5	0					+4·68	0	− ·66	+ ·28	− ·27	− 1·66	+ 4·49	− 4·94
	− ·2466					− ·136	o	− ·0025	+ ·2442	+ ·1381	+ ·9989	− ·1184	− ·052
6	0						+4·68	− ·28	− ·66	+ 1·66	− ·27	+ 4·94	+ 4·49
	o						o	o	o	o	o	o	o
7	0							+5·81	0	+ 1·83	+ ·38	− 1·22	− 1·54
	− ·004484							− ·00004	+ ·00444	+ ·0025	+ ·0182	− ·00215	− ·00094
8	0								+5·81	− ·38	+ 1·83	+ 1·54	− 1·22
	+ ·4439								− ·4395	− ·2486	− 1·798	+ ·2131	+ ·0932
9	0									+10·96	0	− ·71	+ 1·38
	+ ·2511									− ·1406	− 1·017	+ ·1205	+ ·0527
10	0										+10·96	− 1·36	− ·71
	+1·816										− 7·355	+ ·8717	+ ·3814
11	0											+37·47	0
	− ·2152											− ·1033	− ·0452
12	0												+37·47
	− ·09417												− ·0198
13	0 o	+1·00 +3·77446	+2·23 (1) (2)	0 o o	+ ·83 − ·2801 − ·4318	0 o o	+ ·55 − ·00941 − ·01716	+ ·99 + ·18603 + ·08799	+ ·01 − ·00089 + ·00188	+ 4·05 + ·4373 − 2·35406	− ·56 − ·3255 − ·06054	− ·21 − ·00099 − ·003034	+ ·48 − ·00694 + ·00226
14	− ·3722	0		+2·041	0	− ·6347	+ ·10	+1·1163	−2·5515	+ ·5284	+ ·3074	− 1·2987	− ·4382
	o	o		o	o	o	o	o	o	o	o	o	o
15	0	0			+2·35	− ·10	− ·43	+2·92	+1·12	+ 1·20	+ ·32	+ ·36	+ 1·12
	o	− ·3722			− ·309	o	− ·2047	− ·3685	− ·0037	− 1·5074	+ ·2084	+ ·0782	− ·1787
16	− ·2466	0				+4·544	0	− ·6625	+ ·5242	− ·1319	− ·6611	+ 4·3716	− 4·992
	o	o				o	o	o	o	o	o	o	o
17	0	0					+4·68	− ·28	− ·66	+ 1·66	− ·27	+ 4·94	+ 4·40
	o	− ·2466					− ·136	− ·2442	− ·0025	− ·9089	+ ·1381	+ ·052	− ·1184
18	− ·004484	0						+5·8100	+ ·00444	+ 1·8325	+ ·3982	− 1·22215	− 1·54094
	o	− ·4439						− ·4395	− ·00444	− 1·798	+ ·2486	+ ·0932	− ·2131
19	+ ·4439	0							+5·3705	− ·6286	+ ·032	+ 1·7531	− 1·1268
	o	− ·004484							− ·0000	− ·0182	+ ·0025	+ ·00094	− ·00215
20	+ ·2511	0								+10·8194	− 1·017	− ·5805	+ 1·4127
	o	−1·816								− 7·355	+ 1·017	+ ·3814	− ·8717
21	+1·816	0									+ 3·605	− ·4883	− ·3286
	o	+ ·2511									− ·1406	− ·0527	+ ·1205
22	− ·2152	0										+37·3667	− ·0452
	o	+ ·09417										− ·0198	+ ·0452
23	− ·09417	0											+37·4502
	o	− ·2152											− ·1033
24	− ·3722 −1·0617	0 + ·68875	+1·00 +3·81166	+2·041 (1) (2) (3)	0 o o o	− ·6347 + ·01866 − ·01085 − ·10384	+ ·10 − ·00171 − ·00294 + ·00240	+1·1163 + ·20976 + ·09922 − ·38032	−2·5515 + ·22678 − ·47945 −2·23044	+ ·5284 + ·05712 − ·30713 − ·06626	+ ·3074 + ·17868 + ·03323 − ·02383	− 1·2087 − ·00612 − ·01877 + ·01375	− ·4382 + ·006332 − ·002064 − ·02312
25	0	− ·3722	0		+2·041	− ·10	− ·6347	+2·5515	+1·1163	− ·3074	+ ·5284	+ ·4382	− 1·2987
	o	o	o		o	o	o	o	o	o	o	o	o
26	− ·2466	0	0			+4·544	0	− ·6625	+ ·5242	− ·1310	− ·6611	+ 4·3716	− 4·992
	− ·1157	o	+ ·311			− ·1974	+ ·0311	+ ·3471	− ·7935	+ ·1643	+ ·0956	− ·4039	− ·1363
27	0	− ·2466	0				+4·544	− ·5242	− ·6625	+ ·6611	− ·1319	+ 4·992	+ 4·3716
	+ ·01824	o	− ·049				− ·0049	− ·0547	+ ·1250	− ·0259	− ·01506	+ ·0636	+ ·0215
28	− ·001484	− ·4439	0					+5·3705	0	+ ·0345	+ ·6468	− 1·12895	− 1·75404
	+ ·2036	o	− ·547					− ·6105	+1·3955	− ·289	− ·1681	+ ·7103	+ ·2397
29	+ ·4439	− ·004484	0						+5·3705	− ·6468	+ ·0345	+ 1·75404	− 1·12895
	− ·4653	o	+1·25						−3·190	+ ·6606	+ ·3843	− 1·6235	− ·5478
30	+ ·2511	−1·816	0							+ 3·4644	0	− ·2081	+ ·541
	+ ·0964	o	− ·2589							− ·1368	− ·0796	+ ·3362	+ ·1134
31	+1·816	+ ·2511	0								+ 3·4644	− ·541	− ·2081
	+ ·0561	o	− ·1506								− ·0463	+ ·1956	+ ·0660
32	− ·2152	+ ·09417	0									+37·3469	0
	− ·2368	o	+ ·6363									− ·8264	− ·2788
33	− ·09417	− ·2152	0										+37·3469
	− ·0799	o	+ ·2147										− ·0941
34	0 − ·68875	− ·3722 −1·0617	0 + ·0006	+1·00 +3·81166	+2·041 (1) (2) (3) (4)	− ·10 + ·00294 − ·00171 − ·01636 + ·00240	− ·6347 + ·01085 + ·01866 − ·01522 − ·10384	+2·5515 + ·47945 + ·22678 − ·86930 −2·23044	+1·1163 − ·09922 + ·20976 + ·97584 − ·38032	− ·3074 − ·03323 + ·17868 + ·03855 − ·02383	+ ·5284 + ·30713 + ·05712 − ·04096 − ·06626	+ ·4382 + ·002064 + ·006332 − ·00464 − ·02312	− 1·2987 + ·01877 − ·00612 − ·06851 + ·01375

TABLE LXI.—(Continued).

Equation Number	Right Hand Side.				Left Hand Side. k_{14}	k_9	k_{10}	k_{15}	k_{16}	k_{19}	k_{20}	k_{11}	k_{12}
35 36 37 38 39 40 41 42	− ·3623 o + ·01824 o + ·199116 o − ·0214 o + ·3475 o +1·8721 o − ·452 o − ·17407 o	0 − ·01824 − ·2466 − ·1157 − ·4439 + ·4653 − ·004484 + ·2036 −1·816 − ·0561 + ·2511 + ·0964 + ·09417 + ·0799 − ·2152 − ·2368	+ ·311 o − ·049 o − ·547 o +1·25 o − ·2589 o − ·1506 o + ·6363 o + ·2147 o	0 + ·049 0 + ·311 0 −1·25 0 − ·547 0 + ·1506 0 − ·2589 0 − ·2147 0 + ·6363		+4·3466 − ·0049	+ ·0311 − ·0311 +4·5391 − ·1974	− ·3154 + ·1250 − ·5789 + ·7935 +4·76 −3·19	− ·2693 + ·0547 − ·5375 + ·3471 +1·3955 −1·3955 +2·181 − ·6105	+ ·0324 − ·01506 + ·6352 − ·0956 − ·2545 + ·3843 + ·0138 + ·1681 + 3·3276 − ·0463	− ·5655 + ·0259 − ·14696 + ·1643 + ·4787 − ·6606 + ·4188 − ·289 − ·0796 + ·0796 + 3·4181 − ·1368	+ 3·9677 + ·0215 + 5·0558 + ·1363 − ·41865 − ·5478 + ·13054 − ·2397 + ·1281 + ·0660 − ·3454 − ·1134 +36·5205 − ·0041	− 5·1283 − ·0036 + 4·8831 − ·4039 − 1·51484 + 1·6235 − 1·67675 + ·7103 + ·0544 − ·1956 − ·1421 + ·3362 − ·2788 + ·2788 +37·2528 − ·8264
43	− ·3623 − ·12764	− ·01824 + ·07423	+ ·311 + ·71022	+ ·049 − ·1041	+1·00 +1·43627	+4·3417 (1) (2) (3) (4) (5)	0 0 0 0 0 0	− ·1904 − ·03578 − ·01692 + ·06487 + ·16644 − ·00408	− ·2146 + ·01907 − ·04033 − ·1876 + ·07311 − ·01512	+ ·01734 + ·00187 − ·01008 − ·00217 − ·001344 − ·00020	− ·5396 − ·31364 − ·05834 − ·04183 − ·06767 − ·02429	− 3·0802 − ·01870 − ·05705 − ·04225 − ·21045 − ·1380	− 5·1019 − ·07503 − ·02445 − ·2730 − ·05408 − ·25308
44 45 46 47 48 49 50	+ ·01824 o + ·199116 − ·01589 − ·0214 − ·0179 + ·3475 + ·00145 +1·8721 − ·0450 − ·452 + ·333 − ·17407 − ·4323	− ·3623 o + ·0214 − ·0008 + ·199116 − ·000901 −1·8721 + ·0001 + ·3475 − ·0023 + ·17407 + ·01675 − ·452 − ·0218	− ·049 o − ·547 + ·01364 +1·25 + ·0154 − ·2589 − ·00124 − ·1506 + ·03865 + ·6363 − ·2858 + ·2147 + ·3713	+ ·311 o −1·25 + ·00215 − ·547 + ·0024 + ·1506 − ·0002 − ·2589 + ·0061 − ·2147 − ·045 + ·6363 + ·0585	0 o 0 + ·04385 0 + ·04943 0 − ·003995 0 + ·1242 0 − ·9188 0 +1·196		+4·3417 o	+ ·2146 o + 1·57 − ·00835	− ·1904 o 0 − ·0094 +1·570 − ·0106	+ ·5396 o + ·1298 + ·00076 + ·1810 + ·0009 + 3·2813 − ·0001	+ ·01734 o − ·1819 − ·0237 + ·1298 − ·0267 0 + ·00215 + 3·2813 − ·0670	+ 5·1019 o − ·00045 + ·1749 − ·10916 + ·1072 + ·1041 − ·01503 − ·4588 + ·4050 +36·4281 − 3·664	+ 3·0802 o + ·10016 − ·2277 − ·08813 − ·2566 + ·4588 + ·0107 + ·1041 − ·0450 0 + 4·760 +30·4264 − 6·2080
51	+ ·01824 − ·07423	− ·3623 − ·12764	− ·049 + ·1041	+ ·311 + ·71022	0 o	+1·00 +1·43627	+4·3417 (1) (2) (3) (4) (5) (6)	+ ·2146 + ·04033 + ·01907 − ·07311 − ·1876 + ·0046 − ·01512	− ·1904 + ·01692 − ·03578 − ·16644 + ·06487 − ·01341 − ·00408	+ ·5396 + ·05834 − ·31304 − ·06767 − ·04183 − ·00600 − ·02429	+ ·01734 + ·01008 + ·00187 − ·001344 − ·00217 + ·00078 − ·00020	+ 5·1019 + ·02445 + ·01503 − ·05408 − ·2740 − ·1808 − ·25308	+ 3·0802 − ·05705 + ·01870 + ·21045 − ·04225 + ·1040 − ·1380
52 53 54 55 56 57	+ ·18323 − ·00090 − ·0393 + ·0008 + ·349 − ·0023 +1·827 − ·0001 − ·119 − ·0218 − ·60637 − ·01675	+ ·0206 + ·0179 + ·1982 − ·0159 −1·872 + ·045 + ·3452 + ·00145 + ·1908 + ·4323 − ·4738 + ·333	− ·5334 + ·0024 +1·2654 − ·00215 − ·2601 + ·0061 − ·112 + ·0002 + ·3505 + ·0585 + ·586 + ·045	−1·24785 − ·0154 − ·5446 + ·01364 + ·1504 − ·0386 − ·2528 − ·00124 − ·2597 − ·3713 + ·6948 − ·2858	+ ·04385 o + ·0494 o − ·00399 o + ·1242 o − ·9188 o +1·196 o	0 − ·04943 0 + ·04385 0 − ·1242 0 − ·00399 0 −1·196 0 − ·9188		+1·5616 − ·0106	− ·0094 + ·0094 +1·5504 − ·00835	+ ·1336 − ·0267 − ·1828 + ·0237 + 3·2812 − ·0670	− ·2056 − ·0009 + ·1031 + ·00076 + ·00215 − ·00215 + 3·2143 − ·0001	− ·7015 − ·2566 + ·088 + ·2277 + ·1782 − ·0450 + ·0368 − ·0207 +32·7024 − 6·2080	− ·1185 − ·1072 − 1·22305 + ·1749 + ·1705 − ·4050 − ·4588 − ·0159 + 4·760 − 4·760 +30·2178 − 3·664
58	+ ·1823 + ·29144	+ ·0385 + ·13785	− ·531 − ·52842	−1·26325 −1·35583	+ ·04385 + ·03326	− ·0494 − ·10925	+1·00 +1·04117	+1·551 (1) (2) (3) (4) (5) (6) (7)	0 0 0 0 0 0 0 0	+ ·1039 + ·01123 − ·06039 − ·01303 + ·00805 − ·00117 − ·00468 − ·00186	− ·2065 − ·1200 − ·03232 + ·01601 + ·0259 − ·0093 + ·00233 − ·00014	− 1·0481 − ·004036 − ·01515 + ·0111 + ·05520 + ·03640 + ·05121 − ·0274	− ·3157 + ·004502 − ·001487 − ·01606 + ·003343 − ·01543 + ·01600 − ·002772
59 60 61 62 63	− ·0385 o + ·3467 − ·0122 +1·827 + ·0243 − ·1408 + ·1231 − ·6231 + ·0371	+ ·1823 o −1·827 − ·0026 + ·3467 + ·0051 + ·6231 + ·0260 − ·1408 + ·0078	+1·26325 o − ·254 + ·0356 − ·1118 − ·0716 + ·409 − ·3588 + ·631 − ·108	− ·531 o + ·1118 + ·0847 − ·254 − ·1681 − ·631 − ·853 + ·409 − ·257	+ ·0494 o − ·00399 − ·00294 + ·1242 + ·00584 − ·9188 + ·0296 +1·196 + ·0089	+ ·04385 o − ·1242 + ·0033 − ·00399 − ·00657 −1·196 − ·0334 − ·9188 − ·0101	0 0 0 − ·06703 0 + ·1331 0 + ·6754 0 + ·2034		+ 1·551 o	+ ·2065 o + 3·2142 − ·0070	+ ·1039 o 0 + ·01384 + 3·2142 − ·0275	+ ·3157 o − ·4668 + ·0702 + ·0161 − ·1305 +26·5538 − ·7078	− 1·0481 o − ·0161 + ·02110 − ·4668 − ·0420 0 − ·2132 +26·5538 − ·0042

TABLE LXI.—(Concluded).

Equation number	Right Hand Side.								Left Hand Side.				
									k_{10}	k_{19}	k_{20}	k_{11}	k_{12}
64	− ·0385	+ ·1823	+ 1·26325	− ·531	+ ·0494	+ ·04385	0	+1·00	+1·551	+ ·2065	+ ·1039	+ ·3157	− 1·0481
	− ·13785								(1)	+ ·02232	+ ·06039	+ ·001487	+ ·01515
		+ ·29144							(2)	− ·120	+ ·01123	+ ·004562	− ·004936
			+ 1·35583						(3)	− ·0259	− ·00805	− ·003343	− ·05529
				− ·52842					(4)	+ ·01601	− ·01303	− ·01666	+ ·01110
					+ ·10925				(5)	− ·00233	+ ·00468	− ·01099	− ·05121
						+ ·03326			(6)	− ·0093	− ·00117	− ·01543	+ ·03649
							0		(7)	− ·00369	+ ·00460	+ ·00825	− ·00920
								+1·04117	(8)	− ·00914	− ·00186	− ·002772	− ·0274
65	+ ·3345	− 1·8296	− ·2184	+ ·1965	− ·00693	− ·1209	− ·06703	0		+3·2072	+ ·01384	− ·3966	+ ·00506
	+ ·0051	− ·0243	− ·1681	+ ·0716	− ·00657	− ·00584	0	− ·1331		− ·0275	− ·01384	− ·0420	+ ·1395
66	+ 1·8518	+ ·3518	− ·1834	− ·4221	+ ·13 04	− ·01056	+ ·1331	0			+3·1867	− ·1234	− ·5088
	+ ·0026	− ·0122	− ·0847	+ ·0356	− ·0033	− ·00294	0	− ·06703			− ·0070	− ·02116	+ ·0702
67	− ·0177	+ ·6491	+ ·0502	− 1·484	− ·8892	− 1·2294	+ ·6754	0				+25·846	− ·2132
	+ ·0078	− ·0371	− ·257	+ ·108	− ·0101	− ·0089	0	− ·2034				− ·0642	+ ·2132
68	− ·586	− ·133	+ ·523	+ ·152	+1·2049	− ·9289	+ ·2034	0					+26·4896
	− ·026	+ ·1231	+ ·853	− ·3588	+ ·0334	+ ·0296	0	+ ·6754					− ·7078
69	+ ·3396	−1·8539	− ·3865	+ ·2681	− ·0135	− ·12674	− ·06703	− ·1331	+1·00	+3·1797	0	− ·4386	+ ·14456
	+ ·34375									(1)	0	− ·002065	− ·002089
		−1·8482								(2)	0	− ·006338	+ ·000681
			− ·39877							(3)	0	+ ·004645	+ ·007626
				+ ·24649						(4)	0	+ ·02314	− ·001531
					− ·03583					(5)	0	+ ·01527	+ ·007063
						− ·14314				(6)	0	+ ·02143	− ·005034
							− ·05683			(7)	0	− ·01147	+ ·001269
								− ·14073		(8)	0	+ ·003851	+ ·003779
									+1·00261	(9)	0	− ·00235	− ·000256
	+1·8539	+ ·3396	− ·2681	− ·3865	+ ·12674	− ·0135	+ ·1331	− ·06703	0		+3·1797	− ·14456	− ·4386
70	0	0	0	0	0	0	0	0	0		0	0	0
	− ·0099	+ ·612	− ·2068	−1·376	− ·8993	− 1·2383	+ ·6754	− ·2034	0			+25·7818	0
71	+ ·0468	− ·2558	− ·0533	+ ·037	− ·00186	− ·0175	− ·00924	− ·01836	+ ·13795			− ·0605	+ ·01995
	− ·612	− ·0099	+1·376	− ·2068	+1·2383	− ·8993	+ ·2034	+ ·6754	0				+25·7818
72	− ·0154	+ ·0842	+ ·0176	− ·0122	+ ·0006	+ ·00576	+ ·00305	+ ·00605	− ·04545				− ·0066
73	+ 1·8539	+ ·3396	− ·2681	− ·3865	+ ·12674	− ·0135	+ ·1331	− ·06703	0	+1·00	+3·1797	− ·14456	− ·4386
	+ 1·8482										(1)	− ·000681	+ ·006338
		+ ·34375									(2)	− ·002089	− ·002065
			− ·24649								(3)	+ ·001531	− ·02314
				− ·39877							(4)	+ ·007626	+ ·004645
					+ ·14314						(5)	+ ·005034	− ·02143
						− ·03583					(6)	+ ·007063	+ ·01527
							+ ·14073				(7)	− ·003779	− ·003851
								− ·05683			(8)	+ ·001269	− ·01147
									0		(9)	− ·00078	+ ·00078
										+1·00261	(10)	− ·000256	− ·00235
74	+ ·0369	+ ·3562	− ·2601	− 1·339	− ·90116	− 1·2558	+ ·66616	− ·22276	+ ·13795	0		+25·7213	+ ·01995
	+ ·0842	+ ·0154	− ·0122	− ·0176	+ ·00576	− ·0006	+ ·00605	− ·00305	0	+ ·04545		− ·0066	− ·01995
75	− ·6274	+ ·0743	+ 1·3936	− ·219	+1·2389	− ·89354	+ ·20646	+ ·68145	− ·04545	0			+25·7752
	+ ·2558	+ ·0468	− ·0370	− ·0533	+ ·0175	− ·00186	+ ·01836	− ·00924	0	+ ·13795			− ·0605
76	+ ·1211	+ ·3716	− ·2723	− 1·3566	− ·8954	− 1·2564	+ ·6722	− ·2258	+ ·13795	+ ·04545	+1·00	+25·7147	0*
77	− ·3716	+ ·1211	+ 1·3566	− ·2723	+ 1·2564	− ·8954	+ ·2258	+ ·6722	− ·04545	+ ·13795	0		+25·7147
	0	0	0	0	0	0	0	0	0	0	0		0
78	− ·3716	+ ·1211	+ 1·3566	− ·2723	+ 1·2564	− ·8954	+ ·2258	+ ·6722	− ·04545	+ ·13795	0	+ 1·00	+25·7147

	k_{17}	k_{18}	k_{13}	k_{14}	k_{15}	k_{10}	k_{13}	k_{16}	k_{19}	k_{20}	k_{11}	k_{12}
Case I	+1·6926	0	− ·5202	− ·33746	− ·0294	− ·0171	+ ·1879	− ·08888	+ ·1081	+ ·58125	+ ·004709	− ·01445
II		+1·6926	+ ·33746	− ·5202	+ ·0171	− ·0294	+ ·08888	+ ·1879	− ·58125	+ ·1081	+ ·01445	+ ·004709
III			+1·8675		+ ·1636	+ ·02398	− ·3407	+ ·87417	− ·1254	− ·07752	− ·01059	+ ·052756
IV				+1·8675	− ·02398	+ ·1636	− ·87417	− ·3407	+ ·07752	− ·1254	− ·052756	− ·01059
V					+ ·3308	0	+ ·02144	+ ·07044	− ·01127	+ ·04502	− ·03482	+ ·04886
VI						+ ·3308	− ·07044	+ ·02144	− ·04502	− ·01127	− ·04886	− ·03482
VII							+ ·6713	0	− ·01787	+ ·04426	+ ·02614	+ ·008781
VIII								+ ·6713	− ·04426	− ·01787	− ·008781	+ ·02614
IX									+ ·3153	0	+ ·0053646	− ·0017675
X										+ ·3153	+ ·0017675	+ ·0053646
XI											+ ·038888	0
XII												+ ·038888

* As the coefficient of k_{12} vanishes no elimination is required, and this equation gives k_{11} direct.

TABLE LXII.

Verification of solution of 12 equations.

+·3554460	0	− ·1092420	− ·0708686	− ·0061740	− ·0035910	+ ·0394590	−·0186648	+·0227010	+·1220625	+·00098889	− ·00303450
0	+·8124480	+ ·1619808	− ·2496960	+ ·0082080	− ·0141120	+ ·0426624	+·0901920	−·2790000	+·0518880	+·00693600	+ ·00226032
+·1872720	−·1214856	− ·6723000	0	− ·0588960	− ·0086328	+ ·1226520	−·3147012	+·0451440	+·0279072	+·00381240	− ·01899216
+·3779552	+·5826240	0	−2·0916000	+ ·0268576	− ·1832320	+ ·9790704	+·3815840	−·0868224	+·1404480	+·05908672	+ ·01186080
+·1452360	−·0844740	− ·8081840	+ ·1184612	−1·6341520	0	− ·1059136	−·3479736	+·0556738	−·2223988	+·17201080	− ·24136840
−·0767790	−·1320060	+ ·1076702	+ ·7345640	0	+1·4852920	− ·3162756	+·0962656	−·2021398	−·0506023	−·21938140	− ·15634180
−·2893660	−·1368752	+ ·5246780	+1·3462218	− ·0330176	+ ·1084776	−1·0338020	0	+·0275198	−·0681604	−·04025560	− ·01352274
+·1084336	−·2292380	−1·0664874	+ ·4156540	− ·0859368	− ·0261568	0	−·8189860	+·0539972	+·0218014	+·01071282	− ·03189080
+·1470160	−·7905000	− ·1705440	+ ·1054272	− ·0153272	− ·0612272	− ·0243032	−·0601936	+·4288080	0	+·00729586	− ·00240380
−·4126875	−·0767510	+ ·0550392	+ ·0890340	− ·0810642	+ ·0080017	− ·0314246	+·0126877	0	−·2238630	−·00125493	− ·00380887
0	0	0	0	0	0	0	0	0	0	0	0
−·5414415	+·17614623	+1·97676732	− ·3968073	+1·8307842	−1·3047054	+ ·32902407	+·9794658	−·06622823	+·20101156	0	+1·45713336
+·0010848	+·00018843	− ·00062188	+ ·0004903	+ ·0003820	+ ·0001141	+ ·00114887	−·0003241	−·00034663	+·00009416	+·00015156	+ ·99989141

16. The residuals in table LXII are sufficiently small. Accordingly the values of k_9 . . . k_{20} have been found satisfactorily for the 12 latter cases. It remains to find their values for the first 8 cases, and also values of k_1 to k_8 for all cases. In this the work is much simplified by the known symmetry of the solution. The introduction of cases 1 to 8 —*i.e.* giving the R.H.S. of the first 8 equations values 1,0 . . . , (case 1) 0, 1, . . . (case 2) etc. causes the R.H.S. of the latter 12 equations to take the values $-\Sigma^s (s, t)\, {}_rK_s$ for case r and equation t, s being taken from 1 to 8: and these quantities accordingly have to be found for each of the cases 1 to 8. Values of these quantities with sign reversed have already been given in table LIX. It is necessary then to combine these cases 9 to 20 in such a way as to give the solution for k_9 to k_{20} for these related cases. For case r the value of k_u is ${}_rk_u = -\Sigma^t\, {}_tk_u \left\{ \Sigma^s (s, t)\, {}_rK_s \right\}$, t being given all values from 9 to 20: but in fact $\Sigma^s (s,t)\, {}_rK_s$ vanishes for values of t above 16. The process is carried out in table LXIII.

The next step is to find ${}_tk_u$ for values of t and u from 1 to 8. Having found values of ${}_tk_u$, for all values of u and values of t from 9 to 20, it is possible to write down values of ${}_uk_t$ by symmetry. Equation (15) then enables the remaining quantities to be found as is done in table LXIV. The symmetry occurring largely simplifies the process while still affording a check.

This leads up to the solution of the combined 20 equations for all the 20 fundamental cases. The results of the solution, compiled from tables LXI, LXIII, LXIV, are given in table LXV. The solution of these 20 equations enables the probable errors of the N.W. Quadrilateral after adjustment of circuit conditions only, to be written down. By the incorporation of the next three equations, corresponding results can be given for the case when circuits and base line closures have been made (the actual adjustment carried out). Finally by incorporation of the last three equations the corresponding results obtainable if Laplace closures were introduced at each extra base can be given. Accordingly before giving the application of table LXV, the further solution of 23 and 26 equations will be carried out.

17. In table LXV are also shown certain multipliers. They are the coefficients of k_{21}, k_{22}, k_{23} in table XLVII. It is necessary to find k_u in terms of k_{21}, k_{22}, k_{23}, for all values of u between 1 and 20 by means of (15), with a view to substituting in equations 21, 22, 23. For this values of $\Sigma^s\, {}_sk_r\, (s,t)$ are required, where ${}_sk_r$ are the values given in table LXV and t has values 21, 22, 23, so that (s,t) are the multipliers just alluded to. Each of these products ${}_sk_r\, (s,t)$ are given in table LXVI, and their sums $\Sigma^s\, {}_sk_r\, (s,t)$ for values of s from 1 to 20, the latter in old face type. These are the coefficients of $-k_u$ in the expressions for k_1 . . . k_{20} as found from the first 20 equations. These quantities have to be substituted in equations 21, 22, 23, and accordingly multiplied by the respective coefficients. The process is carried out in table LXVII, where the coefficients of the three equations giving k_{21}, k_{22}, k_{23}, are formed. The process has been carried out with full accuracy to illustrate the complete symmetry of the resulting equations. The solution of these equations is very simple and is given in table LXVIII, with verification at the foot of the table.

TABLE LXIII.

r	t	9	10	11	12	13	14	15	16	17 to 20
1	$-\Sigma^s\ {}_sK_r(s,t)$ from table *LIX*	+ ·1516	− ·0778	+ ·3226	− ·2509	+ ·0975	− ·0308	+ ·0513	− ·1528	Columns 17 to 20 comprise only zeros
2		+ ·0778	+ ·1516	+ ·2509	+ ·3226	+ ·0308	+ ·0975	+ ·1528	+ ·0513	
3		− ·0342	+ ·0021	− ·0418	+ ·0399	− ·0086	− ·0222	− ·0357	− ·0095	
4		− ·0021	− ·0342	− ·0399	− ·0418	+ ·0222	− ·0086	+ ·0095	− ·0357	
5		− ·2593	+ ·0602	− ·6208	− ·6395	− ·6469	+ ·0037	− ·5993	+ ·8145	
6		− ·0602	− ·2593	+ ·6395	− ·62 8	− ·0037	− ·6469	− ·8145	− ·5993	
7		+ ·0252	− ·1433	− ·4363	− ·3867	− ·0034	+ ·1509	+ ·1826	+ ·1453	
8		+ ·1433	+ ·0252	+ ·3867	− ·4363	− ·1509	− ·0034	− ·1453	+ ·1826	
${}_tK_u$	u=9	+ ·3308	0	− ·0348	+ ·0489	+ ·1636	− ·0240	+ ·0214	+ ·0704	K_{17} to K_{20} are not required
	10	0	+ ·3308	− ·0489	− ·0348	+ ·0240	+ ·1636	− ·0704	+ ·0214	
	11	− ·0348	− ·0489	+ ·0389	0	− ·0106	− ·0528	+ ·0261	− ·0088	
	12	+ ·0489	− ·0348	0	+ ·0389	+ ·0528	− ·0106	+ ·0088	+ ·0261	
	13	+ ·1636	+ ·0240	− ·0106	+ ·0528	+1·8675	0	− ·3407	+ ·8742	
	14	− ·0240	+ ·1636	− ·0528	− ·0106	0	+1·8675	− ·8742	− ·3407	
	15	+ ·0214	− ·0704	+ ·0261	+ ·0088	− ·3407	− ·8742	+ ·6713	0	
	16	+ ·0704	+ ·0214	− ·0088	+ ·0261	+ ·8742	− ·3407	0	+ ·6713	
	17	− ·0294	− ·0171	+ ·0047	− ·0145	− ·5202	− ·3375	+ ·1879	− ·0889	
	18	+ ·0171	− ·0294	+ ·0145	+ ·0047	+ ·3375	− ·5202	+ ·0889	+ ·1879	
	19	− ·0113	− ·0450	+ ·0054	− ·0018	− ·1254	+ ·0775	− ·0179	− ·0443	
	20	+ ·0450	− ·0113	+ ·0018	+ ·0054	− ·0775	− ·1254	+ ·0443	− ·0179	

u	r	9	10	11	12	13	14	15	16	${}_rk_u$
9	1	+ ·05015	0	− ·01123	− ·01227	+ ·01596	+ ·00074	+ ·00110	− ·01076	= + ·03369
	2	+ ·02575	0	− ·00873	+ ·01578	+ ·00504	− ·00234	+ ·00327	+ ·00361	= + ·04238
	3	− ·01131	0	+ ·00145	+ ·00195	− ·00141	− ·00053	− ·00076	− ·00067	= − ·01022
	4	− ·00068	0	+ ·00139	− ·00204	+ ·00364	+ ·00021	+ ·00020	− ·00251	= + ·00021
	5	− ·08576	0	+ ·02160	− ·03127	− ·10583	− ·00009	− ·01262	+ ·05733	= − ·15684
	6	− ·01991	0	− ·02225	− ·03036	− ·00061	+ ·01552	− ·01744	− ·04218	= − ·11723
	7	+ ·00833	0	+ ·01518	− ·01891	− ·00056	− ·00362	+ ·00391	+ ·01023	= + ·01456
	8	+ ·04740	0	− ·01846	− ·02134	− ·02469	+ ·00008	− ·00311	+ ·01285	= − ·00227
10	1	0	− ·02575	− ·01578	+ ·00873	+ ·00234	− ·00504	− ·00361	− ·00327	= − ·04238
	2	0	+ ·05015	− ·01227	− ·01123	+ ·00074	+ ·01596	− ·01076	+ ·00110	= + ·03369
	3	0	+ ·00068	+ ·00204	− ·00139	− ·00021	− ·00364	+ ·00251	− ·00020	= − ·00021
	4	0	− ·01131	+ ·00195	+ ·00145	− ·00053	− ·00141	− ·00067	− ·00076	= − ·01022
	5	0	+ ·01991	+ ·03036	+ ·02225	− ·01552	+ ·00061	+ ·04218	+ ·01744	= + ·11723
	6	0	− ·08576	− ·03127	+ ·02160	− ·00009	− ·10583	+ ·05733	− ·01282	= − ·15684
	7	0	− ·04740	+ ·02134	+ ·01346	− ·00008	+ ·02469	− ·01285	+ ·00311	= + ·00227
	8	0	+ ·00833	− ·01891	+ ·01518	− ·00362	− ·00056	+ ·01023	+ ·00391	= + ·01456
11	1	− ·00528	+ ·00381	+ ·01255	0	− ·00103	+ ·00163	+ ·00134	+ ·00184	= + ·01436
	2	− ·00271	− ·00741	+ ·00976	0	− ·00033	− ·00515	+ ·00399	− ·00045	= − ·00230
	3	+ ·00119	− ·00010	− ·00163	0	+ ·00009	+ ·00117	− ·00093	+ ·00006	= − ·00013
	4	+ ·00007	+ ·00167	− ·00155	0	− ·00024	+ ·00045	+ ·00025	− ·00031	= + ·00096
	5	+ ·00962	− ·00294	− ·02415	0	+ ·00686	− ·00020	− ·01564	− ·00717	= − ·03422
	6	+ ·00209	+ ·01268	+ ·02488	0	+ ·00004	+ ·03416	− ·02127	+ ·00527	= + ·05785
	7	− ·00088	+ ·00701	− ·01697	0	+ ·00004	− ·00797	+ ·00477	− ·00128	= − ·01528
	8	− ·00499	− ·00123	+ ·01504	0	+ ·00160	+ ·00018	− ·00879	− ·00161	= + ·00520
12	1	+ ·00741	+ ·00271	0	− ·00976	+ ·00515	+ ·00033	+ ·00045	− ·00399	= + ·00230
	2	+ ·00381	− ·00528	0	+ ·01255	+ ·00163	− ·00103	+ ·00134	+ ·00134	= + ·01436
	3	− ·00167	− ·00007	0	+ ·00155	− ·00045	+ ·00024	− ·00031	− ·00025	= − ·00096
	4	− ·00010	+ ·00119	0	− ·00163	+ ·00117	− ·00009	+ ·00008	− ·00093	= − ·00013
	5	− ·01268	− ·00209	0	− ·02488	− ·03416	− ·00004	− ·00527	+ ·02127	= − ·05785
	6	− ·00294	+ ·00902	0	− ·02415	− ·00020	+ ·00686	− ·00717	− ·01564	= − ·03422
	7	+ ·00123	+ ·00499	0	− ·01504	− ·00018	− ·00160	+ ·00161	+ ·00379	= − ·00520
	8	+ ·00701	− ·00088	0	− ·01697	− ·00797	+ ·00004	− ·00128	+ ·00477	= − ·01528
13	1	+ ·02480	− ·00187	− ·00346	− ·01324	+ ·18208	0	− ·01748	− ·13358	= + ·03725
	2	+ ·01274	+ ·00364	− ·00266	+ ·01703	+ ·05752	0	− ·05204	+ ·04485	= + ·08108
	3	− ·00559	+ ·00005	+ ·00044	+ ·00211	− ·01606	0	+ ·01216	− ·00830	= − ·01519
	4	− ·00034	− ·00082	+ ·00042	− ·00221	+ ·04146	0	− ·00324	− ·03121	= + ·00406
	5	− ·04241	+ ·00145	+ ·00659	− ·03378	−1·20809	0	+ ·20418	+ ·71204	= − ·36002
	6	− ·00985	− ·00622	− ·00678	− ·03279	− ·00691	0	+ ·27750	− ·52391	= − ·30896
	7	+ ·00412	− ·00344	+ ·00462	− ·02042	− ·00635	0	− ·06221	+ ·12702	= + ·04334
	8	+ ·02344	+ ·00060	− ·00410	− ·02303	− ·28181	0	+ ·04950	+ ·15968	= − ·07577

u	r	9	10	11	12	13	14	15	16	${}_rk_u$
14	1	− ·00364	− ·01274	− ·01703	+ ·00266	0	− ·05752	− ·04485	+ ·05204	= − ·08108
	2	− ·00187	+ ·02480	− ·01324	− ·00346	0	+ ·18208	− ·13358	− ·01748	= + ·03725
	3	+ ·00082	+ ·00034	+ ·00221	− ·00042	0	− ·04146	+ ·03121	+ ·00324	= − ·00406
	4	+ ·00005	− ·00559	+ ·00211	+ ·00044	0	− ·01606	− ·00830	+ ·01216	= − ·01519
	5	+ ·00622	+ ·00985	+ ·03279	+ ·00678	0	+ ·00691	+ ·52391	− ·27750	= + ·30896
	6	+ ·00145	− ·04241	− ·03378	+ ·00659	0	−1·20809	+ ·71204	+ ·20418	= − ·36002
	7	− ·00060	− ·02344	+ ·02303	+ ·00410	0	+ ·28181	− ·15968	− ·04950	= + ·07577
	8	− ·00344	+ ·00412	− ·02042	+ ·00462	0	− ·00635	+ ·12702	− ·06221	= + ·04334
15	1	+ ·00324	+ ·00548	+ ·00842	− ·00221	− ·03322	+ ·02693	+ ·03444	0	= + ·04308
	2	+ ·00166	− ·01067	+ ·00655	+ ·00284	− ·01049	− ·08523	+ ·10257	0	= + ·00723
	3	− ·00073	− ·00014	− ·00109	+ ·00035	+ ·00293	+ ·01941	− ·02397	0	= − ·00324
	4	− ·00004	+ ·00241	− ·00104	− ·00037	− ·00756	+ ·00752	+ ·00638	0	= + ·00730
	5	− ·00555	− ·00424	− ·01620	− ·00563	+ ·22040	− ·00323	− ·40231	0	= − ·21676
	6	− ·00129	+ ·01825	+ ·01670	− ·00546	+ ·00126	+ ·56552	− ·54677	0	= + ·04821
	7	+ ·00054	+ ·01009	− ·01139	− ·00340	+ ·00116	− ·13192	+ ·12258	0	= − ·01234
	8	+ ·00307	− ·00177	+ ·01009	− ·00384	+ ·05141	+ ·00297	− ·09754	0	= − ·03561
16	1	+ ·01067	− ·00166	− ·00284	− ·00655	+ ·08523	+ ·01049	0	− ·10257	= − ·00723
	2	+ ·00548	+ ·00324	− ·00221	+ ·00842	+ ·02693	− ·03322	0	+ ·03444	= + ·04308
	3	− ·00241	+ ·00004	+ ·00037	+ ·00104	− ·00752	+ ·00756	0	− ·00638	= − ·00730
	4	− ·00014	− ·00073	+ ·00035	− ·00109	+ ·01941	+ ·00293	0	− ·02397	= − ·00324
	5	− ·01825	+ ·00129	+ ·00546	− ·01670	− ·56552	− ·00126	0	+ ·54677	= − ·04821
	6	− ·00424	− ·00555	− ·00563	− ·01620	− ·00323	+ ·22040	0	− ·40231	= − ·21676
	7	+ ·00177	− ·00307	+ ·00384	− ·01009	− ·00297	− ·05141	0	+ ·09754	= + ·03561
	8	+ ·01009	+ ·00054	− ·00340	− ·01139	− ·13192	+ ·00116	0	+ ·12258	= − ·01234
17	1	− ·00446	+ ·00133	+ ·00152	+ ·00364	− ·05072	+ ·01040	+ ·00964	+ ·01358	= − ·01507
	2	− ·00229	− ·00259	+ ·00118	− ·00468	− ·01602	− ·03291	+ ·02871	− ·00456	= − ·03316
	3	+ ·00101	− ·00004	− ·00020	− ·00058	+ ·00447	+ ·00749	− ·00671	+ ·00084	= + ·00628
	4	+ ·00006	+ ·00058	− ·00019	+ ·00061	− ·01155	+ ·00290	+ ·00179	+ ·00817	= − ·00263
	5	+ ·00762	− ·00103	− ·00292	+ ·00927	+ ·33652	− ·00125	− ·11261	− ·07241	= + ·16319
	6	+ ·00177	+ ·00443	+ ·00301	+ ·00900	+ ·00192	+ ·21833	− ·15304	+ ·05328	= + ·13870
	7	− ·00074	+ ·00245	− ·00205	+ ·00561	+ ·00177	− ·05093	+ ·03431	− ·01292	= − ·02250
	8	− ·00421	− ·00043	+ ·00182	+ ·00633	+ ·07850	+ ·00115	− ·02730	− ·01623	= + ·03963
18	1	+ ·00259	+ ·00229	+ ·00468	− ·00118	+ ·03291	+ ·01602	+ ·00456	− ·02871	= + ·03316
	2	+ ·00133	− ·00446	+ ·00364	+ ·00152	+ ·01040	− ·05072	+ ·01358	+ ·00964	= − ·01507
	3	− ·00058	− ·00006	− ·00061	+ ·00019	− ·00290	+ ·01155	− ·00317	− ·00179	= + ·00263
	4	− ·00004	+ ·00101	− ·00058	− ·00020	+ ·00749	+ ·00447	+ ·00084	− ·00671	= + ·00628
	5	− ·00443	− ·00177	− ·00900	− ·00301	− ·21833	− ·00192	− ·05328	+ ·15304	= − ·13870
	6	− ·00103	+ ·00762	+ ·00927	− ·00292	− ·00125	+ ·33652	− ·07241	− ·11261	= + ·16319
	7	+ ·00043	+ ·00421	− ·00633	− ·00182	− ·00115	− ·07850	+ ·01623	+ ·02730	= − ·03963
	8	+ ·00245	− ·00074	+ ·00561	− ·00205	− ·05093	+ ·00177	− ·01292	+ ·03431	= − ·02250
19	1	− ·00171	+ ·00350	+ ·00174	+ ·00045	− ·01223	− ·00239	− ·00092	+ ·00677	= − ·00479
	2	− ·00068	− ·00682	+ ·00135	− ·00058	− ·00386	+ ·00756	− ·00274	− ·00227	= − ·00824
	3	+ ·00039	− ·00009	− ·00023	− ·00007	+ ·00108	− ·00172	+ ·00064	+ ·00042	= + ·00042
	4	+ ·00002	+ ·00154	− ·00022	+ ·00008	− ·00278	− ·00067	− ·00017	+ ·00157	= − ·00063
	5	+ ·00293	− ·00271	− ·00335	+ ·00115	+ ·08112	+ ·00029	+ ·01073	− ·03608	= + ·05408
	6	+ ·00068	+ ·01167	+ ·00345	+ ·00112	+ ·00046	− ·05013	+ ·01458	+ ·02655	= + ·00838
	7	− ·00028	+ ·00645	− ·00235	+ ·00070	+ ·00043	+ ·01169	− ·00327	− ·00644	= + ·00693
	8	− ·00162	− ·00113	+ ·00209	+ ·00079	+ ·01892	− ·00026	+ ·00260	− ·00809	= + ·01330
20	1	+ ·00622	+ ·00088	+ ·00058	− ·00135	− ·00756	+ ·00386	+ ·00227	+ ·00274	= + ·00824
	2	+ ·00350	− ·00171	+ ·00045	+ ·00174	− ·00239	− ·01223	+ ·00677	− ·00092	= − ·00479
	3	− ·00154	− ·00002	− ·00006	+ ·00022	+ ·00067	+ ·00278	− ·00157	+ ·00017	= + ·00068
	4	− ·00009	+ ·00039	− ·00007	− ·00023	− ·00172	+ ·00108	+ ·00042	+ ·00064	= + ·00042
	5	− ·01167	− ·00068	− ·00112	− ·00345	+ ·05013	− ·00046	− ·02655	− ·01458	= − ·00838
	6	− ·00271	+ ·00293	+ ·00115	− ·00335	+ ·00029	+ ·08112	− ·03608	+ ·01073	= + ·05408
	7	+ ·00113	+ ·00162	− ·00079	− ·00209	+ ·00026	− ·01892	+ ·00809	− ·00260	= − ·01330
	8	+ ·00645	− ·00028	+ ·00070	− ·00235	+ ·01169	+ ·00043	− ·00644	− ·00327	= + ·00693

TABLE LXIV.

t	9	10	11	12	13	14	15	16	
r=1	+·1516	−·0778	+·3226	−·2509	+·0975	−·0308	+·0513	−·1528	$\Sigma^s\,{}_rK_s\,(s,t)$ from table LIX.
2	+·0778	+·1516	+·2509	+·3226	+·0308	+·0975	+·1528	+·0513	
3	−·0342	+·0021	−·0418	+·0399	−·0086	−·0222	−·0357	−·0095	
4	−·0021	−·0342	−·0399	−·0418	+·0222	−·0086	+·0095	−·0357	
5	−·2593	+·0602	−·6208	−·6395	−·6469	+·0037	−·5993	+·8145	
6	−·0602	−·2593	+·6395	−·6208	−·0037	−·6469	−·8145	−·5993	
7	+·0252	−·1433	−·4363	−·3867	−·0034	+·1509	+·1826	+·1453	
8	+·1433	+·0252	+·3867	−·4363	−·1509	−·0034	−·1453	+·1826	
u=1	+·0337	−·0424	+·0144	+·0023	+·0373	−·0811	+·0431	−·0072	${}_tK_u$ or ${}_uK_t$
2	+·0424	+·0337	−·0023	+·0144	+·0811	+·0373	+·0072	+·0431	
3	−·0102	−·0002	−·0001	−·0010	−·0152	−·0041	−·0032	−·0073	
4	+·0002	−·0102	+·0010	−·0001	+·0041	−·0152	+·0073	−·0032	
5	−·1568	+·1172	−·0342	−·0579	−·3600	+·3090	−·2168	−·0482	
6	−·1172	−·1568	+·0579	−·0342	−·3090	−·3600	+·0482	−·2168	
7	+·0146	+·0023	−·0153	−·0052	+·0433	+·0758	−·0123	+·0356	
8	−·0023	+·0146	+·0052	−·0153	−·0758	+·0433	−·0356	−·0123	

u	r	${}_tK_u\,\Sigma^s\,{}_rK_s\,(s,t)$								${}_rK_u$ from LVIII.	${}_rk_u$
1	1	+·00511	+·00330	+·00464	−·00058	+·00364	+·00250	+·00221	+·00110	+1·13250	=+1·15442
	2	+·00262	−·00643	+·00341	+·00074	+·00115	−·00791	+·00658	−·00037	+·00001	0
	3	−·00115	−·00009	−·00060	+·00009	−·00032	+·00180	−·00154	+·00007	−·06144	=−·06318
	4	−·00007	+·00145	−·00057	−·00010	+·00083	+·00070	+·00041	+·00026	+·28814	=+·29104
	5	−·00874	−·00255	−·00894	−·00147	−·02413	−·00030	−·02582	−·00586	−·05514	=−·13294
	6	−·00203	+·01100	+·00921	−·00143	−·00014	+·05247	−·03510	+·00431	+·01627	=+·05454
	7	+·00085	+·00608	−·00628	−·00089	−·00013	−·01224	+·00787	−·00105	−·00170	=−·00748
	8	+·00483	+·00107	+·00557	−·00100	−·00563	+·00028	−·00626	−·00132	−·00884	=−·01346
2	1	+·00642	−·00262	−·00058	−·00360	+·00790	−·00115	+·00037	−·00658	+·00001	0
	2	+·00330	+·00511	−·00074	+·00463	+·00249	+·00363	+·00110	+·00221	+1·13250	=+1·15442
	3	−·00145	+·00007	+·00009	+·00057	−·00070	−·00083	−·00026	−·00041	−·28814	=−·29104
	4	−·00009	−·00115	+·00010	−·00060	+·00180	−·00032	+·00007	−·00154	−·06144	=−·06318
	5	−·01099	+·00203	+·00147	−·00918	−·05245	+·00014	−·00433	+·03509	−·01627	=−·05454
	6	−·00255	−·00873	−·00143	−·00891	−·00030	−·02410	−·00589	−·02582	−·05514	=−·13294
	7	+·00107	−·00483	+·00089	−·00555	−·00028	+·00562	+·00132	+·00626	+·00884	=+·01346
	8	+·00607	+·00085	−·00100	−·00626	−·01224	−·00013	−·00105	+·00787	−·00170	=−·00748
3	1	−·00155	+·00002	−·00004	+·00024	−·00148	+·00012	−·00017	+·00112	−·06144	=−·06318
	2	−·00080	−·00003	−·00003	−·00031	−·00047	−·00040	−·00049	−·00037	−·28814	=−·29104
	3	+·00035	−·00000	+·00001	−·00004	+·00013	+·00009	+·00012	+·00007	+·10155	=+·10227
	4	+·00002	+·00001	+·00000	+·00004	−·00034	+·00004	−·00003	+·00026	0	0
	5	+·00265	−·00001	+·00008	+·00061	+·00983	−·00002	+·00194	−·00595	+·00408	=+·01322
	6	+·00062	+·00005	−·00008	+·00060	+·00006	+·00263	+·00264	+·00437	+·01394	=+·02484
	7	−·00026	+·00003	+·00006	+·00037	+·00005	−·00061	−·00059	−·00106	−·00402	=−·00604
	8	−·00146	−·00001	−·00005	+·00042	+·00229	+·00001	+·00047	−·00133	−·00011	=+·00024
4	1	+·00003	+·00079	+·00032	+·00003	+·00040	+·00047	+·00037	+·00049	+·28814	=+·29104
											=−·06318
											= 0
											=+·10227
		The same numbers occur as for u = 3, differently arranged.									=−·02481
											=+·01322
											=−·00024
											=−·00604
5	1	−·02377	−·00912	−·01104	+·01452	−·03510	−·00952	−·01112	+·00736	−·05514	=−·13294
	2	−·01220	+·01777	−·00858	−·01867	−·01109	+·03012	−·03312	−·00247	−·01627	=−·05454
	3	+·00536	+·00025	+·00143	−·00231	+·00310	−·00686	+·00774	+·00046	+·004 8	=+·01322
	4	+·00033	−·00401	+·00136	+·00242	−·00800	−·00266	−·00206	+·00172	−·01394	=−·02481
	5	+·04065	+·00706	+·02123	+·03703	+·23290	+·00114	+·12993	−·03924	+·39363	=+·82433
	6	+·00944	−·03040	−·02188	+·03594	+·00133	−·19990	+·17658	+·02888	0	0
	7	−·00395	−·01680	+·01492	+·02239	+·00122	+·04663	−·03959	−·00700	+·00309	=+·02091
	8	−·02247	+·00295	−·01322	+·02527	+·05432	−·00105	+·03150	−·00880	+·09941	=+·16791
6											=+·05454
											=−·13294
											=+·02484
		The same numbers occur as for u = 5, differently arranged.									=+·01325
											0
											=+·82433
											=−·16791
											=+·02091
7	1	+·00221	−·00018	−·00493	+·00130	+·00422	−·00233	−·00063	−·00544	−·00170	=−·00748
	2	+·00114	+·00035	−·00384	−·00168	+·00133	+·00739	−·00188	+·00183	+·00884	=+·01346
	3	−·00050	+·00000	+·00064	−·00021	−·00037	−·00168	+·00044	−·00034	−·00402	=−·00604
	4	−·00003	−·00008	+·00061	+·00022	+·00096	−·00065	−·00012	−·00127	+·00011	=−·00024
	5	−·00379	+·00014	+·00950	+·00333	−·02800	+·00028	+·00737	+·02900	+·00309	=+·02091
	6	−·00088	−·00060	−·00978	+·00323	−·00016	−·04904	+·01002	−·02133	−·09941	=−·16791
	7	+·00037	−·00033	+·00667	+·00201	−·00015	+·01144	−·00225	+·00517	+·08424	=+·05717
	8	+·00209	+·00006	−·00592	+·00227	−·00653	−·00026	+·00179	+·00650	0	0
8											=−·01348
											=−·00748
											=+·00025
		The same numbers occur as for u = 7, differently arranged.									=−·00604
											=+·16795
											=+·02092
											0
											=+·05717

TABLE LXV.

Values of ${}_sk_r$ for 20 conditions.

r \ s	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Multipliers k_{21}	k_{22}	k_{23}
1	+1·15442	0	−·06318	+·29104	−·13294	+·05454	−·00748	−·01346	+·03369	−·04238	+·01436	+·00330	+ ·03725	− ·08108	+·04308	−·00723	− ·01507	+ ·03316	−·00479	+·00824	+2·19	+ ·42	0
2	0	+1·15442	−·29104	−·06318	−·05454	−·13294	+·01346	−·00748	+·04238	+·03369	−·00230	+·01436	+ ·08108	+ ·03725	+·00723	+·04308	− ·03316	− ·01507	−·00824	−·00479	0	0	0
3	− ·06318	− ·29104	+·10227	0	+·01322	+·02484	−·00604	+·00024	−·01022	−·00021	−·00013	−·00096	− ·01519	− ·00406	−·00324	−·00730	+ ·00628	+ ·00263	+·00042	+·00063	+1·86	+ ·22	0
4	+ ·29104	− ·06318	0	+·10227	−·02484	+·01322	−·00024	−·00604	+·00021	−·01022	+·00096	−·00013	+ ·00406	− ·01519	+·00730	−·00324	− ·00263	+ ·00628	−·00063	+·00042	−6·62	− ·18	0
5	− ·13294	− ·05454	+·01322	−·02484	+·82433	0	+·02091	+·16791	−·15684	+·11723	−·03422	−·05785	− ·36002	+ ·30896	−·21676	−·04821	+ ·16319	− ·13870	+·05408	−·00838	0	+ ·31	0
6	+ ·05454	− ·13294	+·02484	+·01322	0	+·82433	−·16791	+·02091	−·11723	−·15684	+·05785	−·03422	− ·30896	− ·36002	+·04821	−·21676	+ ·13870	+ ·16319	+·00838	+·05408	0	0	0
7	− ·00748	+ ·01346	−·00604	−·00024	+·02091	−·16791	+·05717	0	+·01456	+·00227	−·01528	−·00520	+ ·04334	+ ·07577	−·01234	+·03561	− ·02250	− ·03963	+·00693	−·01330	0	+ ·10	0
8	− ·01346	− ·00748	+·00024	−·00604	+·16791	+·02091	0	+·05717	−·00227	+·01456	+·00520	−·01528	− ·07577	+ ·04334	−·03561	−·01234	+ ·03963	− ·02250	+·01330	+·00693	0	− ·12	0
9	+ ·03369	+ ·04238	−·01022	+·00021	−·15684	−·11723	+·01456	−·00227	+·33080	0	−·03482	+·04886	+ ·16360	− ·02398	+·02144	+·07044	− ·02940	+ ·01710	−·01127	+·04502	0	0	+ ·68
10	− ·04238	+ ·03369	−·00021	−·01022	+·11723	−·15684	+·00227	+·01456	0	+·33080	−·04886	−·03482	+ ·02398	+ ·16360	−·07044	+·02144	− ·01710	− ·02940	−·04502	−·01127	0	0	0
11	+ ·01436	− ·00230	−·00013	+·00096	−·03422	+·05785	−·01528	+·00520	−·03482	−·04886	+·03889	0	− ·01059	− ·05276	+·02614	−·00578	+ ·00471	+ ·01445	+·00536	+·00177	0	0	− ·94
12	+ ·00230	+ ·01436	−·00096	−·00013	−·05785	−·03422	−·00520	−·01528	+·04886	−·03482	0	+·03889	+ ·05276	− ·01059	+·00578	+·02614	− ·01445	+ ·00471	−·00177	+·00536	0	0	− ·68
13	+ ·03725	+ ·08108	−·01519	+·00406	−·36002	−·30896	+·04334	−·07577	+·16360	+·02398	−·01059	+·05276	+1·86750	0	−·34070	+·87417	− ·52020	+ ·33746	−·12540	−·07752	0	+ ·34	0
14	− ·08108	+ ·03725	−·00406	−·01519	+·30896	−·36002	+·07577	+·04334	−·02398	+·16360	−·05276	−·01059	0	+1·86750	−·87417	−·34070	− ·33746	− ·52020	+·07752	−·12540	0	0	0
15	+ ·04308	+ ·00723	−·00324	+·00730	−·21676	+·04821	−·01234	−·03561	+·02144	−·07044	+·02614	+·00578	− ·34070	− ·87417	+·67130	0	+ ·18790	+ ·08888	−·01787	+·04426	0	+ ·15	0
16	− ·00723	+ ·04308	−·00730	−·00324	−·04821	−·21676	+·03561	−·01234	+·07044	+·02144	−·00578	+·02614	+ ·87417	− ·34070	0	+·67130	− ·08888	+ ·18790	−·04426	−·01787	0	− ·10	0
17	− ·01507	− ·03316	+·00628	−·00263	+·16319	+·13870	−·02250	+·03963	−·02940	−·01710	+·00471	−·01445	− ·52020	− ·33746	+·18790	−·08888	+1·69260	0	+·10810	+·58125	0	+ ·16	+ ·70
18	+ ·03316	− ·01507	+·00263	+·00628	−·13870	+·16319	−·03963	−·02250	+·01710	−·02940	+·01445	+·00471	+ ·33746	− ·52020	+·08888	+·18790	0	+1·69260	−·58125	+·10810	0	0	0
19	− ·00479	− ·00824	+·00042	−·00063	+·05408	+·00838	+·00693	+·01330	−·01127	−·04502	+·00536	−·00177	− ·12540	+ ·07752	−·01787	−·04426	+ ·10810	− ·58125	+·31530	0	0	+ ·02	− ·63
20	+ ·00824	− ·00479	+·00063	+·00042	−·00838	+·05408	−·01330	+·00693	+·04502	−·01127	+·00177	+·00536	− ·07752	− ·12540	+·04426	−·01787	+ ·58125	+ ·10810	0	+·31530	0	− ·02	−1·17

TABLE LXVI. Values of ${}_s k_r\,(s,\,t)$ and $\Sigma^s\,{}_s k_r\,(s,\,t)$, s from 1 to 20, the latter in old face type.

t	s	r=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	1	+2·5281798	0	−·1383642	+·6373776	−·2911386	+·1194426	−·0163812	−·0294774	+·0737811	−·0928122	+·0314484	+·0050370	+·0815775	−·1775652	+·0943452	−·0158337	−·0330033	+·0726204	−·0104901	+·0180456
	3	−0·1175148	−·5413344	+·1902222	0	+·0245892	+·0462024	−·0112344	+·0004464	−·0190092	−·0003906	−·0002418	−·0017856	−·0282534	−·0075516	−·0060264	−·0135780	+·0116808	+·0048918	+·0007812	+·0011718
	4	−1·9266848	+·4182516	0	−·6770274	+·1644408	−·0875164	+·0015888	+·0399848	−·0013902	+·0676564	−·0063552	+·0008605	−·0268772	+·1005578	−·0483260	+·0214488	+·0174106	−·0415736	+·0041706	−·0027804
Sum		+·4839802	−·1230828	+·0518580	−·0396498	−·1021086	+·0781286	−·0260268	+·0109538	+·0533817	−·0255464	+·0248514	+·0041120	+·0264469	−·0845590	+·0399928	−·0079629	−·0039119	+·0359386	−·0055383	+·0164370
22	1	+·4848564	0	−·0265356	+·1222368	−·0558348	+·0229068	−·0031416	−·0056532	+·0141498	−·0177996	−·0060312	+·0009660	+·0156450	−·0340536	+·0180936	−·0030366	−·0063294	+·0139272	−·0020118	+·0034608
	3	−·0138996	−·0640288	+·0224994	0	+·0029084	+·0054648	−·0013288	+·0000528	−·0022484	−·0000462	−·0000286	−·0002112	−·0033418	−·0008932	−·0007128	−·0016060	+·0013816	+·0005786	+·0000924	+·0001386
	4	−·0523872	+·0113724	0	−·0184086	+·0044712	−·0023796	+·0000432	+·0010872	−·0000378	+·0018396	−·0001728	+·0000234	−·0007308	+·0027342	−·0013140	+·0005832	+·0004734	−·0011304	+·0001134	−·0000756
	5	−·0412114	−·0169074	+·0040982	−·0077004	+·2555423	0	+·0064821	+·0520521	−·0486204	+·0363413	−·0106082	−·0179335	−·1116062	+·0957776	−·0671956	−·0149451	+·0505889	−·0429970	+·0167648	−·0025978
	7	−·0007480	+·0013460	−·0006040	−·0000240	+·0020910	−·0167910	+·0057170	0	+·0014560	+·0002270	−·0015280	−·0005200	+·0043340	+·0075770	−·0012340	+·0035610	−·0022500	−·0039630	+·0006930	−·0013300
	8	+·0016152	+·0008926	−·0000288	+·0007248	−·0201492	−·0025092	0	−·0068604	+·0002724	−·0017472	−·0006240	+·0018336	+·0090924	−·0052008	+·0042732	+·0014808	−·0047556	+·0037000	−·0015960	−·0008316
	13	+·0126650	+·0275672	−·0051646	+·0013804	−·1224068	−·1050464	+·0147356	−·0257618	+·0556240	+·0081532	−·0036006	+·0179384	+·6349500	0	−·1158380	+·2972178	−·1768680	+·1147364	−·0426360	−·0263568
	15	+·0064620	+·0010845	−·0004860	+·0010950	−·0325140	+·0072315	−·0018510	−·0053415	+·0032160	−·0105660	+·0039210	+·0013170	−·0511050	−·1311255	+·1006950	0	+·0281850	+·0133320	−·0026805	+·0066390
	16	+·0007230	−·0043080	+·0007300	+·0003240	+·0048210	+·0216760	−·0035610	+·0012340	−·0070440	−·0021440	+·0006780	−·0026140	−·0874170	+·0340700	0	−·0671300	+·0088880	−·0187960	+·0044260	+·0017870
	17	−·0024112	−·0053056	+·0010048	−·0004208	+·0261104	+·0221920	−·0036000	+·0063408	−·0047040	−·0027360	+·0007536	−·0023120	−·0832320	−·0539936	+·0300640	−·0142208	+·2708160	0	+·0172960	+·0930000
	19	−·0000958	−·0001648	+·0000084	−·0000126	+·0010816	+·0001676	+·0001386	+·0002660	−·0002254	−·0009004	+·0001072	−·0000354	−·0025080	+·0015504	−·0003574	−·0008852	+·0021620	−·0116250	+·0063060	0
	20	−·0001648	+·0000958	−·0000126	−·0000084	+·0001676	−·0010816	+·0002660	−·0001386	−·0009004	+·0002254	−·0000354	−·0001072	+·0015504	+·0025080	−·0008852	+·0003574	−·0116250	−·0021620	0	−·0063060
Sum		+·3954036	−·0483511	−·0044908	+·0991862	+·0662887	−·0481691	+·0139001	+·0172774	+·0109378	+·0108471	−·0049066	−·0016549	+·3256310	−·0810495	−·0344112	+·2013765	+·1606669	+·0646068	−·0032327	+·0675276
23	9	+·0229092	+·0288184	−·0069496	+·0001428	−·1066512	−·0797164	+·0099008	−·0015436	+·2249440	0	−·0236776	+·0332248	+·1112480	−·0163064	+·0145792	+·0478992	−·0199920	+·0116280	−·0076636	+·0306136
	11	−·0134984	+·0021620	+·0001222	−·0009024	+·0321668	−·0543790	+·0143632	−·0048880	+·0327308	+·0459284	−·0365566	0	+·0099546	+·0495944	−·0245716	+·0082532	−·0044274	−·0135830	−·0050384	−·0016638
	12	−·0015640	−·0097648	+·0006528	+·0000884	+·0393380	+·0232696	+·0035360	+·0113904	−·0332248	+·0236776	0	−·0264452	−·0358768	+·0072012	−·0059704	−·0177752	+·0998260	−·0032028	+·0012036	−·0036448
	17	−·0105490	−·0232120	+·0043960	−·0018410	+·1142330	+·0970900	−·0157500	+·0277410	−·0205800	−·0119700	+·0032970	−·0101150	−·3641400	−·2362220	+·1315300	−·0622160	+1·1848200	0	+·0756700	+·4068750
	19	+·0030177	+·0051912	−·0002646	+·0003969	−·0340704	−·0052794	−·0043659	−·0033790	+·0071001	+·0283626	−·0033768	+·0011151	+·0790020	−·0488376	+·0112581	+·0278838	−·0681030	+·3661875	−·1986390	0
	20	−·0096408	+·0056043	−·0007371	−·0004914	+·0098046	−·0632736	+·0155610	−·0081081	−·0526734	+·0131859	−·0020709	−·0062712	+·0906984	+·1467180	−·0517842	+·0209079	−·6800625	−·1264770	0	−·3689010
Sum		−·0093253	+·0087991	−·0027803	−·0026067	+·0548208	−·0822888	+·0232451	+·0152127	+·1582967	+·0991845	−·0623840	−·0084915	−·1091138	−·0978524	+·0750411	+·0249520	+·4220611	+·2345527	−·1344674	+·0632790

TABLE LXVII.

Multipliers for equation				From LXVI: coefficients of			Equation 21			Equation 22			Equation 23		
r	21	22	23	k_{21}	k_{22}	k_{23}	21	22	23	21	22	23	21	22	23
1	+2·19	+·42		−·4839802	−·3954036	+·0093253	−1·059916638	−·865933884	+·020422407	−·203271684	−·166069512	+·003916626			
2	0	0		+·1230828	+·0483511	−·0087991	0	0	0		0				
3	+1·86	+·22		−·0518580	+·0044908	+·0027803	−·096455880	+·008352888	+·005171358	−·011408760	+·000987976	+·000611666			
4	−6·62	−·18		+·0396498	−·0991862	+·0026067	−·262481676	+·656612644	−·017256354	−·007136964	+·017853516	−·000469206			
5		+·31		+·1021086	−·0662887	−·0548208				+·031653666	−·020549497	−·016994448			
6		0		−·0781286	+·0481691	+·0822888				+·002602680	−·001390010	−·002324510			
7		+·10		+·0260268	−·0139001	−·0232451				+·001314456	+·002073288	+·001825524			
8		−·12		−·0109538	−·0172774	−·0152127									
9			+·68	−·0533817	−·0109378	−·1582967							−·036299556	−·007437704	−·107641756
10				+·0255464	−·0108471	−·0991845									
11			−·94	−·0248514	+·0049066	+·0623840							+·023360816	−·004612204	−·058641806
12			−·68	−·0041120	+·0016549	+·0084915							+·002796160	−·001125332	−·005774220
13		+·34		−·0264469	−·3256310	+·1091138				−·008991946	−·110714540	+·037098692			
14				+·0845590	+·0810495	+·0978524									
15		+·15		−·0399928	+·0344112	−·0750411				−·005998920	+·005161680	−·011256165			
16		−·10		+·0079629	−·2013765	−·0249520				−·000796290	+·020137650	+·002495290			
17		+·16	+·70	+·0039119	−·1606669	−·4220611				+·000625904	−·025706704	−·067529776	+·002738330	−·112466330	−·295442770
18				−·0359386	−·0646068	−·2345527									
19		+·02	−·63	+·0055383	+·0032327	+·1344674				+·000110766	+·000064654	+·002689348	−·003489129	−·002036601	−·084714462
20		−·02	−1·17	−·0164370	−·0675276	−·0632790				+·000328740	+·001350552	+·001265580	+·019231290	+·079007292	+·074036430
							−1·418854194	−·200968352	+·008337411	−·200968352	−·276800947	−·048671379	+·008337411	−·048671379	−·478178584
				Coefficients from table XLVII			+2·19	0	0	0	+1·22	0	0	0	+1·37

TABLE LXVIII.

No.	Right Hand Side	Left Hand Side. k_{21}	k_{22}	k_{23}
1	+1·00 +1·05885	+·7711	−·2010 −·05885	+·0083 +·0000
2	0 +·2607		+·9432 −·0524	−·0487 +·0022
3	0 −·01076			+·8918 −·0001
4	+·2607 +·2608	+1·00 +1·0027	+·8908	−·0465 −·0001 −·0027
5	−·01076 +·01361	0 +·0522		+·8917 −·0024
6	+·00285	+·0522	+1·00	+·8893
	Solution	+1·37317	+·29277 +1·12562	+·00320 +·05870 +1·12448
	Verification			
	+1·000031177 +·000022346 −·000006828	+1·058851387 −·276007170 +·011397311	−·058846770 +·276140664 −·014257899	+·000026560 −·000155840 +·002853760
	−·000007463 +·999979824 +·000099043	+·225754947 −·05884677 +·002429991	−·022624962 +1·061684784 −·054817694	+·000487210 −·000285869 +·05234866
	+·000002004 −·000039536 +·999979134	+·00246752 −·0006432 +·00002656	−·0117987 +·05536584 −·00285869	+·009333184 −·054762176 +1·002811264

18. To pass to the complete solution of the 23 equations the process is precisely similar to that already followed after the solution of 12 equations in §15 : the notation given in the corresponding tables concerned —*viz* tables LXIX, LXX—explains itself and the solution, keeping only 4 decimal places, is exhibited in table LXXI.

In this table are given also the value of $\Sigma^s {}_sk_r$ for all values of s from 1 to 23. These obviously should correspond to the case where all the R. H. S. of the 23 equations are unity. This solution, which depends on all the fundamental cases, can be verified in the original equations, and affords a complete check of the work. The substitution is carried out in table LXII. It will be seen that the values of the R.H.S. obtained by substitution differ slightly from unity, the greatest difference being ·035, showing that no gross error has been committed. It is of interest to consider to what these differences may be attributed. Each value of ${}_sk_r$ is given to four decimal places: and accordingly may be in error by ·00005. The sum of 23 such quantities *may* be wrong by ·00115: and in substituting in the original equation such an error is multiplied by coefficients, the largest of which is 111 51, which would admit of the corresponding term containing an error of ·1. That this extreme value should be obtained is most unlikely: but it is clear that the actual discrepancies obtained may easily be attributable to this cause. This line of argument shows how many figures it will be necessary to keep to be absolutely sure of all errors being less than a stated amount. For the present object, the solution as found may be considered sufficiently precise.

19. Table LXXI also contains the necessary multipliers to proceed to the solution of the complete 26 equations. The process is exactly similar to that already described in passing from 20 to 23 equations. All results are given in tables LXXIII—LXXIX, and the verification, similar to that of LXXII, in table LXXX. This completes the solution of the equations of table XLVII.

TABLE LXIX.

r		t	21	22	23	
1		$-\Sigma^s {}_sk_r\,(s,t)$ from table LXVI	− ·48398	− ·39540	+ ·00933	
2			+ ·12308	+ ·04835	− ·00880	
3			− ·05186	+ ·00449	+ ·00278	
4			+ ·03965	− ·09919	+ ·00261	
5			+ ·10211	− ·06629	− ·05482	
6			− ·07813	+ ·04817	+ ·08229	
7			+ ·02603	− ·01390	− ·02325	
8			− ·01095	− ·01728	− ·01521	
9			− ·05338	− ·01094	− ·15830	
10			+ ·02555	− ·01085	− ·09918	
11			− ·02485	+ ·00491	+ ·06238	
12			− ·00411	+ ·00165	+ ·00849	
13			− ·02645	− ·32563	+ ·10911	
14			+ ·08456	+ ·08105	+ ·09785	
15			− ·03999	+ ·03441	− ·07504	
16			+ ·00796	− ·20138	− ·02495	
17			+ ·00391	− ·16067	− ·42206	
18			− ·03594	− ·06461	− ·23455	
19			+ ·00554	+ ·00323	+ ·13447	
20			− ·01644	− ·06753	− ·06328	
$_tk_u$		21	+1·37317	+ ·29277	+ ·0032(	
t	u	22	+ ·29277	+1·12562	+ ·05870	Values of $_tk_u$
		23	+ ·00320	+ ·05870	+1·12448	
21	1		−·664587	−·115761	+·000030	=−·780318
	2		+·169010	+·014155	−·000028	=+·183137
	3		−·071213	+·001315	+·000009	=−·069889
	4		+·054446	−·029040	+·000008	=+·025414
	5		+·140214	−·019408	−·000175	=+·120631
	6		−·107286	+·014103	+·000263	=−·092920
	7		+·035744	−·004070	−·000074	=+·031600
	8		−·015036	−·005059	−·000049	=−·020144
	9		−·073300	−·003203	−·000506	=−·077009
	10		+·035084	−·003177	−·000317	=+·031590
	11		−·034123	+·001438	+·000200	=−·032485
	12		−·005644	+·000483	+·000027	=−·005134
	13		−·036320	−·095335	+·000349	=−·131306
	14		+·116115	+·023729	+·000313	=+·140157
	15		−·054913	+·010074	−·000240	=−·045079
	16		+·010930	−·058958	−·000080	=−·048108
	17		+·005369	−·047039	−·001350	=−·043020
	18		−·049352	−·018916	−·000750	=−·069018
	19		+·007607	+·000946	+·000430	=+·008983
	20		−·022575	−·019771	−·000203	=−·042549
22	1		−·141695	−·445070	+·000548	=−·586217
	2		+·036034	+·054424	−·000517	=+·089941
	3		−·015183	+·005054	+·000163	=−·009966
	4		+·011608	−·111650	+·000153	=−·099889
	5		+·029895	−·074617	−·003218	=−·047940
	6		−·022874	+·054221	+·004830	=+·036177
	7		+·007621	−·015646	−·001365	=−·009390
	8		−·003206	−·019451	−·000893	=−·023550
	9		−·015628	−·012314	−·009292	=−·037234
	10		+·007480	−·012213	−·005822	=−·010555
	11		−·007275	+·005527	+·003662	=+·001914
	12		−·001203	+·001857	+·000498	=+·001152
	13		−·007744	−·366536	+·006405	=−·367875
	14		+·024757	+·091232	+·005744	=+·121733
	15		−·011708	+·038733	−·004405	=+·022620
	16		+·002330	−·226677	−·001465	=−·225812
	17		+·001145	−·180853	−·024775	=−·204483
	18		−·010522	−·072726	−·013768	=−·097016
	19		+·001622	+·003636	+·007893	=+·013151
	20		−·004813	−·076013	−·003715	=−·084541
23	1		−·001549	−·023210	+·010491	=−·014268
	2		+·000394	+·002838	−·009895	=−·006663
	3		−·000166	+·000264	+·003126	=+·003224
	4		+·000127	−·005822	+·002935	=−·002760
	5		+·000327	−·003891	−·061644	=−·065208
	6		−·000250	+·002828	+·092533	=+·095111
	7		+·000083	−·000816	−·026144	=−·026877
	8		−·000035	−·001014	−·017103	=−·018152
	9		−·000171	−·000642	−·178005	=−·178818
	10		+·000082	−·000637	−·111526	=−·112081
	11		−·000080	+·000288	+·070145	=+·070353
	12		−·000013	+·000097	+·009547	=+·009631
	13		−·000085	−·019114	+·122692	=+·103493
	14		+·000271	+·004758	+·110030	=+·115059
	15		−·000128	+·002020	−·084381	=−·082489
	16		+·000025	−·011821	−·028056	=−·039852
	17		+·000013	−·009431	−·474598	=−·484016
	18		−·000115	−·003793	−·263747	=−·267655
	19		+·000018	+·000190	+·151209	=+·151417
	20		−·000053	−·003964	−·071157	=−·075174

TABLE LXX.

t	21	22	23	
r=1	− ·4840	− ·3954	+ ·0093	$\Sigma^s\ {}_rk_s\,(s,t)$ from Table LXVI
2	+ ·1231	+ ·0484	− ·0088	
3	− ·0519	+ ·0045	+ ·0028	
4	+ ·0396	− ·0992	+ ·0026	
5	+ ·1021	− ·0663	− ·0548	
6	− ·0781	+ ·0482	+ ·0823	
7	+ ·0260	− ·0139	− ·9232	
8	− ·0110	− ·0173	− ·0152	
9	− ·0534	− ·0109	− ·1583	
10	+ ·0255	− ·0108	− ·0092	
11	− ·0249	+ ·0049	+ ·0624	
12	− ·0041	+ ·0017	+ ·0085	
13	− ·0264	− ·3256	+ ·1091	
14	+ ·0846	+ ·0810	+ ·0979	
15	− ·0400	+ ·0344	− ·0750	
16	+ ·0080	− ·2014	− ·0250	
17	+ ·0039	− ·1607	− ·4221	
18	− ·0359	− ·0646	− ·2346	
19	+ ·0055	+ ·0032	+ ·1345	
20	− ·0164	− ·0675	− ·0633	
u=1	− ·7803	− ·5862	− ·0143	$_tk_u$ from LXX
2	+ ·1831	+ ·0899	− ·0067	
3	− ·0699	− ·0100	+ ·0032	
4	+ ·0254	− ·0999	− ·0028	
5	+ ·1206	− ·0479	− ·0652	
6	− ·0929	+ ·0362	+ ·0951	
7	+ ·0316	− ·0094	− ·0269	
8	− ·0201	− ·0236	− ·0182	
9	− ·0770	− ·0372	− ·1783	
10	+ ·0316	− ·0106	− ·1121	
11	− ·0325	+ ·0019	+ ·0704	
12	− ·0051	+ ·0012	+ ·0096	
13	− ·1313	− ·3679	+ ·1035	
14	+ ·1402	+ ·1217	+ ·1151	
15	− ·0451	+ ·0226	− ·0825	
16	− ·0481	− ·2258	− ·0399	
17	− ·0430	− ·2045	− ·4840	
18	− ·0690	− ·0970	− ·2677	
19	+ ·0090	+ ·0132	+ ·1514	
20	− ·0425	− ·0845	− ·0752	

r	u	$_tk_u\,\Sigma^s\ {}_rk_s\,(s,t)$ (t = 21)	(t = 22)	(t = 23)	$_rk_u$ from LXV	$_rk_u$
1	1	+ ·37767	+ ·23178	− ·00013	+ 1·15442	= + 1·7637
	2	− ·09605	− ·02837	+ ·00013	0	= − ·1242
	3	+ ·04050	− ·00264	− ·00004	− ·06318	= − ·0254
	4	− ·03000	+ ·05815	− ·00004	+ ·29104	= + ·3182
	5	− ·07067	+ ·03887	+ ·00078	− ·13294	= − ·1730
	6	+ ·06094	− ·02825	− ·00118	+ ·05454	= + ·0861
	7	− ·02029	+ ·00815	+ ·00033	− ·00748	= − ·0193
	8	+ ·00858	+ ·01014	+ ·00022	− ·01346	= + ·0055
	9	+ ·04167	+ ·00639	+ ·00226	+ ·03369	= + ·0840
	10	− ·01900	+ ·00633	+ ·00142	− ·04238	= − ·0545
	11	+ ·01043	− ·00287	− ·00089	+ ·01436	= + ·0300
	12	+ ·00320	− ·00100	− ·00012	+ ·00230	= + ·0041
	13	+ ·02060	+ ·19087	− ·00156	+ ·03725	= + ·2472
	14	− ·06601	− ·04748	− ·00140	− ·08108	= − ·1960
	15	+ ·03121	− ·02017	+ ·00107	+ ·04308	= + ·0552
	16	− ·00624	+ ·11806	+ ·00036	− ·00723	= + ·1050
	17	− ·00304	+ ·09420	+ ·00603	− ·01507	= + ·0821
	18	+ ·02801	+ ·03787	+ ·00335	+ ·03316	= + ·1024
	19	− ·00429	− ·00188	− ·00102	− ·00479	= − ·0120
	20	+ ·01280	+ ·03057	+ ·00091	+ ·00824	= + ·0615
2	1	− ·08862	− ·03555	− ·00006	0	= − ·1242
	2	+ ·02254	+ ·00435	+ ·00006	+ 1·15442	= + 1·1814
	3	− ·00950	+ ·00040	− ·00002	− ·29104	= − ·3002
	4	+ ·00725	− ·00892	− ·00002	− ·06318	= − ·0649
	5	+ ·01869	− ·00506	+ ·00037	− ·05454	= − ·0414
	6	− ·01430	+ ·00433	− ·00055	− ·13294	= − ·1435
	7	+ ·00476	− ·00125	+ ·00016	+ ·01346	= + ·0171
	8	− ·00201	− ·00156	+ ·00010	− ·00748	= − ·0109
	9	− ·00978	− ·00098	+ ·00106	+ ·04238	= + ·0327
	10	+ ·00467	− ·00097	+ ·00066	+ ·03309	= + ·0381
	11	− ·00455	+ ·00044	− ·00042	− ·00230	= − ·0068
	12	− ·00075	+ ·00015	− ·00006	+ ·01436	= + ·0137
	13	− ·00483	− ·02927	− ·00073	+ ·08104	= + ·0462
	14	+ ·01550	+ ·00728	− ·00066	+ ·03725	= + ·0594
	15	− ·00732	+ ·00309	+ ·00050	+ ·00723	= + ·0035
	16	+ ·00147	− ·01811	+ ·00017	+ ·04308	= + ·0266
	17	+ ·00071	− ·01445	+ ·00283	− ·03316	= − ·0441
	18	− ·00654	− ·00581	+ ·00157	− ·01507	= − ·0259
	19	+ ·00101	+ ·00029	− ·00090	− ·00824	= − ·0078
	20	− ·00300	− ·00607	+ ·00042	− ·00479	= − ·0134
3	1	+ ·03383	+ ·00395	+ ·00003	− ·06818	= − ·0254
	2	− ·00860	− ·00048	− ·00003	− ·29104	= − ·3002
	3	+ ·00363	− ·00004	+ ·00001	+ ·10227	= + ·1059
	4	− ·00277	+ ·00009	+ ·00001	0	= − ·0018
	5	− ·00714	+ ·00066	− ·00018	+ ·01322	= + ·0066
	6	+ ·00546	− ·00048	+ ·00026	+ ·02484	= + ·0301
	7	− ·00182	+ ·00014	− ·00007	− ·00604	= − ·0078
	8	+ ·00077	+ ·00017	− ·00005	+ ·00024	= + ·0011
	9	+ ·00373	+ ·00011	− ·00051	− ·01022	= − ·0069
	10	− ·00178	+ ·00011	− ·00032	− ·00021	= − ·0022
	11	+ ·00174	− ·00005	+ ·00020	− ·00013	= + ·0018
	12	+ ·00029	− ·00002	+ ·00003	− ·00096	= − ·0007
	13	+ ·00185	+ ·00326	+ ·00035	− ·01519	= − ·0097
	14	− ·00501	− ·00081	+ ·00031	− ·00406	= − ·0105
	15	+ ·00280	− ·00034	− ·00024	− ·00324	= − ·0010
	16	− ·00056	+ ·00201	− ·00008	− ·00730	= − ·0059
	17	− ·00027	+ ·00161	− ·00135	+ ·00628	= + ·0062
	18	+ ·00251	+ ·00065	− ·00075	+ ·00263	= + ·0050
	19	− ·00038	− ·00003	+ ·00043	+ ·00042	= + ·0004
	20	+ ·00115	+ ·00068	− ·00020	+ ·00063	= + ·0023
4	1	− ·01229	+ ·03950	− ·00003	+ ·29104	= + ·3182
	2	+ ·00313	− ·00484	+ ·00002	− ·06318	= − ·0649
	3	− ·00132	− ·00045	− ·00001	0	= − ·0018
	4	+ ·00101	+ ·00091	− ·00001	+ ·10227	= + ·1132
	5	+ ·00259	+ ·00662	+ ·00015	− ·02484	= − ·0155
	6	− ·00198	− ·00481	− ·00023	+ ·01322	= + ·0062
	7	+ ·00066	+ ·00139	+ ·00006	− ·00024	= + ·0019
	8	− ·00028	+ ·00173	+ ·00004	− ·00604	= − ·0046
	9	− ·00136	+ ·00109	+ ·00044	+ ·00021	= + ·0004
	10	+ ·00065	+ ·00108	+ ·00028	− ·01022	= − ·0082
	11	− ·00063	− ·00040	− ·00017	+ ·00096	= − ·0003
	12	− ·00010	− ·00017	− ·00002	− ·00013	= − ·0004
	13	− ·00067	+ ·03253	− ·00031	+ ·00406	= + ·0356
	14	+ ·00215	− ·00809	− ·00027	− ·01519	= − ·0214
	15	− ·00102	− ·00344	+ ·00021	+ ·00730	= + ·0031
	16	+ ·00020	+ ·02012	+ ·00007	− ·00324	= + ·0171
	17	+ ·00010	+ ·01605	+ ·00118	− ·00263	= + ·0147
	18	− ·00091	+ ·00645	+ ·00066	+ ·00628	= + ·0125
	19	+ ·00014	− ·00032	− ·00038	− ·00063	= − ·0012
	20	− ·00042	+ ·00674	+ ·00018	+ ·00042	= + ·0069
5	1	− ·05837	+ ·01894	− ·00061	− ·13294	= − ·1730
	2	+ ·01485	− ·00232	+ ·00057	− ·05454	= − ·0414
	3	− ·00626	− ·00022	− ·00018	+ ·01322	= + ·0066
	4	+ ·00478	+ ·00475	− ·00017	− ·02484	= − ·0165
	5	+ ·01231	+ ·00318	+ ·00357	+ ·82433	= + ·8434
	6	− ·00942	− ·00231	− ·00537	0	= − ·0171
	7	+ ·00314	+ ·00067	+ ·00151	+ ·02091	= + ·0262
	8	− ·00133	+ ·00083	+ ·00099	+ ·16791	= + ·1684
	9	− ·00644	+ ·00052	+ ·01032	− ·15684	= − ·1524
	10	+ ·00308	+ ·00052	+ ·00647	+ ·11723	= + ·1273
	11	− ·00300	− ·00023	− ·00407	− ·03422	= − ·0415
	12	− ·00049	− ·00008	− ·00055	− ·05785	= − ·0590
	13	− ·00318	+ ·01560	− ·00711	− ·36002	= − ·3547
	14	+ ·01020	− ·00388	− ·00638	+ ·30896	= + ·3089
	15	− ·00482	− ·00165	+ ·00489	− ·21676	= − ·2183
	16	+ ·00096	+ ·00965	+ ·00163	− ·04821	= − ·0360
	17	+ ·00047	+ ·00770	+ ·02752	+ ·16319	= + ·1990
	18	− ·00133	+ ·00309	+ ·01530	− ·13870	= − ·1246
	19	+ ·00066	− ·00015	− ·00877	+ ·05408	= + ·0458
	20	− ·00198	+ ·00323	+ ·00413	− ·00838	= − ·0030
6	1	+ ·04496	− ·01431	+ ·00088	+ ·05454	= + ·0861
	2	− ·01144	+ ·00175	− ·00084	− ·13294	= − ·1435
	3	+ ·00482	+ ·00017	+ ·00027	+ ·02484	= + ·0301
	4	− ·00368	− ·00359	+ ·00025	+ ·01322	= + ·0062
	5	− ·00949	− ·00240	− ·00521	0	= − ·0171
	6	+ ·00726	+ ·00174	+ ·00783	+ ·82433	= + ·8412
	7	− ·00242	− ·00050	− ·00221	− ·16791	= − ·1730
	8	+ ·00102	− ·00063	− ·00145	+ ·02091	= + ·0198
	9	+ ·00406	− ·00039	− ·01505	− ·11723	= − ·1277
	10	− ·00237	− ·00039	− ·00943	− ·15684	= − ·1690
	11	+ ·00231	+ ·00018	+ ·00593	+ ·05785	= + ·0663
	12	+ ·00038	+ ·00006	+ ·00081	− ·03422	= − ·0330
	13	+ ·00245	− ·01179	+ ·01038	− ·30896	= − ·3079
	14	− ·00786	+ ·00293	+ ·00931	− ·36002	= − ·3556
	15	+ ·00872	+ ·00125	− ·00713	+ ·04821	= + ·0460
	16	− ·00074	− ·00729	− ·00238	− ·21676	= − ·2272
	17	− ·00036	− ·00582	− ·04014	+ ·13870	= + ·0924
	18	+ ·00334	− ·00234	− ·02231	+ ·16319	= + ·1419
	19	− ·00051	+ ·00012	+ ·01279	+ ·00838	= + ·0208
	20	+ ·00152	− ·00244	− ·00602	+ ·05408	= + ·0471
7	1	− ·01529	+ ·00372	− ·00025	− ·00748	= − ·0193
	2	+ ·00389	− ·00046	+ ·00024	+ ·01346	= + ·0171
	3	− ·00164	− ·00004	− ·00008	− ·00604	= − ·0078
	4	+ ·00125	+ ·00003	− ·00007	− ·00024	= + ·0019
	5	+ ·00323	+ ·00062	+ ·00147	+ ·02091	= + ·0262
	6	− ·00247	− ·00045	− ·00222	− ·16791	= − ·1730
	7	+ ·00082	+ ·00013	+ ·00062	+ ·05717	= + ·0587
	8	− ·00035	+ ·00016	+ ·00041	0	= + ·0002
	9	− ·00169	+ ·00010	+ ·00426	+ ·01456	= + ·0172
	10	+ ·00081	+ ·00010	+ ·00267	+ ·00227	= + ·0058
	11	− ·00079	− ·00005	− ·00168	− ·01528	= − ·0178
	12	− ·00013	− ·00002	− ·00023	− ·00520	= − ·0056
	13	− ·00083	+ ·00306	− ·00294	+ ·04334	= + ·0426
	14	+ ·00267	− ·00076	− ·00263	+ ·07577	= + ·0751
	15	− ·00126	− ·00032	+ ·00202	− ·01234	= − ·0119
	16	+ ·00025	+ ·00189	+ ·00067	+ ·03561	= + ·0384
	17	+ ·00012	+ ·00151	+ ·01136	− ·02250	= − ·0095
	18	− ·00113	+ ·00061	+ ·00631	− ·03963	= − ·0339
	19	+ ·00017	− ·00003	− ·00362	+ ·00693	= + ·0035
	20	− ·00052	+ ·00063	+ ·00170	− ·01330	= − ·0115
8	1	+ ·00973	+ ·00933	− ·00017	− ·01346	= + ·0055
	2	− ·00247	− ·00114	+ ·00017	− ·00748	= − ·0109
	3	+ ·00104	− ·00011	− ·00005	+ ·00924	= + ·0101
	4	− ·00080	+ ·00234	− ·00005	− ·00604	= − ·0046
	5	− ·00205	+ ·00157	+ ·00100	+ ·16791	= + ·1684
	6	+ ·00157	− ·00114	− ·00150	+ ·02091	= + ·0198
	7	− ·00052	+ ·00033	+ ·00042	0	= + ·0002
	8	+ ·00022	+ ·00041	+ ·00028	+ ·05717	= + ·0581
	9	+ ·00107	+ ·00026	+ ·00288	− ·00227	= + ·0019
	10	− ·00051	+ ·00026	+ ·00181	+ ·01456	= + ·0161
	11	+ ·00050	− ·00012	− ·00114	+ ·00520	= + ·0045
	12	+ ·00008	− ·00004	− ·00015	− ·01528	= − ·0154
	13	+ ·00053	+ ·00768	− ·00199	− ·07577	= − ·0695
	14	− ·00170	− ·00191	− ·00178	+ ·04334	= + ·0379
	15	+ ·00080	− ·00081	+ ·00137	− ·03561	= − ·0343
	16	− ·00016	+ ·00475	+ ·00045	− ·01234	= − ·0073
	17	− ·00008	+ ·00379	+ ·00768	+ ·03963	= + ·0510
	18	+ ·00072	+ ·00152	+ ·00427	− ·02250	= − ·0160
	19	− ·00011	− ·00008	− ·00245	+ ·01330	= + ·0107
	20	+ ·00033	+ ·00159	+ ·00115	+ ·00693	= + ·0100
9	1	+ ·03727	+ ·01471	− ·00166	+ ·03360	= + ·0840
	2	− ·00948	− ·00180	+ ·00157	+ ·04238	= + ·0327
	3	+ ·00400	− ·00017	− ·00050	− ·01022	= − ·0069
	4	− ·00305	+ ·00309	− ·00016	+ ·00021	= + ·0004
	5	− ·00786	+ ·00247	+ ·00980	− ·15684	= − ·1524
	6	+ ·00601	− ·00179	− ·01472	− ·11723	= − ·1277
	7	− ·00200	+ ·00053	+ ·00415	+ ·01456	= + ·0172
	8	+ ·00085	+ ·00064	+ ·00272	− ·00227	= + ·0019
	9	+ ·00411	+ ·00041	+ ·02830	+ ·33080	= + ·3636
	10	− ·00196	+ ·00040	+ ·01774	0	= + ·0162
	11	+ ·00192	− ·00018	− ·01116	− ·03482	= − ·0442
	12	+ ·00032	− ·00006	− ·00152	+ ·04886	= + ·0476
	13	+ ·00196	+ ·01211	− ·01951	+ ·16360	= + ·1582
	14	− ·00651	− ·00301	− ·01750	− ·02398	= − ·0510
	15	+ ·00308	− ·00128	+ ·01341	+ ·02144	= + ·0367
	16	− ·00062	+ ·00749	+ ·00447	+ ·07044	= + ·0818
	17	− ·00030	+ ·00508	+ ·07547	− ·02940	= + ·0518
	18	+ ·00276	+ ·00240	+ ·04195	+ ·01710	= + ·0642
	19	− ·00042	− ·00012	− ·02405	− ·01127	= − ·0350
	20	+ ·00126	+ ·00251	+ ·01132	+ ·04502	= + ·0601
10	1	− ·01520	+ ·00419	− ·00104	− ·04238	= − ·0545
	2	+ ·00389	− ·00051	+ ·00099	+ ·03309	= + ·0381
	3	− ·00164	− ·00005	− ·00031	− ·00021	= − ·0022
	4	+ ·00125	+ ·00105	− ·00029	− ·01022	= − ·0082
	5	+ ·00323	+ ·00070	+ ·00614	+ ·11723	= + ·1273

TABLE LXX.—(Continued).

r	u	t = 21	t = 22	t = 23	$_rk_u$ from LXV	$_rk_u$
		$_tk_u\ \Sigma^s\ _rk_s\ (s,t)$				
10	6	−·00247	−·00051	−·00923	−·15684	=−·1690
	7	+·00082	+·00015	+·00260	+·00227	=+·0058
	8	−·00035	+·00018	+·00170	+·01456	=+·0161
	9	−·00169	+·00012	+·01775	0	=+·0162
	10	+·00081	+·00011	+·01112	+·33080	=+·3428
	11	−·00079	−·00005	−·00700	−·04886	=−·0567
	12	−·00013	−·00002	−·00095	−·03482	=−·0359
	13	−·00083	+·00345	−·01223	+·02398	=+·0143
	14	+·00267	−·00086	−·01097	+·16360	=+·1544
	15	−·00126	−·00036	+·00841	−·07044	=−·0637
	16	+·00025	+·00214	+·00280	+·03144	=+·0266
	17	+·00012	+·00170	+·04732	−·01710	=+·0320
	18	−·00113	+·00068	+·02680	−·02940	=−·0036
	19	+·00017	−·00003	−·01509	−·04502	=−·0600
	20	−·00052	+·00072	+·00710	−·01127	=−·004
11	1	+·01574	−·00075	+·00065	+·01436	=+·0300
	2	−·00400	+·00009	−·00062	−·00230	=−·0068
	3	+·00169	+·00001	+·00020	−·00013	=+·0018
	4	−·00129	−·00019	+·00018	+·00096	=−·0003
	5	−·00332	−·00013	−·00386	−·03422	=−·0415
	6	+·00254	+·00009	+·00579	+·05785	=+·0663
	7	−·00085	−·00003	−·00163	−·01528	=−·0178
	8	+·00036	−·00003	−·00107	+·00520	=+·0045
	9	+·00174	−·00002	−·01114	−·03482	=−·0442
	10	−·00085	−·00002	−·00698	−·04886	=−·0567
	11	+·00081	+·00001	+·00439	+·03889	=+·0441
	12	+·00013	+·00000	+·00060	0	=+·0007
	13	+·00086	−·00062	+·00768	−·01059	=−·002.
	14	−·00275	+·00015	+·00680	−·05276	=−·0485
	15	+·00130	+·00007	−·00528	+·02614	=+·0222
	16	−·00026	−·00038	−·00176	−·00878	=−·0112
	17	−·00013	−·00031	−·02972	+·00471	=−·0254
	18	+·00117	−·00012	−·01651	+·01445	=−·0010
	19	−·00018	+·00001	+·00947	+·00536	=+·0147
	20	+·00053	−·00013	−·00446	+·00177	=−·0023
12	1	+·00247	−·00047	+·00009	+·00230	=+·0044
	2	−·00063	+·00006	−·00008	+·01436	=+·0137
	3	+·00026	+·00001	+·00003	−·00096	=−·0007
	4	−·00020	−·00012	+·00002	−·00013	=−·0004
	5	−·00052	−·00008	−·00053	−·05785	=−·0590
	6	+·00040	+·00006	+·00079	−·03422	=−·0330
	7	−·00013	−·00002	−·00022	−·00520	=−·0056
	8	+·00006	−·00002	−·00015	−·01528	=−·0154
	9	+·00027	−·00001	−·00152	+·04886	=+·0476
	10	−·00013	−·00001	−·00095	−·03482	=−·0359
	11	+·00013	+·00001	+·00060	0	=+·0007
	12	+·00002	+·00000	+·00008	+·03889	=+·0390
	13	+·00013	−·00039	+·00105	+·05276	=+·0536
	14	−·00043	+·00010	+·00094	−·01059	=−·0100
	15	+·00020	+·00004	−·00072	+·00878	=+·0083
	16	−·00004	−·00024	−·00024	+·02614	=+·0256
	17	−·00002	−·00019	−·00405	−·01445	=−·0187
	18	+·00018	−·00008	−·00225	+·00471	=+·0026
	19	−·00003	+·00000	+·00129	−·00177	=−·0005
	20	+·00008	−·00068	−·00061	−·00536	=+·0048
13	1	+·06355	+·14547	+·00096	+·03725	=+·2472
	2	−·01616	−·01781	−·00091	+·08108	=+·0462
	3	+·00681	−·00166	+·00029	−·01519	=−·0097
	4	−·00520	+·03650	+·00027	+·00406	=+·0356
	5	−·01341	+·02439	−·00567	−·36002	=−·3547
	6	+·01025	−·01773	+·00852	−·30896	=−·3079
	7	−·00341	+·00511	−·00240	+·04334	=+·0426
	8	+·00144	+·00636	−·00157	−·07577	=−·0695
	9	+·00701	+·00401	+·01638	+·16360	=+·1582
	10	−·00335	+·00397	−·01027	+·02398	=+·0143
	11	+·00327	−·00180	+·00646	−·01059	=−·0027
	12	+·00054	−·00063	+·00088	+·05276	=+·0536
	13	+·00347	+·11979	+·01129	+1·86750	=+2·0020
	14	−·01111	−·02980	+·01013	0	=−·0308
	15	+·00525	−·01266	−·00776	−·34070	=−·3559
	16	−·00105	+·07410	−·00259	+·87417	=+·9446
	17	−·00051	+·05912	−·04369	−·52020	=−·5053
	18	+·00471	+·02377	−·02428	+·33746	=+·3417
	19	−·00072	−·00118	+·01392	−·12540	=−·1134
	20	+·00215	+·02483	−·00655	−·07752	=−·0571

r	u	t = 21	t = 22	t = 23	$_rk_u$ from LXV	$_rk_u$
		$_tk_u\ \Sigma^s\ _rk_s\ (s,t)$				
14	1	−·06786	−·04812	+·00107	−·08108	=−·1960
	2	+·01726	+·00589	−·00101	+·03725	=+·0594
	3	−·00728	+·00055	−·00032	−·00406	=−·0105
	4	+·00555	−·01207	+·00030	−·01519	=−·0214
	5	+·01431	−·00807	−·00631	+·30896	=+·3089
	6	−·01095	+·00587	+·00947	−·36002	=−·3556
	7	+·00365	−·00169	−·00267	+·07577	=+·0751
	8	−·00154	−·00211	−·00175	+·04334	=+·0379
	9	−·00749	−·00133	−·01822	−·02398	=−·0510
	10	+·00358	−·00131	−·01142	+·16360	=+·1544
	11	−·00349	+·00060	+·00718	−·05276	=−·0485
	12	−·00057	+·00021	+·00098	−·01059	=−·0100
	13	−·00370	−·03963	+·01256	0	=−·0308
	14	+·01186	+·00986	+·01127	+1·86750	=+1·9005
	15	−·00561	+·00419	−·00863	−·87417	=−·8842
	16	+·00112	−·02451	−·00288	−·34070	=−·3670
	17	+·00055	−·01956	−·04858	−·33746	=−·4050
	18	−·00303	−·00786	−·02700	−·52020	=−·5601
	19	+·00077	+·00039	+·01548	+·07752	=+·0942
	20	−·00230	−·00821	−·00729	−·12540	=−·1432
15	1	+·02181	−·00894	−·00077	+·04308	=+·0552
	2	−·00555	+·00109	+·00073	+·00723	=+·0035
	3	+·00234	+·00010	−·00023	−·00324	=−·0010
	4	−·00179	−·00224	−·00021	+·00730	=+·0031
	5	−·00460	−·00150	+·00452	−·21676	=−·2183
	6	+·00352	+·00109	−·00679	+·04821	=+·0460
	7	−·00117	−·00031	+·00191	−·01234	=−·0119
	8	+·00049	−·00039	+·00125	−·03561	=−·0343
	9	+·00241	−·00025	+·01306	+·02144	=+·0367
	10	−·00115	−·00024	+·00818	−·07044	=−·0637
	11	+·00112	+·00011	−·00515	+·02614	=+·0222
	12	+·00018	+·00004	−·00070	+·00878	=+·0083
	13	+·00119	−·00736	−·00900	−·34070	=−·3559
	14	−·00382	+·00183	−·00808	−·87417	=−·8842
	15	+·00180	+·00078	+·00619	+·67130	=+·6801
	16	−·00036	−·00455	+·00206	0	=−·0029
	17	−·00018	−·00363	+·03484	+·18790	=+·2189
	18	+·00162	−·00146	+·01935	+·08888	=+·1084
	19	−·00025	+·00007	−·01110	−·01787	=−·0291
	20	+·00074	−·00153	+·00522	+·04426	=+·0487
16	1	+·02328	+·08928	−·00037	−·00723	=+·1050
	2	−·00592	−·01093	+·00035	+·04308	=+·0266
	3	+·00250	−·00102	−·00011	−·00730	=−·0059
	4	−·00191	+·02240	−·00010	−·00324	=+·0171
	5	−·00491	+·01497	+·00219	−·04821	=−·0360
	6	+·00376	−·01088	−·00328	−·21676	=−·2272
	7	−·00125	+·00314	+·00093	+·03561	=+·0384
	8	+·00052	+·00891	+·00061	−·01234	=−·0073
	9	+·00257	+·00246	+·00631	+·07044	=+·0818
	10	−·00123	+·00244	+·00396	+·02144	=+·0266
	11	+·00120	−·00111	−·00249	−·00878	=−·0112
	12	+·00020	−·00038	−·00034	+·02614	=+·0256
	13	+·00127	+·07352	−·00435	+·87417	=+·9446
	14	−·00407	−·01829	−·00391	−·34070	=−·3670
	15	+·00192	−·00777	+·00299	0	=−·0029
	16	−·00038	+·04548	+·00100	+·67130	=+·7174
	17	−·00019	+·03629	+·01684	−·08888	=−·0359
	18	+·00173	+·01459	+·00936	+·18790	=+·2136
	19	−·00026	−·00072	−·00537	−·04426	=−·0506
	20	+·00079	+·01524	+·00253	−·01787	=+·0007
17	1	+·02081	+·08086	−·00450	−·01507	=+·0821
	2	−·00529	−·00990	+·00426	−·03316	=−·0441
	3	+·00223	−·00092	−·00136	+·00628	=+·0062
	4	−·00170	+·02029	−·00126	−·00263	=+·0147
	5	−·00439	+·01356	+·02652	+·16319	=+·1990
	6	+·00336	−·00986	−·03983	+·13870	=+·0924
	7	−·00112	+·00284	+·01123	−·02250	=−·0095
	8	+·00047	+·00354	+·00736	+·03963	=+·0510
	9	+·00230	+·00223	+·07662	−·02940	=+·0518
	10	−·00110	+·00221	+·04801	−·01710	=+·0320
	11	+·00107	−·00100	−·03020	+·00471	=−·0254
	12	+·00018	−·00035	−·00411	−·01445	=−·0187
	13	+·00114	+·06659	−·05280	−·52020	=−·5053
	14	−·00364	−·01656	−·04738	−·33746	=−·4050
	15	+·00172	−·00703	+·03630	+·18790	=+·2189

r	u	t = 21	t = 22	t = 23	$_rk_u$ from LXV	$_rk_u$
		$_tk_u\ \Sigma^s\ _rk_s\ (s,t)$				
17	16	−·00034	+·04119	+·01210	−·08888	=−·0359
	17	−·00017	+·03286	+·20430	+1·69260	=+1·9296
	18	+·00154	+·01321	+·11355	0	=+·1283
	19	−·00024	−·00065	−·06510	+·10810	=+·0421
	20	+·00071	+·01380	+·03064	+·58125	=+·6264
18	1	+·03340	+·03836	−·00249	+·03316	=+·1024
	2	−·00850	−·00469	+·00236	−·01507	=−·0259
	3	+·00358	−·00044	−·00075	+·00263	=+·0050
	4	−·00273	+·00962	−·00070	+·00628	=+·0125
	5	−·00705	+·00643	+·01467	−·13870	=−·1246
	6	+·00539	−·00467	−·02203	+·16310	=+·1419
	7	−·00179	+·00135	+·00621	−·03963	=−·0339
	8	+·00075	+·00168	+·00407	−·02250	=−·0160
	9	+·00368	+·00106	+·04238	+·01710	=+·0642
	10	−·00176	+·00105	+·02656	−·02940	=−·0036
	11	+·00172	−·00048	−·01670	+·01445	=−·0010
	12	+·00028	−·00016	−·00228	+·00471	=+·0026
	13	+·00182	+·03158	−·02921	+·33746	=+·3417
	14	−·00584	−·00786	−·02621	−·52020	=−·5601
	15	+·00276	−·00334	+·02008	+·08888	=+·1084
	16	−·00055	+·01954	+·00669	+·18790	=+·2136
	17	−·00027	+·01559	+·11300	0	=+·1283
	18	+·00248	+·00626	+·06280	+1·69260	=+1·7641
	19	−·00038	−·00031	−·03601	−·58125	=−·6179
	20	+·00113	+·00654	+·01695	+·10810	=+·1327
19	1	−·00435	−·00522	+·00141	−·00479	=−·0129
	2	+·00111	−·00064	−·00133	−·00824	=−·0078
	3	−·00047	+·00006	+·00042	+·00042	=+·0004
	4	+·00036	−·00131	+·00039	−·00063	=−·0012
	5	+·00002	−·00088	−·00830	+·05408	=+·0458
	6	−·00070	+·00064	+·01246	+·00838	=+·0208
	7	+·00023	−·00018	−·00351	+·00693	=+·0035
	8	−·00010	−·00023	−·00230	+·01330	=+·0107
	9	−·00048	−·00014	−·02397	−·01127	=−·0359
	10	+·00023	−·00014	−·01502	−·04502	=−·0600
	11	−·00022	+·00006	+·00945	+·00536	=+·0147
	12	−·00004	+·00002	+·00120	−·00177	=−·0005
	13	−·00024	−·00430	+·01652	−·12540	=−·1134
	14	+·00076	+·00107	+·01482	+·07752	=+·0942
	15	−·00036	+·00045	−·01136	−·01787	=−·0291
	16	+·00007	−·00266	−·00379	−·04426	=−·0506
	17	+·00004	−·00212	−·06391	+·10810	=+·0421
	18	−·00032	−·00085	−·03552	−·58125	=−·6179
	19	+·00005	+·00004	+·02036	+·31530	=+·3358
	20	−·00015	−·00089	−·00958	0	=−·0106
20	1	+·02058	+·03341	−·00070	+·00824	=+·0615
	2	−·00523	−·00409	+·00066	−·00479	=−·0134
	3	+·00221	−·00098	−·00021	+·00063	=+·0023
	4	−·00168	+·00838	−·00020	+·00042	=+·0069
	5	−·00434	+·00560	+·00412	−·00838	=−·0030
	6	+·00332	−·00407	−·00619	+·05408	=+·0471
	7	−·00111	+·00117	+·00174	−·01330	=−·0115
	8	+·00146	+·00146	+·00114	+·00603	=+·0100
	9	+·00227	+·00092	+·01190	+·04502	=+·0601
	10	−·00108	+·00091	+·00746	−·01127	=−·0040
	11	+·00106	−·00041	−·00469	+·00177	=−·0023
	12	+·00017	−·00014	−·00064	+·00536	=+·0048
	13	+·00112	+·02751	−·00820	−·07752	=−·0571
	14	−·00360	−·00685	−·00736	−·12540	=−·1432
	15	+·00170	−·00291	+·00564	+·04426	=+·0487
	16	−·00034	+·01703	+·00188	−·01787	=+·0007
	17	−·00017	+·01359	+·03174	+·58125	=+·6264
	18	+·00153	+·00546	+·01764	+·10810	=+·1327
	19	−·00023	−·00027	−·01011	0	=−·0106
	20	+·00070	+·00570	+·00476	+·31530	=+·3265

TABLE LXXI. Values of ${}_sk_r$ for 23 conditions.

r \ s	1	2	3	4	5	6	7	8	9	10	11	12
1	+1·7637	−·1242	−·0254	+·3182	−·1730	+·0861	−·0193	+·0055	+·0840	−·0545	+·0300	+·0044
2	−·1242	+1·1814	−·3002	−·0649	−·0414	−·1435	+·0171	−·0109	+·0327	+·0381	−·0068	+·0137
3	−·0254	−·3002	+·1059	−·0018	+·0066	+·0301	−·0078	+·0011	−·0069	−·0022	+·0018	−·0007
4	+·3182	−·0649	−·0018	+·1132	−·0155	+·0062	+·0019	−·0046	+·0004	−·0082	−·0003	−·0004
5	−·1730	−·0414	+·0066	−·0155	+·8434	−·0171	+·0262	+·1684	−·1524	+·1273	−·0415	−·0590
6	+·0861	−·1435	+·0301	+·0062	−·0171	+·8412	−·1730	+·0198	−·1277	−·1690	+·0663	−·0330
7	−·0193	+·0171	−·0078	+·0019	+·0262	−·1730	+·0587	+·0002	+·0172	+·0058	−·0178	−·0056
8	+·0055	−·0109	+·0011	−·0046	+·1684	+·0198	+·0002	+·0581	+·0019	+·0161	+·0045	−·0154
9	+·0840	+·0327	−·0069	+·0004	−·1524	−·1277	+·0172	+·0019	+·3636	+·0162	−·0442	+·0476
10	−·0545	+·0381	−·0022	−·0082	+·1273	−·1690	+·0058	+·0161	+·0162	+·3428	−·0567	−·0359
11	+·0300	−·0068	+·0018	−·0003	−·0415	+·0663	−·0178	+·0045	−·0442	−·0567	+·0441	+·0007
12	+·0044	+·0137	−·0007	−·0004	−·0590	−·0330	−·0056	−·0154	+·0476	−·0359	+·0007	+·0390
13	+·2472	+·0462	−·0097	+·0356	−·3547	−·3079	+·0426	−·0695	+·1582	+·0143	−·0027	+·0536
14	−·1960	+·0594	−·0105	−·0214	+·3089	−·3556	+·0751	+·0379	−·0510	+·1544	−·0485	−·0100
15	+·0552	+·0035	−·0010	+·0031	−·2183	+·0460	−·0119	−·0343	+·0367	−·0637	+·0222	+·0033
16	+·1050	+·0266	−·0059	+·0171	−·0360	−·2272	+·0384	−·0073	+·0818	+·0266	−·0112	+·0256
17	+·0821	−·0441	+·0062	+·0147	+·1990	+·0924	−·0095	+·0510	+·0518	+·0320	−·0254	−·0187
18	+·1024	−·0259	+·0050	+·0125	−·1246	+·1419	−·0339	−·0160	+·0642	−·0036	−·0010	+·0026
19	−·0129	−·0078	+·0004	−·0012	+·0458	+·0208	+·0035	+·0107	−·0359	−·0600	+·0147	−·0005
20	+·0615	−·0134	+·0023	+·0069	−·0030	+·0471	−·0115	+·0100	+·0601	−·0040	−·0023	+·0045
21	−·7803	+·1831	−·0699	+·0254	+·1206	−·0929	+·0316	−·0201	−·0770	+·0316	−·0325	−·0051
22	−·5862	+·0899	−·0100	−·0999	−·0479	+·0362	−·0094	−·0236	−·0372	−·0106	+·0019	+·0012
23	−·0143	−·0067	+·0032	−·0028	−·0652	+·0951	−·0269	−·0182	−·1785	−·1121	+·0704	+·0096
Sum	+·9592	+·9019	−·2894	+·3342	+·4966	−·1177	−·0083	+·1653	+·3053	+·2247	−·0343	+·0268

r \ s	13	14	15	16	17	18	19	20	21	22	23	Multipliers k_{24}	k_{25}	k_{26}
1	+·2472	−·1960	+·0552	+·1050	+·0821	+·1024	−·0129	+·0615	−·7803	−·5862	−·0143	0	0	0
2	+·0462	+·0594	+·0035	+·0266	−·0441	−·0259	−·0078	−·0134	+·1831	+·0899	−·0067	2·19	+·42	0
3	−·0097	−·0105	−·0010	−·0059	+·0062	+·0050	+·0004	+·0023	−·0699	−·0100	+·0032	+6·62	+·18	0
4	+·0356	−·0214	+·0031	+·0171	+·0147	+·0125	−·0012	+·0069	+·0254	−·0999	−·0028	+1·86	+0·22	0
5	−·3547	+·3089	−·2183	−·0360	+·1990	−·1246	+·0458	−·0030	+·1206	−·0479	−·0652	0	0	0
6	−·3079	−·3556	+·0460	−·2272	+·0924	+·1419	+·0208	+·0471	−·0929	+·0362	+·0951	0	+·31	0
7	+·0426	+·0751	−·0119	+·0384	−·0095	−·0339	+·0035	−·0115	+·0316	−·0094	−·0269	0	+·12	0
8	−·0695	+·0379	−·0343	−·0073	+·0510	−·0160	+·0107	+·0100	−·0201	−·0236	−·0182	0	+·10	0
9	+·1582	−·0510	+·0367	+·0818	+·0518	+·0642	−·0359	+·0601	−·0770	−·0372	−·1788	0	0	0
10	+·0143	+·1544	−·0637	+·0266	+·0320	−·0036	−·0609	−·0040	+·0316	−·0106	−·1121	0	0	+·68
11	−·0027	−·0485	+·0222	−·0112	−·0254	−·0010	+·0147	−·0023	−·0325	+·0019	+·0704	0	0	+·68
12	+·0536	−·0100	+·0033	+·0256	−·0187	+·0026	−·0005	+·0048	−·0051	+·0012	+·0096	0	0	−·94
13	+2·0020	−·0308	−·3559	+·9446	−·5053	+·3417	−·1134	−·0571	−·1313	−·3679	+·1035	0	0	0
14	−·0308	+1·9005	−·8842	−·3670	−·4050	−·5601	+·0942	−·1432	+·1402	+·1217	+·1151	0	+·34	0
15	−·3559	−·8842	+·6801	−·0029	+·2189	+·1084	−·0291	+·0457	−·0451	+·0226	−·0825	0	+·10	0
16	+·9446	−·3670	−·0029	+·7174	−·0359	+·2136	−·0506	+·0007	−·0481	−·2258	−·2399	0	+·15	0
17	−·5053	−·4050	+·2189	−·0359	+1·9296	+·1283	+·0421	+·6264	−·0430	−·2045	−·4840	0	0	0
18	+·3417	−·5601	+·1694	+·2136	+·1283	+1·7641	−·6179	+·1327	−·0690	−·0970	−·2677	0	+·16	+·70
19	−·1134	+·0942	−·0291	−·0506	+·0421	−·6179	+·3358	−·0106	+·0090	+·0132	+·1514	0	+·02	+1·17
20	−·0571	−·1432	+·0457	+·0007	+·6264	+·1327	−·0106	+·3265	−·0425	−·0845	−·0752	0	+·02	−·63
21	−·1313	+·1402	−·0451	−·0481	−·0430	−·0690	+·0090	−·0425	+1·3732	+·2928	+·0032	0	0	0
22	−·3679	+·1217	+·0226	−·2258	−·2045	−·0970	+·0132	−·0845	+·2928	+1·1256	+·0587	0	0	0
23	+·1035	+·1151	−·0325	−·0399	−·4840	−·2677	+·1514	−·0752	+·0032	+·0587	+1·1245	0	0	0
Sum	+1·6833	−·0759	−·4752	+1·1396	+1·6991	+1·2007	−·1983	+·8804	+·7539	−·0407	+·3604			

TABLE LXXII. Verification of solution in Table LXXI.

	1	2	3	4	5	6	7	8	9	10	11	12
1	+3·5107	0	+2·1102	−9·9564	+·6714	0	+·3933	−2·2062				
2	0	+3·3010	+9·3617	+1·9842	0	+·6313	+2·0744	+·3698				
3	−·6367	−3·0040	−11·7728	0	+·0087	−·5122	−2·1271	−·3299				
4	−3·4690	+·7352	0	+13·5953	−·5915	−·0100	−·3810	+2·4564				
5	+·3476	0	−·0149	−·8790	+4·8220	0	−·4171	−14·0141	+3·2130	0	+8·0797	−3·1385
6	0	−·0824	−·2083	+·0035	0	−1·1429	−3·3215	+·0989	0	−·7615	−·7439	−1·9150
7	−·0034	−·0191	−·0610	−·0095	+·0070	−·2342	−·9255	0	+·0110	−·1907	−·2458	−·4905
8	−·3802	+·0678	+·1884	+1·2150	−4·6648	−·1389	0	+18·4326	−3·7969	−·2182	−9·7692	+4·8962
9					+1·9753	0	−·4030	−7·6127	+2·9553	0	+5·1321	−3·4590
10					0	+1·4538	+5·1614	−·2966	0	+2·1751	+2·5459	+3·7772
11					−·5581	−·2168	−1·0160	+2·0271	−·5766	−·3866	−2·7200	0
12					−·1694	+·4360	+1·5839	+·7938	−·3036	+·4505	0	+2·1252
13					+3·3498	0	−·1347	−2·3061	+·4713	0	+1·0268	+·4713
14					0	−·1510	−·1040	+·0061	0	−·0213	+·0213	−·0463
15					−·8791	−1·2308	−·9599	+·6463	−·0570	−·2851	+·1521	−·6843
16					−2·9516	+2·1083	+1·5499	+2·3020	−·6838	+·1368	−1·6410	−·3647
17									+·9345	0	+·8156	+·3568
18									0	+·6604	−·2521	+·5763
19									+·0535	−·3292	+·1408	−·2697
20	0	0	0	0	0	0	0		−1·4615	−·2377	−1·1973	−·6251
21	+1·6510	0	+1·4023	−4·9908	0	0	0	0	0	0	0	0
22	−·0171	0	−·0090	+·0073	−·0126	0	−·0041	+·0049	0	0	0	0
23	0	0	0	0	0	0	0	0	+·2451	0	−·3388	−·2451
Sum	+1·0029	+·9985	+·9966	+·9886	+1·0071	+·9926	+·9690	+·9723	+1·0043	+·9925	+1·0062	+·9648

	13	14	15	16	17	18	19	20	21	22	23
1									+2·1006	+·4029	0
2									0	0	0
3									−·5383	−·0637	0
4									−2·2124	−·0602	0
5	+·9882	0	+·9187	−1·2862					0	+·1539	0
6	0	−·2342	−·3048	−·2177					0	0	0
7	+·0007	−·0114	−·0168	−·0113					0	−·0008	0
8	−·2265	−·0132	−·2248	+·3339					0	−·0198	0
9	+·0855	0	+·0366	−·1832	+·1679	0	−·0824	−·5068	0	0	+·2076
10	0	+·0629	+·1348	+·0270	0	+·1236	+·3730	−·0607	0	0	0
11	−·0209	+·0096	+·0110	+·0494	−·0165	+·0072	+·0244	+·0486	0	0	+·0322
12	+·0075	+·0163	+·0386	−·0086	+·0056	+·0129	+·0364	−·0190	0	0	−·0182
13	+5·7737	0	+3·5854	−7·2045	+1·3971	0	+·5387	−2·0200	0	+·5723	0
14	0	−·2603	−·3249	−·1617	0	−·0630	−·0911	−·0243	0	0	0
15	−1·0122	−2·0336	−4·0202	0	−·0048	−·4704	−·8696	−·1806	0	−·0713	0
16	−4·8775	+2·4273	0	+9·6410	−1·1282	+·0114	−·4330	+2·0855	0	−·1140	0
17	+1·4103	0	+·0170	−1·6821	+3·7890	0	−·9515	−6·8814	0	+·2719	+1·1894
18	0	+·9966	+1·1887	+·0120	0	+2·6776	+4·8628	−·6724	0	0	0
19	−·0635	−·2380	−·3629	+·0754	+·1110	−·8031	−2·1784	0	0	−·0040	+·1249
20	−1·0565	+·2817	+·3346	+1·6111	−3·5656	−·4930	0	+9·6492	0	−·0176	+1·0301
21	0	0	0	0	0	0	0	0	+1·6510	0	0
22	−·0138	0	−·0061	+·0041	−·0065	0	−·0008	+·0008	0	−·0497	0
23	0	0	0	0	+·2523	0	−·2271	−·4217	0	0	+·4937
Sum	+·9950	+1·0037	+1·0049	+·9986	+1·0013	+1·0032	+1·0064	+·9952	+1·0009	+·9999	+·9985

TABLE LXXIII. Values of ${}_s k_r(s, t)$ and $\Sigma^s\, {}_s k_r(s, t)$, s from 1 to 23, the latter in old face type.

t	s	r=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
24	2	−·271998	+2·587266	−·657438	−·142181	−·090666	−·314265	+·037449	−·023871	+·071613	+·083439	−·014892	+·030003	+·101178	+·130086	+·007665	+·058254	−·096579	−·056721	−·017082	−·029348	+·400989	+·196881	−·014673
	3	−·168148	−1·987334	+·701058	−·011916	+·043692	+·199262	−·051636	+·007282	−·045678	−·014564	+·011916	−·004634	−·064214	−·069510	−·006620	−·039058	+·041044	+·033100	+·002648	+·015226	−·462738	−·066200	+·021184
	4	+·591852	−·120714	−·003348	+·210552	−·028830	+·011532	+·003534	−·008550	+·000744	−·015252	−·000558	−·000744	+·066216	−·039804	+·005766	+·031806	+·027342	+·023250	−·002232	+·012834	+·047244	−·185814	−·005208
Sum		+·151706	+·479228	+·040272	+·056505	−·075804	−·103471	−·010653	−·025145	+·026679	+·053623	−·003534	+·024625	+·103180	+·020772	+·006811	+·051002	−·028193	−·000371	−·016666	−·001286	−·014505	−·055133	+·001303
25	2	−·052164	+·496188	−·126084	−·027258	−·017388	−·060270	+·007182	−·004578	+·013734	+·016002	−·002856	+·005754	+·019404	+·024948	+·001470	+·011172	−·018522	−·010878	−·008276	−·005628	+·076902	+·037758	−·002814
	3	−·004572	−·054036	+·019062	−·000324	+·001188	+·005418	−·001404	+·000198	−·001242	−·000396	+·000324	−·000126	−·001746	−·001890	−·000180	−·001062	+·001116	+·00 900	+·000072	+·000414	−·012582	−·001800	+·000576
	4	+·070004	−·014278	−·000396	+·024904	−·003410	+·001364	+·000418	−·001012	+·000088	−·001804	−·000066	−·000088	+·007832	−·004708	+·000682	+·003762	+·003234	+·002750	−·000264	+·001518	+·005588	−·021978	−·000616
	6	+·026691	−·044485	+·009331	+·001922	−·005301	+·260772	−·053630	+·006138	−·039587	−·052390	+·020553	−·010230	−·095449	−·110236	+·014260	−·070432	+·028644	+·043989	+·006448	+·014601	−·028799	+·011222	+·029481
	7	−·002316	+·002052	−·000936	+·000228	+·003144	−·020760	+·007044	+·000024	+·002064	+·000696	−·002136	−·000672	+·005112	+·009012	−·001428	+·004608	−·001140	−·004068	+·000420	−·001380	+·003792	−·001128	−·003228
	8	+·000550	−·001090	+·000110	−·000460	+·016840	+·001980	+·000020	+·005810	+·000190	+·001610	+·000450	−·001540	−·006950	+·003790	−·003430	−·000730	+·005100	−·001600	+·001070	+·001000	−·002010	−·002360	−·001820
	14	−·066640	+·020196	−·003570	−·007276	+·105026	−·120904	+·025534	+·012586	−·017340	+·052496	−·016490	−·003400	−·010472	+·646170	−·300628	−·124780	−·187700	−·190434	+·032028	−·048688	+·047668	+·041378	+·039134
	15	+·005520	+·000350	−·000100	+·000310	−·021830	+·004600	−·001190	−·003430	+·003670	−·006370	+·002220	+·000830	−·035590	−·038420	+·068010	−·000290	+·021890	+·010840	−·002910	+·004870	−·004510	+·002260	−·008250
	16	+·015750	+·003990	−·000885	+·002565	−·005400	−·034080	+·005760	−·001095	+·012270	+·003990	−·001680	+·003840	+·141690	−·055050	−·000435	+·107610	−·005385	+·032040	−·007590	+·000105	−·007215	−·033870	−·005985
	18	+·016384	−·004144	+·000800	+·002000	−·019936	+·022704	−·005424	−·002560	+·010272	−·000576	−·000160	+·000416	+·054672	−·089616	+·017344	+·034176	+·020528	+·282256	−·098864	+·021232	−·011040	−·015520	−·042832
	19	−·000258	−·000156	+·000008	−·000024	+·000916	+·000416	+·000070	+·000214	−·000718	−·001200	+·000294	−·000010	−·002268	+·001884	−·000582	−·001012	+·000842	−·012358	+·006716	−·000212	+·000180	+·000264	+·003028
	20	+·001230	−·000268	+·000046	+·000138	−·000060	+·000942	−·000230	+·000200	+·001202	−·000080	−·000046	+·000096	−·001142	−·002864	+·000974	+·000014	+·012528	+·002654	−·000212	+·006530	−·000850	−·001690	−·001504
Sum		+·010179	+·404319	−·102614	−·003275	+·053789	+·062182	−·015850	+·012795	−·015397	+·011978	+·000407	−·005130	+·075093	+·333020	−·203943	−·036964	−·068865	+·156091	−·066362	−·005638	+·067124	+·014536	+·005170
26	10	−·037060	+·025908	−·001496	−·005576	+·086564	−·114920	+·003944	+·010948	+·011016	+·233104	−·038556	−·024412	+·009724	+·104992	−·043316	+·018088	+·021760	−·002448	−·040800	−·002720	+·021488	−·007208	−·076228
	11	−·020400	−·004624	+·001224	−·000204	−·028220	+·045084	−·012104	+·003060	−·030056	−·038556	+·029988	+·000476	−·001836	−·032980	+·015096	−·007616	−·017272	−·000680	+·009996	−·001564	−·022100	+·001292	+·047872
	12	−·004136	−·012878	+·000658	+·000376	+·055460	+·031020	+·005264	+·014476	−·044744	+·033746	−·000658	−·036660	−·050384	+·009400	−·007802	−·024064	+·017578	−·002414	+·000470	−·004512	+·004794	−·001128	−·009024
	18	+·071680	−·018130	+·003500	+·008750	−·087220	+·099330	−·023730	−·011200	+·044940	−·002520	−·000700	+·001820	+·239190	−·392070	+·075880	+·149520	+·089810	+1·234870	−·432530	+·092890	−·048300	−·067900	−·187390
	19	−·015093	−·009126	+·000468	−·001404	+·053586	+·024336	+·004095	+·012519	−·042003	−·070200	+·017199	−·000585	−·132678	+·110214	−·034047	−·059202	+·049257	−·722943	+·392886	−·012402	+·010530	+·015444	+·177138
	20	−·038745	+·003442	−·001449	−·004347	+·001890	−·029673	+·007245	−·003300	−·037863	+·002520	+·001449	−·003024	+·035973	+·090216	−·030681	−·000441	−·394632	−·083601	+·006678	−·205695	+·026775	+·053235	+·047376
Sum		−·002054	−·010408	+·002905	−·002405	+·083060	+·055177	−·015286	+·023503	−·098710	+·158094	+·008722	−·062385	+·099989	−·110228	−·024870	+·076285	−·233499	+·422754	−·063300	−·134003	−·006813	−·006265	−·000256

TABLE LXXIV.

r	Multipliers for equation 24	25	26	From LXXIII: coefficients of k_{24}	k_{25}	k_{26}	Equation 24: 24	25	26	Equation 25: 24	25	26	Equation 26: 24	25	26
1	0	0		−·151706	−·010179	+·002954									
2	+2·19	+·42		−·479228	−·404319	+·010408	−1·04950932	−·88545861	+·02279352	−·20127576	−·16981398	+·00437136			
3	+6·62	+·18		−·040272	+·102614	−·002905	−·26660064	+·67930468	−·01923110	−·00724896	+·01847052	−·00052290			
4	+1·86	+·22		−·056505	+·003275	+·002405	−·10509930	+·00609150	+·00447330	−·01243110	+·00072050	+·00052910			
5				+·075804	−·053789	−·083060									
6		+·31		+·103471	−·062182	−·055177				+·03207601	−·01927642	−·01710487			
7		+·12		+·010653	+·015850	+·015286				+·00127836	+·00190200	+·00183432			
8		+·10		+·025145	−·012795	−·023503				+·00251450	−·00127950	−·00235030			
9				−·026679	+·015397	+·098710									
10			+·68	−·053623	−·011978	−·158094							−·03646364	−·00814504	−·10750392
11			+·68	+·003534	−·000407	−·008722							+·00240312	−·00027676	−·00593096
12			−·94	−·024625	+·005130	+·062385							+·02314750	−·00482220	−·05864190
13				−·103180	−·075093	−·099989									
14		+·34		−·020772	−·333020	+·110228				−·00706248	−·11322680	+·03747752			
15		+·10		−·006811	+·203943	+·024870				−·00068110	+·02039430	+·00248700			
16		+·15		−·051002	+·036964	−·076285				−·00765030	+·00554460	−·01144275			
17				+·028193	+·062865	+·233499									
18		+·16	+·70	+·000371	−·156091	−·422754				+·00005936	−·02497456	−·06764064	+·00025970	−·10926370	−·29592780
19		+·02	+1·17	+·016666	+·066362	+·063300				+·00033332	+·00132724	+·00126600	+·01949922	+·07764354	+·07406100
20		+·02	−·63	+·001286	+·005638	+·134003				+·00002572	+·00011276	+·00268006	−·00081018	−·00355194	−·08442189
21				+·014505	−·067124	+·006813									
22				+·055133	−·014536	+·006265	+2·19000000	0	0	0	+1·22000000	0	0	0	+1·37000000
23				−·001303	−·005170	+·000256	+·76879074	−·20006243	+·00803572	−·20006243	+·93990066	−·04841610	+·00803572	−·04841610	+·89163453

TABLE LXXV.

No.	Right Hand Side	Left Hand Side: k_{24}	k_{25}	k_{26}
1	+1·0000 +1·05868	+·7688	−·2001 −·05871	+·0080 +·00003
2	0 +·2603		+·9399 −·0521	−·0484 +·0021
3	0. −·0104			+·8916 −·0001
4	+·2603 +·26047	+1·0000 +1·00272	+·8878	−·0463 −·00017 −·00272
5	−·0104 +·0136	0 +·05215		+·8915 −·0024
6	+·0032	+·05215	+1·0000	+·8891
	Solution	+1·3771	+·2934 +1·1294	+·0036 +·0587 +1·1247

Verification

+1·00003394	+1·05871448	−·05870934	+·00002880
+·00004258	+·22556592	−·22599294	+·00046960
+·00001941	+·00276768	−·01174587	+·0089976
+·00003471	−·27555771	+·27576666	−·00017424
+·99997264	−·05870934	+1·06152306	−·00284108
+·00001629	−·00072036	+·05517213	−·05443548
+·00002600	+·01101680	−·01420056	+·00320976
+·00002116	+·00234720	−·05466296	+·05233692
+·99997024	+·00002880	−·00284108	+1·00278252

TABLE LXXVI.

$\Sigma^s {}_sk_r(s,t)$

r	t=24	25	26
1	− ·1517	− ·0102	+ ·0030
2	− ·4792	− ·4043	+ ·0104
3	− ·0403	+ ·1026	− ·0029
4	− ·0565	+ ·0033	+ ·0024
5	+ ·0758	− ·0538	− ·0831
6	+ ·1035	− ·0622	− ·0552
7	+ ·0107	+ ·0159	+ ·0153
8	+ ·0251	− ·0128	− ·0235
9	− ·0267	+ ·0154	+ ·0987
10	− ·0536	− ·0120	− ·1581
11	+ ·0035	− ·0004	− ·0087
12	− ·0246	+ ·0051	+ ·0624
13	− ·1032	− ·0751	− ·1000
14	− ·0208	− ·3330	+ ·1102
15	− ·0068	+ ·2039	+ ·0249
16	− ·0510	+ ·0370	− ·0763
17	+ ·0282	+ ·0689	+ ·2335
18	+ ·0004	− ·1561	− ·4228
19	+ ·0167	+ ·0664	+ ·0633
20	+ ·0013	+ ·0056	+ ·1340
21	+ ·0145	− ·0671	+ ·0068
22	+ ·0551	− ·0145	+ ·0063
23	− ·0013	− ·0052	+ ·0003

$_tk_u$

u			
24	+1·3771	+ ·2934	+ ·0036
25	+ ·2934	+1·1294	+ ·0587
26	+ ·0036	+ ·0587	+1·1247

u	t=24	25	26	$_{24}k_u$
1	−·208006	−·002993	+·000011	=−·211888
2	−·659906	−·118622	+·000037	=−·778491
3	−·055497	+·030103	−·000010	=−·025404
4	−·077806	+·000968	+·000009	=−·076829
5	+·104384	−·015785	−·000299	=+·088300
6	+·142530	−·018249	−·000199	=+·124082
7	+·014666	+·004650	+·000055	=+·019371
8	+·034563	−·003756	−·000085	=+·030722
9	−·036766	+·004518	+·000355	=−·031893
10	−·073807	−·003521	−·000569	=−·077897
11	+·004819	−·000117	−·000031	=+·004671
12	−·033874	+·001496	+·000225	=−·032153
13	−·142106	−·022034	−·000360	=−·164500
14	−·028642	−·097702	+·000397	=−·125947
15	−·009364	+·059824	+·000090	=+·050550
16	−·070232	+·010856	−·000275	=−·059651
17	+·038834	+·020215	+·000840	=+·059889
18	+·000551	−·045800	−·001522	=−·046771
19	+·022998	+·019482	+·000228	=+·042708
20	+·001790	+·001643	+·000482	=+·003915
21	+·019968	−·019687	+·000024	=+·000257
22	+·075878	−·004254	+·000023	=+·071647
23	−·001790	−·001526	+·000001	=−·003315

u	t=24	25	26	$_{25}k_u$
1	−·044509	−·011520	+·000176	=−·055853
2	−·140597	−·456616	+·000610	=−·596603
3	−·011824	+·115876	−·000170	=+·103882
4	−·016577	+·003727	+·000141	=−·012709
5	+·022240	−·060762	−·004878	=−·043400
6	+·030367	−·070249	−·003240	=−·043122
7	+·003125	+·017901	+·000898	=+·021924
8	+·007364	−·014456	−·001379	=−·008471
9	−·007834	+·017393	+·005794	=+·015353
10	−·015726	−·013553	−·009280	=−·038559
11	+·001027	−·000452	−·000511	=+·000064
12	−·007218	+·005760	+·003663	=+·002205
13	−·030279	−·084818	−·005870	=−·120967
14	−·006103	−·376090	+·006469	=−·375724
15	−·001995	+·230285	+·001462	=+·229752
16	−·014963	+·041788	−·004479	=+·022346
17	+·008274	+·077816	+·013706	=+·099796
18	+·000117	−·176299	−·024818	=−·201000
19	+·004900	+·074092	+·003716	=+·083608
20	+·000381	+·006325	+·007866	=+·014572
21	+·004254	−·075783	+·000399	=−·071130
22	+·016166	−·016376	+·000370	=+·000160
23	−·000381	−·005873	+·000018	=−·006236

u	t=24	25	26	$_{26}k_u$
1	−·000545	−·000509	+·003374	=+·002230
2	−·001725	−·023732	+·011697	=−·013760
3	−·000145	+·006023	−·003262	=+·002616
4	−·000203	+·000194	+·002699	=+·002690
5	+·000273	−·003158	−·093463	=−·096348
6	+·000373	−·003151	−·062083	=−·064861
7	+·000038	+·000930	+·017208	=+·018176
8	+·000090	−·000751	−·026430	=−·027091
9	−·000096	+·000904	+·111008	=+·111816
10	−·000193	−·000704	−·177815	=−·178712
11	+·000013	−·000023	−·009785	=−·009795
12	−·000089	+·000299	+·070181	=+·070391
13	−·000372	−·004408	−·112470	=−·117250
14	−·000075	−·019547	+·123942	=+·104320
15	−·000024	+·011969	+·028005	=+·039950
16	−·000184	+·002172	−·085815	=−·083827
17	+·000102	+·004044	+·262617	=+·266763
18	+·000001	−·009163	−·475523	=−·484685
19	+·000060	+·003898	+·071194	=+·075152
20	+·000005	+·000329	+·150710	=+·151044
21	+·000052	−·003939	+·007648	=+·003761
22	+·000198	−·000851	+·007086	=+·006433
23	−·000005	−·000305	+·000337	=+·000027

TABLE LXXVII.

t	21	22	23	
r=1	− ·1517	− ·0102	+ ·0030	$\Sigma^s {}_rk_s(s,t)$ from Table LXXIII
2	− ·4792	− ·4043	+ ·0104	
3	− ·0403	+ ·1026	− ·0029	
4	− ·0565	+ ·0033	+ ·0024	
5	+ ·0758	− ·0538	− ·0831	
6	+ ·1035	− ·0622	− ·0552	
7	+ ·0107	+ ·0159	+ ·0153	
8	+ ·0251	− ·0128	− ·0235	
9	− ·0267	+ ·0154	+ ·0987	
10	− ·0536	− ·0120	− ·1581	
11	+ ·0035	− ·0004	− ·0087	
12	− ·0246	+ ·0051	+ ·0624	
13	− ·1032	− ·0751	− ·1000	
14	− ·0208	− ·3330	+ ·1102	
15	− ·0068	+ ·2039	+ ·0249	
16	− ·0510	+ ·0370	− ·0763	
17	+ ·0282	+ ·0689	+ ·2335	
18	+ ·0004	− ·1561	− ·4228	
19	+ ·0167	+ ·0664	+ ·0633	
20	+ ·0013	+ ·0056	+ ·1340	
u=1	− ·2119	− ·0559	+ ·0022	$_tk_u$ from LXXVI
2	− ·7785	− ·5966	− ·0138	
3	− ·0254	+ ·1039	+ ·0026	
4	− ·0768	− ·0127	+ ·0027	
5	+ ·0883	− ·0434	− ·0963	
6	+ ·1241	− ·0431	− ·0649	
7	+ ·0194	+ ·0219	+ ·0182	
8	+ ·0307	− ·0085	− ·0271	
9	− ·0319	+ ·0154	+ ·1118	
10	− ·0779	− ·0386	− ·1787	
11	+ ·0047	+ ·0001	− ·0098	
12	− ·0322	+ ·0022	+ ·0704	
13	− ·1645	− ·1210	− ·1172	
14	− ·1259	− ·3757	+ ·1043	
15	+ ·0505	+ ·2298	+ ·0400	
16	− ·0597	+ ·0223	− ·0838	
17	+ ·0590	+ ·0998	+ ·2668	
18	− ·0468	− ·2010	− ·4847	
19	+ ·0427	+ ·0836	+ ·0752	
20	+ ·0039	+ ·0146	+ ·1510	

t		21	22	23	$_rk_u$ from LXXI	$_rk_u$
r	u	$_tk_u \Sigma^s {}_rk_s(s,t)$				
1	1	+·03215	+·00057	+·00001	+1·7637	=+1·7960
	2	+·10154	+·02260	+·00002	− ·1242	= ·0000
	3	+·00854	−·00574	−·00001	− ·0254	=− ·0228
	4	+·01197	−·00018	+·00001	+ ·3182	=+ ·3300
	5	−·01606	+·00301	−·00018	− ·1730	=− ·1862
	6	−·02193	+·00348	−·00012	+ ·0861	=+ ·0673
	7	−·00227	−·00089	+·00003	− ·0193	=− ·0224
	8	−·00532	+·00072	−·00005	+ ·0055	=+ ·0009
	9	+·00566	−·00086	+·00022	+ ·0840	=+ ·0891
	10	+·01136	+·00067	−·00035	− ·0545	=− ·0429
	11	−·00074	+·00002	−·00002	+ ·0300	=+ ·0291
	12	+·00521	−·00029	+·00014	+ ·0044	=+ ·0003
	13	+·02187	+·00420	−·00022	+ ·2472	=+ ·2729
	14	+·00441	+·01861	+·00024	− ·1960	=− ·1727
	15	+·00144	−·01140	+·00005	+ ·0552	=+ ·0453
	16	+·01081	−·00207	−·00017	+ ·1050	=+ ·1134
	17	−·00598	−·00385	+·00051	+ ·0821	=+ ·0728
	18	−·00008	+·00873	−·00093	+ ·1024	=+ ·1102
	19	−·00354	−·00371	+·00014	− ·0129	=− ·0198
	20	−·00028	−·00031	+·00029	+ ·0615	=+ ·0612
2	1	+·11810	+·00609	−·00004	− ·1242	= ·0000
	2	+·87306	+·24121	−·00014	+1·1814	=+1·7960
	3	+·03137	−·06121	+·00004	− ·3002	=− ·3300
	4	+·04399	−·00197	−·00003	− ·0649	=− ·0228
	5	−·05901	+·03210	+·00115	− ·0414	=− ·0673
	6	−·08057	+·03711	+·00076	− ·1435	=− ·1862
	7	−·00833	−·00949	−·00021	+ ·0171	=− ·0009
	8	−·01954	+·00764	+·00032	− ·0109	=− ·0224
	9	+·02079	−·00919	−·00136	+ ·0327	=+ ·0429
	10	+·04173	+·00716	+·00218	+ ·0381	=+ ·0891
	11	−·00272	+·00024	+·00012	− ·0068	=− ·0093
	12	+·01015	−·00304	−·00086	+ ·0137	=+ ·0291
	13	+·08034	+·04480	+·00138	+ ·0462	=+ ·1727
	14	+·01619	+·19807	−·00152	+ ·0594	=+ ·2729
	15	+·00529	−·12165	−·00034	+ ·0035	=− ·1134
	16	+·03970	−·02207	+·00105	+ ·0266	=+ ·0453
	17	−·02195	−·04111	−·00322	− ·0441	=− ·1102
	18	−·00031	+·09313	+·00584	− ·0259	=+ ·0728
	19	−·01300	−·03961	−·00087	− ·0078	=− ·0612
	20	−·00101	−·00334	−·00185	− ·0134	=− ·0198
3	3	+·0010	+·0107	·0000	+ ·1059	=+·1175
	4	+·0014	+·0008	·0000	− ·0018	= ·0000
	5	−·0019	−·0056	−·0002	+ ·0066	=−·0010
	6	−·0026	−·0065	−·0001	+ ·0301	=+·0208
	7	−·0003	+·0016	·0000	− ·0078	=−·0065
	8	−·0006	−·0013	−·0001	+ ·0011	=−·0009
	9	+·0007	+·0016	+·0003	− ·0069	=−·0043
	10	+·0014	−·0012	−·0004	− ·0022	=−·0025
	11	−·0001	·0000	·0000	+ ·0018	=+·0016
	12	+·0006	+·0005	+·0002	− ·0007	=+·0006
	13	+·0028	−·0078	−·0003	− ·0097	=−·0152
	14	+·0005	−·0346	+·0003	− ·0105	=−·0442
	15	+·0002	+·0212	+·0001	− ·0010	=+·0204
	16	+·0013	+·0038	−·0002	− ·0059	=−·0010
	17	−·0007	+·0072	+·0006	+ ·0062	=+·0133
	18	·0000	−·0162	−·0011	+ ·0050	=−·0122
	19	−·0004	+·0069	+·0002	+ ·0004	=+·0071
	20	·0000	+·0006	+·0003	+ ·0023	=+·0031
4	3	+·0031	−·0013	·0000	− ·0018	= ·0000
	4	+·0043	·0000	·0000	+ ·1132	=+·1175
	5	−·0058	+·0007	−·0002	− ·0155	=−·0208
	6	−·0079	+·0008	−·0001	+ ·0062	=−·0010
	7	−·0008	−·0002	·0000	+ ·0019	=+·0009
	8	−·0010	+·0002	−·0001	− ·0046	=−·0065
	9	+·0021	−·0002	+·0003	+ ·0004	=+·0025
	10	+·0041	+·0002	−·0004	− ·0082	=−·0043
	11	−·0003	·0000	·0000	− ·0003	=−·0006
	12	+·0019	−·0001	+·0002	− ·0004	=+·0016
	13	+·0079	+·0010	−·0003	+ ·0356	=+·0442
	14	+·0016	+·0042	+·0003	− ·0214	=−·0152
	15	+·0005	−·0026	+·0001	+ ·0031	=+·0010
	16	+·0039	−·0005	−·0002	+ ·0171	=+·0204
	17	−·0022	−·0009	+·0006	+ ·0147	=+·0122
	18	·0000	+·0020	−·0012	+ ·0125	=+·0133
	19	−·0013	−·0008	+·0002	− ·0012	=−·0031
	20	−·0001	−·0001	+·0004	+ ·0069	=+·0071

TABLE LXXVII.—(Continued).

t		21	22	23	$_rk_u$ from LXXI	$_rk_u$
r	u	$_tk_u\ \Sigma^s\ _rk_s\ (s,t)$				
5	5	+ ·0067	+ ·0023	+ ·0080	+ ·8434	= + ·8604
	6	+ ·0091	+ ·0027	+ ·0053	− ·0171	= ·0000
	7	+ ·0009	− ·0007	− ·0015	+ ·0262	= + ·0250
	8	+ ·0022	+ ·0006	+ ·0023	+ ·1684	= + ·1734
	9	− ·0024	− ·0007	− ·0095	− ·1524	= − ·1649
	10	− ·0047	+ ·0005	+ ·0152	+ ·1273	= + ·1382
	11	+ ·0003	·0000	+ ·0008	− ·0415	= − ·0403
	12	− ·0022	− ·0002	− ·0060	− ·0590	= − ·0673
	13	− ·0091	+ ·0083	+ ·0096	− ·3547	= − ·3510
	14	− ·0018	+ ·0144	− ·0106	+ ·3089	= + ·3110
	15	− ·0006	− ·0088	− ·0024	− ·2183	= − ·2301
	16	− ·0045	− ·0016	+ ·0073	− ·0360	= − ·0348
	17	+ ·0025	− ·0030	− ·0225	+ ·1990	= + ·1760
	18	·0000	+ ·0068	+ ·0407	− ·1246	= − ·0771
	19	+ ·0015	− ·0029	− ·0061	+ ·0458	= + ·0383
	20	+ ·0001	− ·0002	− ·0129	− ·0030	= − ·0160
6	5	+ ·0094	+ ·0023	+ ·0054	− ·0171	= ·0000
	6	+ ·0128	+ ·0027	+ ·0036	+ ·8412	= + ·8604
	7	+ ·0013	− ·0007	− ·0010	− ·1730	= − ·1734
	8	+ ·0031	+ ·0006	+ ·0015	+ ·0198	= + ·0250
	9	− ·0033	− ·0007	− ·0064	− ·1277	= − ·1382
	10	− ·0067	+ ·0005	+ ·0103	− ·1690	= − ·1649
	11	+ ·0004	·0000	+ ·0006	+ ·0663	= + ·0673
	12	− ·0030	− ·0002	− ·0041	− ·0330	= − ·0403
	13	− ·0128	+ ·0032	+ ·0065	− ·3079	= − ·3110
	14	− ·0026	+ ·0144	− ·0072	− ·3556	= − ·3510
	15	− ·0008	− ·0088	− ·0016	+ ·0460	= + ·0348
	16	− ·0063	− ·0016	+ ·0050	− ·2272	= − ·2301
	17	+ ·0035	− ·0030	− ·0153	+ ·0924	= + ·0777
	18	·0000	+ ·0067	+ ·0274	+ ·1419	= + ·1760
	19	+ ·0021	− ·0029	− ·0041	+ ·0208	= + ·0160
	20	+ ·0002	− ·0002	− ·0087	+ ·0471	= + ·0383
7	7	+ ·0002	+ ·0003	+ ·0003	+ ·0587	= + ·0595
	8	+ ·0005	− ·0003	− ·0004	+ ·0002	= ·0000
	9	− ·0005	+ ·0003	+ ·0018	+ ·0172	= + ·0188
	10	− ·0010	− ·0003	− ·0029	+ ·0058	= + ·0016
	11	+ ·0001	·0000	− ·0002	− ·0178	= − ·0179
	12	− ·0005	+ ·0001	+ ·0011	− ·0056	= − ·0048
	13	− ·0020	− ·0016	− ·0018	+ ·0426	= + ·0371
	14	− ·0004	− ·0073	+ ·0020	+ ·0751	= + ·0694
	15	− ·0001	+ ·0045	+ ·0005	− ·0119	= − ·0070
	16	− ·0010	+ ·0008	− ·0014	+ ·0384	= + ·0368
	17	+ ·0005	+ ·0015	+ ·0042	− ·0095	= − ·0032
	18	·0000	− ·0034	− ·0077	− ·0339	= − ·0450
	19	+ ·0003	+ ·0015	+ ·0012	+ ·0035	= + ·0064
	20	·0000	+ ·0001	+ ·0024	− ·0115	= − ·0089
8	7	+ ·0003	− ·0001	− ·0004	+ ·0002	= ·0000
	8	+ ·0008	+ ·0001	+ ·0006	+ ·0581	= + ·0595
	9	− ·0008	− ·0001	− ·0027	+ ·0019	= − ·0016
	10	− ·0016	+ ·0001	+ ·0043	+ ·0161	= + ·0188
	11	+ ·0001	·0000	+ ·0002	+ ·0045	= + ·0048
	12	− ·0008	·0000	− ·0017	− ·0154	= − ·0179
	13	− ·0032	+ ·0006	+ ·0027	− ·0695	= − ·0694
	14	− ·0006	+ ·0028	− ·0030	+ ·0379	= + ·0371
	15	− ·0002	− ·0017	− ·0007	− ·0343	= − ·0368
	16	− ·0016	− ·0003	+ ·0021	− ·0073	= − ·0070
	17	+ ·0009	− ·0006	− ·0063	+ ·0510	= + ·0450
	18	·0000	+ ·0013	+ ·0115	− ·0160	= − ·0032
	19	+ ·0005	− ·0006	− ·0017	+ ·0107	= + ·0089
	20	·0000	·0000	− ·0036	+ ·0100	= + ·0064
9	9	+ ·0009	+ ·0002	+ ·0110	+ ·3636	= + ·3757
	10	+ ·0017	− ·0002	− ·0177	+ ·0162	·0000
	11	− ·0001	·0000	− ·0010	− ·0442	= − ·0453
	12	+ ·0008	+ ·0001	+ ·0070	+ ·0176	= + ·0554
	13	+ ·0033	− ·0012	− ·0112	+ ·1582	= + ·1491
	14	+ ·0007	− ·0051	+ ·0123	− ·0510	= − ·0431
	15	+ ·0002	+ ·0031	+ ·0028	+ ·0367	= + ·0428
	16	+ ·0016	+ ·0006	− ·0085	+ ·0818	= + ·0755
	17	− ·0009	+ ·0011	+ ·0261	+ ·0518	= + ·0780
	18	·0000	− ·0024	− ·0473	+ ·0642	= + ·0145
	19	− ·0005	+ ·0010	+ ·0071	− ·0359	= − ·0282
	20	·0000	+ ·0001	+ ·0150	+ ·0601	= + ·0752
10	9	+ ·0021	− ·0006	− ·0176	+ ·0162	= ·0000
	10	+ ·0042	+ ·0005	+ ·0283	+ ·3428	= + ·3757
	11	− ·0003	·0000	+ ·0016	− ·0567	= − ·0554
	12	+ ·0019	− ·0002	− ·0111	− ·0359	= − ·0453
	13	+ ·0080	+ ·0029	+ ·0179	+ ·0143	= + ·0431
	14	+ ·0016	+ ·0129	− ·0197	+ ·1544	= + ·1491
	15	+ ·0005	− ·0079	− ·0045	− ·0637	= − ·0755
	16	+ ·0040	− ·0014	+ ·0136	+ ·0266	= + ·0428
	17	− ·0022	− ·0027	− ·0417	+ ·0320	= − ·0145
	18	·0000	+ ·0060	+ ·0756	− ·0036	= + ·0780
	19	− ·0013	− ·0026	− ·0113	− ·0600	= − ·0752
	20	− ·0001	− ·0002	− ·0239	− ·0040	= − ·0282
11	11	·0000	0	+ ·0001	+ ·0441	= + ·0441
	12	− ·0001	0	− ·0006	+ ·0007	·0000
	13	− ·0005	0	+ ·0010	− ·0027	= − ·0022
	14	− ·0001	0	− ·0011	− ·0485	= − ·0497
	15	0	0	− ·0002	+ ·0222	= + ·0220
	16	− ·0002	0	+ ·0007	− ·0112	= − ·0107
	17	+ ·0001	0	− ·0023	− ·0254	= − ·0275
	18	0	0	+ ·0041	− ·0010	= + ·0030
	19	+ ·0001	0	− ·0006	+ ·0147	= + ·0142
	20	0	0	− ·0013	− ·0023	= − ·0036
12	11	− ·0001	0	− ·0006	+ ·0007	·0000
	12	+ ·0008	0	+ ·0044	+ ·0390	= + ·0441
	13	+ ·0033	− ·0002	− ·0070	+ ·0536	= + ·0497
	14	+ ·0007	− ·0007	+ ·0078	− ·0100	= − ·0022
	15	+ ·0002	+ ·0004	+ ·0018	+ ·0083	= + ·0107
	16	+ ·0016	+ ·0001	− ·0054	+ ·0256	= + ·0220
	17	− ·0009	+ ·0002	+ ·0164	− ·0187	= − ·0030
	18	·0000	− ·0003	− ·0298	+ ·0026	= − ·0275
	19	− ·0005	+ ·0001	+ ·0045	− ·0005	= + ·0036
	20	0	·0000	+ ·0094	+ ·0048	= + ·0142
13	13	+ ·0170	+ ·0091	+ ·0117	+ 2·0020	= + 2·0397
	14	+ ·0034	+ ·0403	− ·0129	− ·0308	·0000
	15	+ ·0011	− ·0247	− ·0029	− ·3559	= − ·3824
	16	+ ·0084	− ·0045	+ ·0089	+ ·9446	= + ·9573
	17	− ·0046	− ·0083	− ·0274	− ·5053	= − ·5456
	18	− ·0001	+ ·0189	+ ·0496	+ ·3417	= + ·4101
	19	− ·0027	− ·0080	− ·0074	− ·1134	= − ·1315
	20	− ·0002	− ·0007	− ·0157	− ·0571	= − ·0738
14	13	+ ·0130	+ ·0282	− ·0104	− ·0308	·0000
	14	+ ·0026	+ ·1251	+ ·0115	+ 1·9005	= + 2·0397
	15	+ ·0009	− ·0766	+ ·0026	− ·8842	= − ·9573
	16	+ ·0064	− ·0139	− ·0080	− ·3670	= − ·3824
	17	− ·0066	− ·0259	+ ·0244	− ·4050	= − ·4101
	18	− ·0001	+ ·0586	− ·0441	− ·5601	= − ·5456
	19	− ·0021	− ·0249	+ ·0066	+ ·0942	= + ·0738
	20	− ·0002	− ·0021	+ ·0140	− ·1432	— — ·1315
15	15	− ·0008	+ ·0469	+ ·0010	+ ·6801	= + ·7276
	16	− ·0026	+ ·0085	− ·0031	− ·0029	·0000
	17	+ ·0014	+ ·0158	+ ·0093	+ ·2189	= + ·2455
	18	·0000	− ·0859	− ·0169	+ ·1084	= + ·0556
	19	+ ·0008	+ ·0153	+ ·0025	− ·0291	= − ·0105
	20	+ ·0001	+ ·0013	+ ·0054	+ ·0487	= + ·0556
16	15	+ ·0004	+ ·0045	− ·0021	− ·0029	·0000
	16	+ ·0030	+ ·0008	+ ·0064	+ ·7174	= + ·7276
	17	− ·0017	+ ·0015	− ·0196	− ·0359	= − ·0556
	18	0	− ·0085	+ ·0354	+ ·2136	= + ·2455
	19	− ·0010	+ ·0015	− ·0053	− ·0506	= − ·0555
	20	− ·0001	+ ·0001	− ·0112	+ ·0007	= − ·0105
17	17	+ ·0017	+ ·0069	+ ·0633	+ 1·9296	= + 2·0006
	18	0	− ·0156	− ·1129	+ ·1283	= ·0000
	19	+ ·0010	+ ·0066	+ ·0169	+ ·0421	= + ·0666
	20	+ ·0001	+ ·0006	+ ·0358	+ ·6264	= + ·6628
18	17	− ·0013	− ·0138	− ·1132	+ ·1283	·0000
	18	0	+ ·0314	+ ·2049	+ 1·7641	= + 2·0006
	19	− ·0008	− ·0133	− ·0307	− ·6179	= − ·6628
	20	− ·0001	− ·0011	− ·0649	+ ·1327	= + ·0666
19	19	+ ·0007	+ ·0056	+ ·0048	+ ·3358	= + ·3468
	20	·0000	+ ·0005	+ ·0101	− ·0106	·0000
20	19	+ ·0001	+ ·0010	+ ·0096	− ·0106	= ·0000
	20	0	+ ·0001	+ ·0202	+ ·3265	= + ·3468

TABLE LXXVIII. Values of ${}_s k_r$ for 26 conditions.

r \ s	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	+1·7960	0	−·0228	+·3300	−·1862	+·0673	−·0224	+·0009	+·0891	−·0429	+·0291	+·0093	+·2729	−·1727	+·0453	+·1134	+·0728	+·1102	−·0198	+·0612	−·7785	−·5966	−·0138	−·2119	−·0559	+·0022
2	0	+1·7960	−·3300	−·0228	−·0673	−·1862	−·0009	−·0224	+·0429	+·0891	−·0093	+·0291	+·1727	+·2729	−·1134	+·0453	−·1102	+·0728	−·0612	−·0198	+·2119	+·0559	−·0022	−·7785	−·5966	−·0138
3	−·0228	−·3300	+·1175	0	−·0010	+·0208	−·0065	−·0009	−·0043	−·0025	+·0016	+·0006	−·0152	−·0442	+·0204	−·0010	+·0133	−·0122	+·0071	+·0031	−·0768	−·0127	+·0027	−·0254	+·1039	+·0026
4	+·3300	−·0228	0	+·1175	−·0208	−·0010	+·0009	−·0065	+·0025	−·0043	−·0006	+·0016	+·0442	−·0152	+·0010	+·0204	+·0122	+·0133	−·0031	+·0071	+·0254	−·1039	−·0026	−·0768	−·0127	+·0027
5	−·1862	−·0673	−·0010	−·0208	+·8604	0	+·0250	+·1734	−·1649	+·1382	−·0403	−·0673	−·3510	+·3110	−·2301	−·0348	+·1760	−·0774	+·0383	−·0160	+·1241	−·0431	−·0649	+·0883	−·0434	−·0963
6	+·0673	−·1862	+·0208	−·0010	0	+·8604	−·1734	+·0250	−·1382	−·1649	+·0673	−·0403	−·3110	−·3510	+·0348	−·2301	+·0774	+·1760	+·0160	+·0383	−·0883	+·0434	+·0963	+·1241	−·0431	−·0649
7	−·0224	−·0009	−·0065	+·0009	+·0250	−·1734	+·0595	0	+·0188	+·0016	−·0179	−·0048	+·0371	+·0694	−·0070	+·0368	−·0032	−·0450	+·0064	−·0089	+·0307	−·0085	−·0271	+·0194	+·0219	+·0182
8	+·0009	−·0224	−·0009	−·0065	+·1734	+·0250	0	+·0595	−·0016	+·0188	+·0048	−·0179	−·0694	+·0371	−·0368	−·0070	+·0450	−·0032	+·0089	+·0064	−·0194	−·0219	−·0182	+·0307	−·0085	−·0271
9	+·0891	+·0429	−·0043	+·0025	−·1649	−·1382	+·0188	−·0016	+·3757	0	−·0453	+·0554	+·1491	−·0431	+·0428	+·0755	+·0780	+·0145	−·0282	+·0752	−·0779	−·0386	−·1787	−·0319	+·0154	+·1118
10	−·0429	+·0891	−·0025	−·0043	+·1382	−·1649	+·0016	+·0188	0	+·3757	−·0554	−·0453	+·0431	+·1491	−·0755	+·0428	−·0145	+·0780	−·0752	−·0282	+·0319	−·0154	−·1118	−·0779	−·0386	−·1787
11	+·0291	−·0093	+·0016	−·0006	−·0403	+·0673	−·0179	+·0048	−·0453	−·0554	+·0440	0	−·0022	−·0497	+·0220	−·0107	−·0275	+·0030	+·0142	−·0036	−·0322	+·0022	+·0704	+·0047	+·0001	−·0098
12	+·0093	+·0291	+·0006	+·0016	−·0673	−·0403	−·0048	−·0179	+·0554	−·0453	0	+·0441	+·0497	−·0022	+·0107	+·0220	−·0030	−·0275	+·0036	+·0142	−·0047	−·0001	+·0098	−·0322	+·0022	+·0704
13	+·2729	+·1727	−·0152	+·0442	−·3510	−·3110	+·0371	−·0694	+·1491	+·0431	−·0022	+·0497	+2·0397	0	−·3824	+·9573	−·5456	+·4101	−·1315	−·0738	−·1259	−·3757	+·1043	−·1645	−·1210	−·1172
14	−·1727	+·2729	−·0442	−·0152	+·3110	−·3510	+·0694	+·0371	−·0431	+·1491	−·0497	−·0022	0	+2·0397	−·9573	−·3824	−·4101	−·5456	+·0738	−·1315	+·1645	+·1210	+·1172	−·1259	−·3757	+·1043
15	+·0453	−·1134	+·0204	+·0010	−·2301	+·0348	−·0070	−·0368	+·0428	−·0755	+·0220	+·0107	−·3824	−·9573	+·7276	0	+·2455	+·0556	−·0105	+·0555	−·0597	+·0223	−·0838	+·0505	+·2298	+·0400
16	+·1134	+·0453	−·0010	+·0204	−·0348	−·2301	+·0368	−·0070	+·0755	+·0428	−·0107	+·0220	+·9573	−·3824	0	+·7276	−·0556	+·2455	−·0555	−·0105	−·0505	−·2298	−·0400	−·0597	+·0223	−·0838
17	+·0728	−·1102	+·0133	+·0122	+·1760	+·0774	−·0032	+·0450	+·0780	−·0145	−·0275	−·0030	−·5456	−·4101	+·2455	−·0556	+2·0006	0	+·0666	+·6628	−·0468	−·2010	−·4847	+·0599	+·0998	+·2668
18	+·1102	+·0728	−·0122	+·0133	−·0774	+·1760	−·0450	−·0032	+·0145	+·0780	+·0030	−·0275	+·4101	−·5456	+·0556	+·2455	0	+2·0006	−·6628	+·0666	−·0599	−·0998	−·2668	−·0468	−·2010	−·4847
19	−·0198	−·0612	+·0071	−·0031	+·0383	+·0160	+·0064	+·0089	−·0282	−·0752	+·0142	+·0036	−·1315	+·0738	−·0105	−·0555	+·0666	−·6628	+·3468	0	+·0039	+·0146	+·1510	+·0427	+·0836	+·0752
20	+·0612	−·0198	+·0031	+·0071	−·0160	+·0383	−·0089	+·0064	+·0752	−·0282	−·0036	+·0142	−·0738	−·1315	+·0555	−·0105	+·6628	+·0666	0	+·3468	−·0427	−·0836	−·0752	+·0039	+·0146	+·1510
21	−·7785	+·2119	−·0768	+·0254	+·1241	−·0883	+·0307	−·0194	−·0779	+·0319	−·0322	−·0047	−·1259	+·1645	−·0597	−·0505	−·0468	−·0599	+·0039	−·0427	+1·3771	+·2934	+·0036	+·0003	−·0711	+·0038
22	−·5966	+·0559	−·0127	−·1039	−·0431	+·0434	−·0085	−·0219	−·0386	−·0154	+·0022	−·0001	−·3757	+·1210	+·0223	−·2298	−·2010	−·0998	+·0146	−·0836	+·2934	+1·1294	+·0587	+·0716	+·0002	+·0064
23	−·0138	−·0022	+·0027	−·0026	−·0649	+·0963	−·0271	−·0182	−·1787	−·1118	+·0704	+·0098	+·1043	+·1172	−·0838	−·0400	−·4847	−·2668	+·1510	−·0752	+·0036	+·0587	+1·1247	−·0033	−·0062	0
24	−·2119	−·7785	−·0254	−·0768	+·0883	+·1241	+·0194	+·0307	−·0319	−·0779	+·0047	−·0322	−·1645	−·1259	+·0505	−·0597	+·0599	−·0468	+·0427	+·0039	+·0003	+·0716	−·0033	+1·3771	+·2934	+·0036
25	−·0559	−·5966	+·1039	−·0127	−·0434	−·0431	+·0219	−·0085	+·0154	−·0386	+·0001	+·0022	−·1210	−·3757	+·2298	+·0223	+·0998	−·2010	+·0836	+·0146	−·0711	+·0002	−·0062	+·2934	+1·1294	+·0587
26	+·0022	−·0138	+·0026	+·0027	−·0963	−·0649	+·0182	−·0271	+·1118	−·1787	−·0098	+·0704	−·1172	+·1043	+·0400	−·0838	+·2668	−·4847	+·0752	+·1510	+·0038	+·0064	0	+·0036	+·0587	+1·1247
Sum	+·8762	+·4540	−·2619	+·3085	+·4299	−·1453	+·0201	+·1497	+·3940	+·0372	−·0411	+·0774	+1·4938	−·1466	−·3527	+1·0575	+1·9745	+·7135	−·0951	+1·0129	+·7362	−·0116	+·3594	+·5354	+·5015	+·9661

TABLE LXXIX. Verification of solution in Table *LXXVIII.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	+3·2069	0	+1·9276	−9·0950	+·6133	0	+·3592	−2·0153													+1·9189	+·3680	0	0	0	0
2	0	+1·6616	+4·7125	+·9988	0	+·3178	+1·0442	+·1861													0	0	0	+·9943	+·1907	0
3	−·5763	−2·7185	−10·6541	0	+·0079	−·4636	−1·9250	−·2986													−·4871	−·0576	0	−1·7338	−·0471	0
4	−3·2022	+·6787	0	+12·5498	−·5460	−·0093	−·3517	+2·2675													−2·0423	−·0555	0	+·5738	+·0679	0
5	+·3009	0	−·0129	−·7609	+4·1743	0	−·3611	−12·1318	+2·7815	0	+6·9945	−2·7170	+·8555	0	+·7953	−1·1134					0	+·1333	0	0	0	0
6	0	−·1017	−·2572	+·0044	0	−1·4109	−4·1004	+·1221	0	−·9401	−·9183	−2·3640	0	−·2891	−·3763	−·2688					0	0	0	0	−·0450	0
7	+·0082	+·0462	+·1477	−·0229	−·0169	+·5672	+2·2414	0	−·0265	+·4617	+·5954	+1·1879	−·0016	+·0275	+·0406	+·0273					0	+·0020	0	0	+·0024	0
8	−·3443	+·0614	+·1707	+1·1003	−4·2245	−·1257	0	+16·6930	−3·4386	−·1976	−8·8473	+4·4341	−·2051	−·0120	−·2036	+·3024					0	−·0180	0	0	+·0150	0
9					+2·5492	0	−·5201	−9·0502	+3·8139	0	+6·6231	−4·4640	+·1103	0	+·0473	−·2364	+·2167	0	−·1064	−·6540	0	0	+·2679	0	0	0
10					0	+·2407	+·8545	−·0491	0	+·3601	+·4215	+·6253	0	+·0104	+·0223	+·0045	0	+·0205	+·0618	−·0100	0	0	0	0	0	+·0253
11					−·6687	−·2598	−1·2174	+2·4291	−·6909	−·4657	−3·2592	0	−·0251	+·0115	+·0132	+·0592	−·0197	+·0086	+·0292	+·0559	0	0	+·0386	0	0	−·0279
12					−·4892	+1·2593	+4·5749	+2·2926	−·8769	+1·3011	0	+6·1378	+·0217	+·0472	+·1115	−·0248	+·0163	+·0372	+·1053	−·0550	0	0	−·0526	0	0	−·0728
13					+2·9727	0	−·1195	−2·0465	+·4183	0	+·9112	+·4183	+5·1237	0	+3·1818	−6·3935	+1·2399	0	+·4780	−1·7926	0	+·5079	0	0	0	0
14					0	−·2917	−·2008	+·0117	0	−·0410	+·0410	−·0894	0	−·5028	−·6274	−·3128	0	−·1217	−·1759	−·0469	0	0	0	0	−·0498	0
15					−·6525	−·9135	−·7125	+·4797	−·0423	−·2116	+·1129	−·5079	−·7513	−1·5096	−2·9838	0	−·0035	−·3492	−·6454	−·1340	0	−·0529	0	0	−·0353	0
16					−2·7389	+1·9564	+1·4382	+2·1362	−·6345	+·1269	−1·5228	−·3384	−4·5261	+2·2525	0	+8·9465	−1·0469	+·0106	−·4019	+1·9352	0	−·1058	0	0	+·1586	0
17									+1·0860	0	+·9478	+·4146	+1·6388	0	+·0197	−1·9548	+4·4081	0	−1·1057	−7·9967	0	+·3159	+1·3822	0	0	0
18									0	+·3924	−·1498	+·3425	0	+·5922	+·7064	+·0071	0	+1·5911	+2·8897	−·8996	0	0	0	0	+·1142	+·4995
19									+·0257	−·1579	+·0675	−·1293	−·0304	−·1141	−·1740	+·0361	+·0533	−·3852	−1·0423	0	0	−·0019	+·0599	0	−·0019	−·1113
20									−1·6814	−·2735	−1·3775	−·7192	−1·2155	+·3241	+·3849	+1·8536	−4·1022	−·5672	0	+11·1014	0	−·0203	−1·1851	0	+·0203	−·6381
21	+1·6123	0	+1·3693	−4·8736	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1·6123	0	0	0	0	0
22	−·0049	0	−·0026	+·0021	−·0036	0	−·0012	+·0014	0	0	0	0	−·0039	0	−·0017	+·0012	−·0019	0	−·0002	+·0002	0	−·0142	0	0	0	0
23	0	0	0	0	0	0	0	0	+·2443	0	−·3377	−·2443	0	0	0	0	+·2515	0	−·2264	−·4204	0	0	+·4922	0	0	0
24	0	+1·1725	+3·5443	+·9958	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1·1725	0	0
25	0	+·2106	+·0903	+·1103	0	+·1555	+·0602	+·0502	0	0	0	0	0	+·1705	+·0502	+·0752	0	+·0802	+·0100	+·0100	0	0	0	0	+·6118	0
26	0	0	0	0	0	0	0	0	0	+·6570	+·6570	−·9082	0	0	0	0	0	+·6763	+1·1805	−·6087	0	0	0	0	0	+1·3237
Sum	+1·0007	+1·0108	+1·0356	+1·0051	+·9771	+1·0224	+1·0623	+1·0781	+·9786	+1·0118	+0·9593	+1·0788	+·9910	+1·0083	+1·0064	+1·0091	+1·0066	+1·0012	+1·0003	+·9848	+1·0018	+1·0009	+1·0029	+1·0068	+1·0018	+·9984

Probable errors after adjustment.

20. Having obtained the solution of the normal equations corresponding to either 20, 23 or 26 conditions it is now possible to determine the probable errors of side, azimuth, easting and northing as explained briefly at the end of Chapter VII. This is first done for the point U_1 for which the necessary quantities forming the R. H. S. of the normal equations have already been found and given in table *L*. The computations are given in full for this point: after which other points are considered in less detail. It is necessary to form the quantities $[uff]$ and $[uff] - k_1\,[uaf] - k_2\,[ubf] - \ldots$ which may be denoted by u_f and u_F respectively [*vide* Chapter VII, equation (14)]. These are the reciprocal weights before and after adjustment and their square roots when multiplied by 33·2, 1·575, 4·08, as the case may be, give the probable errors in 7th place of log side, azimuth (in seconds), easting or northing (in feet) as explained in § 8 of Chapter VII. The factor $k = \sqrt{\frac{u_F}{u_f}}$ shows the improvement (or otherwise) caused by the adjustment.

Side closure at U_1

$$[uff] = (u\,1f) + (u\,2f) = 2\cdot61 + 2\cdot30 = 4\cdot91$$
$$[uaf] = 2\cdot61$$
$$[ubf] = 0$$

etc. as given in Table *L*.

For 20 conditions

R.H.S. of normal equation (13) are 2·61 0

$$k_r = 2\cdot61\,{}_1k_r + 2\cdot08\,{}_3k_r - 6\cdot80\,{}_4k_r + 2\cdot30\,{}_5k_r + \cdot02\,{}_7k_r - 1\cdot49\,{}_8k_r + 1\cdot99\,{}_{13}k_r + 1\cdot85\,{}_{15}k_r - 2\cdot59\,{}_{16}k_r.$$

and this is required for values of r 1, 3, 4, 5, 7, 8, 13, 15, 16, ${}_sk_r$ being taken from table *LXV*.

Putting in the coefficients of 2·61 etc. from table *LXV* the computation of k_r stands as follows :—

TABLE LXXX.

	r	1	3	4	5	7	8	13	15	16
s	[usf]	Values of $[usf]\,{}_sk_r$								
1	+2·61	+3·013	− ·165	+ ·760	− ·347	− ·020	− ·035	+ ·097	+ ·112	− ·019
3	+2·08	− ·131	+ ·218	0	+ ·028	− ·013	+ ·000	− ·032	− ·007	− ·015
4	−6·80	−1·979	0	− ·695	+ ·169	+ ·002	+ ·041	− ·028	− ·050	+ ·022
5	+2·30	− ·306	+ ·030	− ·057	+1·896	+ ·048	+ ·386	− 828	− ·499	− ·111
7	+ ·02	·000	·000	·000	·000	+ ·001	0	+ 001	·000	+ ·001
8	−1·49	+ ·020	·000	+ ·009	− ·250	0	− ·085	+ ·113	+ ·053	+ ·018
13	+1·99	+ ·074	− ·030	+ ·008	− ·716	+ ·086	− ·151	+3·716	− ·678	+1·740
15	+1·85	+ ·080	− ·006	+ ·014	− ·401	− ·023	− ·066	− ·630	+1·242	0
16	−2·59	+ ·019	+ ·019	+ ·008	+ ·125	− ·092	+ ·032	−2 264	0	−1·739
Sum = k_r		+0·790	+0·061	+0·047	+0·504	−0·011	+0·122	+0·145	+0 173	−0·103
Multiplier = [urf]		+2·61	+2·08	−6·80	+2·30	+ ·02	−1·49	+1·99	+1·85	−2 59
$[urf]k_r$		+2·062	+ ·127	− ·320	+1·159	·000	− ·182	+ ·289	+ ·320	+ ·267

$u_f = 4\cdot91 \quad u_F = v_f - \Sigma\,[urf]\,k_r = 4\cdot91 - 3\cdot72 = 1\cdot19 \quad K = \sqrt{\frac{u_F}{u_f}} = \cdot49$

TABLE LXXXI.

Values of ${}_{s}k_r$ from Table *LXXI.*

For 23 conditions ***Side closure***

s	r [usf]	1	3	4	5	7	8	13	15	16	21	22
		Values of $[usf]\,{}_{s}k_r$										
1	+2·61	+4·603	− ·066	+ ·831	− ·451	− ·050	+ ·014	+ ·645	+ ·144	+ ·274	−2·037	−1·530
3	+2·08	− ·053	+ ·220	− ·004	+ ·014	− ·016	+ ·002	− ·020	− ·002	− ·012	− ·145	− ·021
4	−6·80	−2·164	+ ·012	− ·770	+ ·105	− ·013	+ ·031	− ·242	− ·021	− ·116	− ·173	+ ·679
5	+2·30	− ·398	+ ·015	− ·036	+1·940	+ ·060	+ ·387	− ·816	− ·502	− ·083	+ ·277	− ·110
7	+0·02	·000	·000	·000	+ ·001	+ ·001	·000	+ ·001	·000	+ ·001	+ ·001	·000
8	−1·49	− ·008	− ·002	+ ·007	− ·251	− ·000	− ·087	+ ·104	+ ·051	+ ·011	+ ·030	+ ·035
13	+1·99	+ ·492	− ·019	+ ·071	− ·706	+ ·085	− ·138	+3·984	− ·708	+1·880	− ·261	− ·732
15	+1·85	+ ·102	− ·002	+ ·006	− ·404	− ·022	− ·063	− ·658	+1·258	− ·005	− ·083	+ ·042
16	−2·59	− ·272	+ ·015	− ·044	+ ·093	− ·099	+ ·019	−2·447	+ ·008	−1·858	+ ·125	+ ·585
21	+2·19	−1·709	− ·153	+ ·056	+ ·264	+ ·069	− ·044	− ·288	− ·099	− ·105	+3·007	+ ·641
22	+0·73	− ·428	− ·007	− ·078	− ·035	− ·007	− ·017	− ·269	+ ·016	− ·165	+ ·214	+ ·822
Sum = k_r		+ ·165	+ ·013	+ ·044	+ ·570	+ ·008	+ ·104	− ·006	+ ·145	− ·178	+ ·955	+ ·411
Multiplier = $[urf]$		+2·61	+2·08	−6 80	+2·30	+0·02	−1·49	+1·99	+1·85	−2·59	+2·19	+0·73
$[urf]k_r$		+ ·431	+ ·027	− ·299	+1·311	·000	− ·155	− ·012	+ ·268	+ ·461	+2·091	+ ·300

$u_f = 4\cdot91$ $\qquad u_F = u_f - \Sigma\,[urf]\,k_r = 4\cdot91 - 4\cdot42 = \cdot49$ $\qquad K = \sqrt{\dfrac{u_F}{u_f}} = \cdot32$

Azimuth closure at U_1

$[uff] = 2\cdot61 + 2\cdot30 = 4\cdot91$

$$k_r = 2\cdot61\ {}_{2}k_r\ + 6\cdot80\ {}_{3}k_r\ + 2\cdot08\ {}_{4}k_r\ + 2\cdot30\ {}_{6}k_r\ + 1\cdot49\ {}_{7}k_r\ + 0\cdot02\ {}_{8}k_r\ + 1\cdot99\ {}_{14}k_r$$
$$+ 2\cdot59\ {}_{15}k_r\ + 1\cdot85\ {}_{16}k_r.$$

This holds for either 20 or 23 conditions. But the values of ${}_{s}k_r$ are different in the two cases. Values of r required are 2, 3, 4, 6 . . . . 16.

For 20 conditions.

Comparing terms in k_1 side closure and k_2 azimuth closure

$$\begin{aligned} 2\cdot61\ {}_{1}k_1 &= 2\cdot61\ {}_{2}k_2 \quad \text{since } {}_{1}k_1 = {}_{2}k_2 \\ 2\cdot08\ {}_{3}k_1 &= 2\cdot08\ {}_{4}k_2 \quad ,,\ \ {}_{3}k_1 = {}_{4}k_2 \\ -6\cdot80\ {}_{4}k_1 &= 6\cdot80\ {}_{3}k_2 \quad ,,\ \ {}_{4}k_1 = -\,{}_{3}k_2 \\ 2\cdot30\ {}_{5}k_1 &= 2\cdot30\ {}_{6}k_2 \quad ,,\ \ {}_{5}k_1 = {}_{6}k_2 \end{aligned}$$

etc.

so that k_2 (azimuth) is same as k_1 (side).

As regards k (side) and k_4 (azimuth)

$$2\cdot61\ {}_{1}k_3 = 2\cdot61\ {}_{2}k_4$$

etc.

and k_4 (azimuth) = k_3 (side).

For k_4 (side) and k_3 (azimuth)

$$2\cdot61\ {}_{1}k_4 = -\ 2\cdot61\ {}_{2}k_3$$

so that k_3 (azimuth) = k_4 (side)

etc.

No further computation in this case is necessary; and the multipliers being similarly related numerically and with regard to sign the same value of k holds for azimuth as for side.

For 23 conditions.—The symmetry is lost by the introduction of base-line conditions without corresponding Laplace conditions and the values of k_2, k_3, k_4, k_6 . . . k_{16} have to be computed.

TABLE LXXXII.

For 23 conditions *Azimuth closure*

s \ r	$[usf]$	2	3	4	6	7	8	14	15	16
		Values of $[usf]\,{}_s\mathrm{k}_r$								
2	+2·61	+3·086	− ·784	− ·169	− ·374	+ ·045	− ·028	+ ·155	+ ·009	+ ·069
3	+6·80	−2·042	+ ·720	− ·012	+ ·205	− ·053	+ ·007	− ·071	− ·007	− ·040
4	+2·08	− ·135	− ·004	+ ·236	+ ·013	+ ·004	− ·010	− ·045	+ ·006	+ ·036
6	+2·30	− ·330	+ ·069	+ ·014	+1·940	− ·398	+ ·046	− ·819	+ ·106	− ·523
7	+1·49	+ ·025	− ·012	+ ·003	− ·258	+ ·088	·000	+ ·112	− ·018	+ ·057
8	+0·02	·000	·000	·000	·000	·000	+ ·001	+ ·001	− ·001	− ·001
14	+1·99	+ ·118	− ·209	− ·043	− ·708	+ ·150	+ ·075	+3·790	−1·760	− ·732
15	+2·59	+ ·009	− ·003	+ ·008	+ ·119	− ·031	− ·089	−2·290	+1·765	− ·008
16	+1·85	+ ·049	− ·011	+ ·032	− ·420	+ ·071	− ·014	− ·680	− ·005	+1·329
Sum = k_r		+ ·780	− ·234	+ ·069	+ ·517	− ·124	− ·012	+ ·153	+ ·095	+ ·187
Multiplier = $[urf]$		+2·61	+6·80	+2·08	+2·30	+1·49	+0·02	+1·99	+2·59	+1·85
$[urf]\mathrm{k}_r$		+2·039	−1·591	+ ·144	+1·189	− ·185	·000	+ ·305	+ ·246	+ ·346

$u_f = 4 \cdot 91 \quad u_F = u_f - \Sigma[urf]\mathrm{k}_r = 4 \cdot 91 - 2 \cdot 49 = 2 \cdot 42 \quad \mathrm{K} = \sqrt{\frac{u_F}{u_f}} = \cdot 70$

TABLE LXXXIII.

Values of ${}_s\mathrm{k}_r$ from Table *LXXVIII.*

For 26 conditions *Side closure*

s \ r	$[usf]$	1	3	4	5	7	8	13	15	16	21	22
		Values of $[usf]\,{}_s\mathrm{k}_r$										
1	+2·61	+4·688	− ·059	+ ·861	− ·486	− ·058	+ ·002	+ ·712	+ ·118	+ ·296	−2·032	−1·557
3	+2·08	− ·047	+ ·244	0	− ·002	− ·014	− ·002	− ·032	+ ·042	− ·002	− ·161	− ·026
4	−6·80	−2·244	0	− ·799	+ ·141	− ·006	+ ·044	− ·301	− ·007	− ·139	− ·173	+ ·706
5	+2·30	− ·428	− ·002	− ·048	+1·979	+ ·057	+ ·399	− ·808	− ·529	− ·080	+ ·285	− ·099
7	+ ·02	0	0	0	+ ·001	+ ·001	0	+ ·001	0	+ ·001	+ ·001	0
8	−1·49	− ·001	+ ·001	+ ·010	− ·258	0	− ·089	+ ·103	+ ·055	+ ·010	+ ·029	+ ·033
13	+1·99	+ ·543	− ·032	+ ·088	− ·698	+ ·074	− ·138	+4·059	− ·761	+1·905	− ·251	− ·748
15	+1·85	+ ·084	+ ·038	+ ·002	− ·426	− ·013	− ·068	− ·707	+1·346	0	− ·110	+ ·041
16	−2·59	− ·294	+ ·003	− ·053	+ ·090	− ·095	+ ·018	−2·479	0	−1·884	+ ·131	+ ·595
21	+2·19	−1·705	− ·168	+ ·056	+ ·272	+ ·067	− ·042	− ·276	− ·131	− ·111	+3·016	+ ·643
22	+0·73	− ·436	− ·009	− ·076	− ·031	− ·006	− ·016	− ·274	+ ·016	− ·168	+ ·214	+ ·824
Sum = k_r		+0·160	+ ·016	+ ·041	+ ·582	+ ·007	+ ·108	− ·002	+ ·149	− ·172	+ ·949	+ ·412
Multiplier = $[urf]\mathrm{k}_r$		+2·61	+ 2·08	−6·80	+2·30	+ ·02	−1·49	+1·99	+ 1·85	−2·59	+2·19	+0·73
$[urf]\mathrm{k}_r$		+ ·418	+ ·033	− ·279	+1·339	·000	− ·161	− ·004	+ ·276	+ ·445	+2·078	+ ·301

$u_f = 4{\cdot}91 \quad u_F = u_f - \Sigma[urf]\mathrm{k}_r = 4 \cdot 91 - 4{\cdot}45 = {\cdot}46 \quad \mathrm{K} = \sqrt{\frac{u_F}{u_f}} = \cdot 31$

No special explanation is required for tables *LXXXIV-LXXXVII,* which are now given.

TABLE LXXXIV.

For 20 conditions — *At U_1* — *Easting closure*

r		1	2	3	4	5	6	7	8	13	14	15	16
s	[usf]	Values of $[usf]_s k_r$											
1	− ·30	− ·346	0	+ ·019	− ·087	+ ·040	− ·016	+ ·002	+ ·004	− ·011	+ ·024	− ·013	+ ·002
2	− 6·72	0	−7·758	+ 1·956	+ ·425	+ ·367	+ ·803	− ·090	+ ·050	− ·545	− ·250	− ·049	− ·289
3	−11·83	+ ·747	+3·443	− 1·210	0	− ·156	− ·204	+ ·071	− ·003	+ ·180	+ ·048	+ ·038	+ ·086
4	− 4·59	−1·336	+ ·290	0	− ·469	+ ·114	− ·061	+ ·001	+ ·028	− ·019	+ ·070	− ·034	+ ·015
5	− 3·51	+ ·467	+ ·191	− ·046	+ ·087	−2·893	0	− ·073	− ·589	+1·264	− 1·084	+ ·761	+ ·169
6	−12·15	− ·663	+1·615	− ·302	− ·161	0	−10·016	+2·040	− ·254	+3·754	+ 4·374	− ·586	+2·634
7	− 7·53	+ ·056	− ·101	+ ·045	+ ·002	− ·157	+ 1·264	− ·430	0	− ·326	− ·571	+ ·093	− ·268
8	+ 2·18	− ·029	− ·016	+ ·001	− ·013	+ ·366	+ ·046	0	+ ·125	− ·165	+ ·004	− ·078	− ·027
13	− 3·13	− ·117	− ·254	+ ·048	− ·013	+1·127	+ ·967	− ·136	+ ·237	−5·845	0	+ 1·066	−2·736
14	−10·44	+ ·846	− ·389	+ ·042	+ ·159	−3·226	+ 3·759	− ·791	− ·452	0	−10·497	+ 9·120	+3·557
15	−16·22	− ·699	− ·117	+ ·053	− ·118	+3·516	− ·782	+ ·200	+ ·578	+5·526	+14·179	−10·888	0
16	− 5·64	+ ·041	− ·243	+ ·041	+ ·018	+ ·272	+ 1·223	− ·201	+ ·070	−4·930	+ 1·922	0	−3·786
Sum = k_r		−1·033	−3·339	+ ·647	− ·170	− ·630	− 3·017	+ ·593	− ·206	−1·117	− ·691	− ·564	− ·643
Multiplier = [urf]		− ·30	− 6·72	−11·83	−4·59	−3·51	−12·15	−7·53	+2·18	−3·13	−10·44	−16·22	−5·64
$[urf]k_r$		+ ·310	+22·438	− 7·654	+ ·780	+2·211	+36·657	−4·465	− ·449	+3·496	+ 7·214	+ 9·148	+3·627

$u_f = 93{\cdot}29 \qquad u_F = u_f - \Sigma[urf]k_r = 93{\cdot}29 - 73{\cdot}31 = 19{\cdot}98 \qquad K = \sqrt{\dfrac{u_F}{u_f}} = {\cdot}46$

TABLE LXXXV.

For 23 conditions — *At U_1* — *Easting closure*

r		1	2	3	4	5	6	7	8	13	14	15	16	21	22
s	[usf]	Values of $[usf]_s k_r$													
1	− ·30	− ·529	+ ·037	+ ·008	− ·005	+ ·052	− ·026	+ ·006	− ·002	− ·074	+ ·059	− ·017	− ·032	+ ·234	+ ·176
2	− 6·72	+ ·835	− 7·939	+ 2·017	+ ·436	+ ·278	+ ·964	− ·115	+ ·073	− ·310	− ·399	− ·024	− ·179	−1·230	− ·604
3	−11·83	+ ·300	+ 3·551	− 1·253	+ ·021	− ·078	− ·356	+ ·092	− ·013	+ ·115	+ ·124	+ ·012	+ ·070	+ ·827	+ ·118
4	− 4·59	−1·461	+ ·298	+ ·008	− ·520	+ ·071	− ·028	− ·009	+ ·021	− ·163	+ ·098	− ·014	− ·078	− ·117	+ ·459
5	− 3·51	+ ·607	+ ·145	− ·023	+ ·054	−2·960	+ ·060	− ·092	− ·591	+1·245	− 1·084	+ ·766	+ ·126	− ·423	+ ·168
6	−12·15	−1·046	+ 1·744	− ·366	− ·075	+ ·208	−10·221	+2·102	− ·241	+3·741	+ 4·321	− ·559	+2·760	+1·129	− ·440
7	− 7·53	+ ·145	− ·129	+ ·059	− ·014	− ·197	+ 1·303	− ·442	− ·002	− ·321	− ·566	+ ·090	− ·289	− ·238	+ ·071
8	+ 2·18	+ ·012	− ·024	+ ·002	− ·010	+ ·367	+ ·043	0	+ ·127	− ·152	+ ·083	− ·075	− ·016	− ·044	− ·051
13	− 3·13	− ·774	− ·145	+ ·080	− ·111	+1·110	+ ·964	− ·133	+ ·218	−6·266	+ ·096	+ 1·114	−2·957	+ ·411	+1·152
14	−10·44	+2·046	− ·620	+ ·110	+ ·223	−3·225	+ 3·712	− ·784	− ·396	+ ·322	−19·841	+ 9·231	+3·831	−1·464	−1·271
15	−16·22	− ·895	− ·057	+ ·016	− ·050	+3·541	− ·746	+ ·193	+ ·556	+5·773	+14·342	−11·031	+ ·047	+ ·732	− ·367
16	− 5·64	− ·592	− ·150	+ ·033	− ·096	+ ·203	+ 1·281	− ·217	+ ·041	−5·328	+ 2·070	+ ·016	−4·046	+ ·271	+1·274
21	− ·14	+ ·109	− ·026	+ ·011	− ·004	− ·017	+ ·013	− ·004	+ ·003	+ ·018	− ·020	+ ·006	+ ·007	− ·192	− ·041
22	− ·53	+ ·311	− ·048	+ ·005	+ ·053	+ ·025	− ·019	+ ·005	+ ·013	+ ·195	− ·065	− ·012	+ ·120	− ·155	− ·597
Sum = k_r		− ·932	− 3·363	+ ·657	− ·188	− ·522	− 3·056	+ ·602	− ·193	−1·205	− ·782	− ·497	− ·636	− ·259	+ ·047
Multiplier = [urf]		− ·30	− 6·72	−11·83	−4·59	−3·51	−12·15	−7·53	+2·18	−3·13	−10·44	−16·22	−5·64	− ·14	− ·53
$[urf]k_r$		+ ·280	+22·599	− 7·772	+ ·863	+1·832	+37·130	−4·533	− ·421	+3·772	+ 8·164	+ 8·061	+3·587	+ ·036	− ·025

$u_f = 93{\cdot}29 \qquad u_F = u_f - \Sigma[urf]k_r = 93{\cdot}29 - 73{\cdot}57 = 19{\cdot}72 \qquad K = \sqrt{\dfrac{u_F}{u_f}} = {\cdot}46$

TABLE LXXXVI.

For 23 conditions — *At U_1* — *Northing closure*

s \ r	$[usf]$	1	2	3	4	5	6	7	8	13	14	15	16	21	22
		Values of $[usf]\,_s\mathrm{k}_r$													
1	+ 6·72	+11·852	− ·835	− ·171	+ 2·138	− 1·163	+ ·579	− ·130	+ ·037	+ 1·661	− ·132	+ ·371	+ ·071	− 5·244	−3·939
2	− ·30	+ ·087	− ·354	+ ·090	+ ·019	+ ·012	+ ·043	− ·005	+ ·003	− ·014	− ·018	− ·001	− ·008	− ·055	− ·027
3	+ 4·59	− ·117	−1·378	+ ·486	− ·008	+ ·030	+ ·138	− ·036	+ ·005	− ·045	− ·048	− ·005	− ·027	− ·321	− ·046
4	− 11·83	− 3·764	+ ·768	+ ·021	− 1·339	+ ·183	− ·073	− ·022	+ ·054	− ·421	+ ·253	− ·037	− ·202	− ·300	+1·182
5	+ 12·15	− 2·102	− ·503	+ ·080	− ·188	+10·247	− ·208	+ ·318	+2·046	− 4·310	+3·753	−2·652	− ·437	+ 1·465	− ·582
6	− 3·51	− ·302	+ ·504	− ·106	− ·022	+ ·060	−2·953	+ ·607	− ·069	+ 1·081	+1·248	− ·161	+ ·797	+ ·326	− ·127
7	− 2·18	+ ·042	− ·037	+ ·017	− ·004	− ·057	+ ·377	− ·128	·000	− ·093	− ·164	+ ·026	− ·084	− ·069	+ ·020
8	− 7·53	− ·041	+ ·082	− ·008	+ ·035	− 1·268	− ·149	− ·002	− ·437	+ ·523	− ·285	+ ·258	+ ·055	+ ·151	+ ·178
13	+ 10·44	+ 2·581	+ ·482	− ·101	+ ·372	− 3·703	−3·214	+ ·445	− ·726	+20·901	− ·322	−3·716	+ 9·862	− 1·371	−3·841
14	− 3·13	+ ·613	− ·186	+ ·033	+ ·067	− ·967	+1·113	− ·235	− ·119	+ ·096	−5·949	+2·768	+ 1·149	− ·439	− ·381
15	+ 5·64	+ ·311	+ ·020	− ·006	+ ·017	− 1·231	+ ·259	− ·067	− ·193	− 2·007	−4·987	+3·836	− ·016	− ·254	+ ·127
16	− 16·22	− 1·703	− ·431	+ ·096	− ·277	+ ·584	+3·685	− ·623	+ ·118	−15·321	+5·953	+ ·047	−11·636	+ ·780	+3·662
21	+ 4·73	− 3·691	+ ·866	− ·331	+ ·120	+ ·570	− ·439	+ ·149	− ·095	− ·621	+ ·663	− ·213	− ·228	+ 6·495	+1·385
22	+ 3·69	− 2·163	+ ·332	− ·037	− ·369	− ·177	+ ·134	− ·035	− ·087	− 1·358	+ ·449	+ ·083	− ·833	+ 1·080	+4·153
Sum$=\mathrm{k}_r$		+ 1·553	− ·670	+ ·063	+ ·561	+ 3·120	− ·708	+ ·236	+ ·537	+ ·072	+ ·414	+ ·604	− 1·537	+ 2·244	+1·764
Multiplier$=[urf]$		+ 6·72	− ·30	+4·59	−11·83	+12·15	−3·51	−2·18	−7·53	+10·44	−3·13	+5·64	−16·22	+ 4·73	+3·69
$[urf]\,\mathrm{k}_r$		+10·436	+ ·201	+ ·289	− 6·637	+37·908	+2·485	− ·514	−4·044	+ ·752	−1·296	+3·407	+24·930	+10·614	+6·509

$u_f = 93·29$ $\qquad u_F = u_f - \Sigma[urf]\ \mathrm{k}_r = 93·29 - 85·04 = 8·25$ $\qquad \mathrm{K} = \sqrt{\dfrac{u_F}{u_f}} = ·30$

TABLE LXXXVII.

For 26 conditions — *At U_1* — *Easting closure*

s \ r	$[usf]$	1	2	3	4	5	6	7	8	13	14	15	16	21	22	24	25
		Values of $[usf]\,_s\mathrm{k}_r$															
1	− ·30	− ·539	0	+ ·007	− ·099	+ ·056	− ·020	+ ·007	·000	− ·082	+ ·052	− ·014	− ·034	+ ·234	+ ·179	+ ·064	+ ·017
2	− 6·72	0	−12·069	+ 2·218	+ ·153	+ ·452	+ 1·251	+ ·006	+ ·151	−1·161	− 1·834	+ ·762	− ·304	−1·424	− ·376	+ 5·232	+4·009
3	−11·83	+ ·270	+ 3·904	− 1·390	0	+ ·012	− ·246	+ ·077	+ ·011	+ ·180	+ ·523	− ·241	+ ·012	+ ·909	+ ·150	+ ·300	−1·229
4	− 4·59	−1·515	+ ·105	0	− ·539	+ ·095	+ ·005	− ·004	+ ·030	− ·203	+ ·070	− ·005	− ·094	− ·117	+ ·477	+ ·353	+ ·058
5	− 3·51	+ ·654	+ ·236	+ ·004	+ ·073	−3·020	0	− ·088	− ·609	+1·232	− 1·092	+ ·808	+ ·122	− ·436	+ ·151	− ·310	+ ·152
6	−12·15	− ·818	+ 2·262	− ·253	+ ·012	0	−10·454	+2·107	− ·304	+3·779	+ 4·265	− ·423	+2·796	+1·073	− ·527	− 1·508	+ ·524
7	− 7·53	+ ·169	+ ·007	+ ·049	− ·007	− ·188	+ 1·306	− ·448	0	− ·279	− ·523	+ ·053	− ·277	− ·231	+ ·064	− ·146	− ·165
8	+ 2·18	+ ·002	− ·049	− ·002	− ·014	+ ·378	+ ·055	0	+ ·130	− ·151	+ ·081	− ·080	− ·015	− ·042	− ·048	+ ·067	− ·019
13	− 3·13	− ·854	− ·541	+ ·048	− ·138	+1·099	+ ·973	− ·116	+ ·217	−6·384	0	+ 1·197	−2·996	+ ·394	+1·176	+ ·515	+ ·379
14	−10·44	+1·803	− 2·849	+ ·461	+ ·159	−3·247	+ 3·664	− ·725	− ·387	0	−21·294	+ 9·994	+3·992	−1·717	−1·263	+ 1·314	+3·922
15	−16·22	− ·735	+ 1·839	− ·331	− ·016	+3·732	− ·564	+ ·114	+ ·597	+6·203	+15·527	−11·802	0	+ ·968	− ·362	− ·819	−3·727
16	− 5·64	− ·640	− ·255	+ ·006	− ·115	+ ·196	+ 1·298	− ·208	+ ·039	−5·399	+ 2·157	0	−4·104	+ ·285	+1·296	+ ·337	− ·126
21	− ·14	+ ·109	− ·030	+ ·018	− ·004	− ·017	+ ·012	− ·004	+ ·003	+ ·018	− ·023	+ ·008	+ ·007	− ·193	− ·041	·000	+ ·010
22	− ·53	+ ·316	− ·030	+ ·007	+ ·055	+ ·023	− ·023	+ ·005	+ ·012	+ ·199	− ·064	− ·012	+ ·122	− ·156	− ·599	− ·038	·000
24	− 4·73	+1·002	+ 3·682	+ ·120	+ ·363	− ·418	− ·587	− ·092	− ·015	+ ·778	+ ·596	− ·239	+ ·282	− ·001	− ·339	− 6·514	−1·388
25	− 3·69	+ ·206	+ 2·201	− ·383	+ ·047	+ ·160	+ ·159	− ·081	+ ·031	+ ·446	+ 1·386	− ·848	− ·082	+ ·262	− ·001	− 1·083	−4·167
Sum$=\mathrm{k}_r$		− ·570	− 1·587	+ ·579	− ·070	− ·687	− 3·171	+ ·550	− ·094	− ·824	− ·173	− ·842	− ·573	− ·192	− ·063	− 2·236	−1·750
Multiplier$=[urf]$		− ·30	− 6·72	−11·83	−4·59	−3·51	−12·15	−7·53	+2·18	−3·13	−10·44	− 16·22	− 5·64	− ·14	− ·53	− 4·73	− 3·69
$[urf]\,\mathrm{k}_r$		+ ·171	+10·665	− 6·850	+ ·321	+2·411	+38·528	−4·142	− ·205	+2·579	+ 1·806	+13·657	+3·232	+ ·027	+ ·033	+10·576	+6·458

$u_f = 93·29$ $\qquad u_F = u_f - \Sigma[urf]\ \mathrm{k}_r = 93·29 - 79·27 = 14·02$ $\qquad \mathrm{K} = \sqrt{\dfrac{u_F}{u_f}} = ·39$

To consider the probable errors at some other points the values of the R.H.S. of the corresponding equations have to be found as was done in table *L* for U_1. The details are given below in table *LXXXVIII.*

TABLE LXXXVIII.

Circuits	Equations	At N_1 computed along route $A_1\ T_1\ S_1\ Q_1\ P_1\ N_1$													
		A_1T_1	T_1S_1	S_1Q_1	Q_1P_1	P_1N_1	Side	Az.	A_1T_1	T_1S_1	S_1Q_1	Q_1P_1	P_1N_1	Easting	Northing
I	1	+·35					+·35	0	−·17					−·17	+·02
	2	0					0	+·35	−·02					−·02	−·17
	3	+·15					+·15	+1·81	−·13					−·13	−·87
	4	−1·81					−1·81	+·15	+·87					+·87	−·13
II	5		+·25				+·25	0		−·34				−·34	+·04
	6		0				0	+·25		−·04				−·04	−·34
	7		+·05				+·05	+1·46		−·30				−·30	−1·95
	8		−1·46				−1·46	+·05		+1·95				+1·95	−·30
III	9			+·53	+·37	+·82	+1·72	0			−1·27	−1·33	−4·32	−6·92	+·95
	10			0	0	0	0	+1·72			−·15	−·19	−·61	−·95	−6·92
	11			+·07	+·24	−·82	+·39	+4·82			−2·69	−1·38	+3·04	−1·03	−19·22
	12			−1·62	−1·05	−2·15	−4·82	+·39			+3·64	+3·66	+11·92	+19·22	−1·03
VIII	23														
	26														

Circuits	Equations	At J_1 along A_1 to N_1 as before and $L_1\ J_1$							
		N_1L_1	L_1J_1	Side	Az.	N_1L_1	L_1J_1	Easting	Northing
I	1			+·35	0			−·17	+·02
	2			0	+·35			−·02	−·17
	3			+·15	+1·81			−·13	−·87
	4			−1·81	+·15			+·87	−·13
II	5			+·25	0			−·34	+·04
	6			0	+·25			−·04	−·34
	7			+·05	+1·46			−·30	−1·95
	8			−1·46	+·05			+1·95	−·30
III	9	+·31	+·37	+2·40	0	−1·93	−1·90	−10·75	+2·55
	10	0	0	0	+2·40	−·52	−1·08	−2·55	−10·75
	11	−·61	−·33	−·55	+5·50	+2·99	+1·36	+3·32	−25·28
	12	−·52	−·16	−5·50	−·55	+4·27	+1·79	+25·28	+3·32
VIII	23	+·31	+·37	+·68	0	−1·93	−1·90	−3·88	+1·60
	26			0	+·68	−·52	−1·08	−1·60	−3·88

Circuits	Equations	At G_1 along A_1 to J_1 as before and $H_1\ G_1$							
		J_1H_1	H_1G_1	Side	Az.	J_1H_1	H_1G_1	Easting	Northing
I	1			+·35	0			−·17	+·02
	2			0	+·35			−·02	−·17
	3			+·15	+1·81			−·13	−·87
	4			−1·81	+·15			+·87	−·13
II	5			+·25	0			−·34	+·04
	6			0	+·25			−·04	−·34
	7			+·05	+1·46			−·30	−1·95
	8			−1·46	+·05			+1·95	−·30
III	9			+2·40	0			−10·75	+2·55
	10			0	+2·40			−2·55	−10·75
	11			−·55	+5·50			+3·32	−25·28
	12			−5·50	−·55			+25·28	+3·32
V	17	+0·38	+·32	−·70	0	−1·53	−1·10	−2·63	+3·57
	18	0	0	0	+·70	−1·63	−1·94	−3·57	−2·63
	19	−·45	−·18	−·63	+1·17	−2·04	−·75	−2·79	−7·65
	20	−·93	−·24	−1·17	−·63	+5·74	+1·91	+7·65	−2·79
VIII	23	+·38	+·32	+1·38	0	−1·53	−1·10	−6·46	+5·17
	26			0	+1·38	−1·63	−1·94	−5·17	−6·46

Circuits	Equations	At C_1 along $A_1\ B_1\ C_1$							
		A_1B_1	B_1C_1	Side	Az.	A_1B_1	B_1C_1	Easting	Northing
I	1	+1·08	+1·11	+2·19	0	−·11	−·03	−·14	+4·73
	2	0	0	0	+2·19	−1·15	−3·58	−4·73	−·14
	3	+·88	+·98	+1·86	+6·62	−4·40	−6·56	−10·96	+3·61
	4	−4·46	−2·16	−6·62	+1·86	−·50	−3·11	−3·61	−10·96
VI	21	+1·08	+1·11	+2·19	0	−·11	−·03	−·14	+4·73
	24	0	0	0	+2·19	−1·15	−3·58	−4·73	−·14

Note.—The values, in columns of Side, Azimuth, Easting and Northing at J_1 and G_1 have been brought from the corresponding columns at N_1 by adding to them the quantities $N_1\ L_1$, $L_1\ J_1$ to find those at J_1; and similarly the quantities J_1H_1, H_1G_1 have been added to the quantities at J_1 to find those at G_1.

The computations of k_r for the following are omitted, and only the values of u_f and k_r for Side, Azimuth, Easting and Northing closures are given in the following Table.

TABLE LXXXIX

C_1

20 conditions—S. closure

r	k_r	$[urf]$	$[urf]k_r$
1	+ ·483	+ 2·19	+ 1·058
3	+ ·052	+ 1·86	+ ·097
4	− ·040	− 6·62	+ ·265

$\Sigma [urf] k_r = + 1·420$
$u_f = 2·19$ $K = ·59$

23 conditions—A. closure

r	k_r	$[urf]$	$[urf]k_r$
2	+ ·479	+ 2·19	+ 1·049
3	+ ·041	+ 6·62	+ ·271
4	+ ·057	+ 1·86	+ ·106

$\Sigma [urf] k_r = + 1·426$
$u_f = 2·19$ $K = ·59$

20 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·521	− ·14	+ ·073
2	−2·042	− 4·73	+ 9·659
3	+ ·265	−10·96	− 2·904
4	− ·111	− 3·61	+ ·401

$\Sigma [urf] k_r = + 7·229$
$u_f = 13·67$ $K = ·69$

23 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·421	− ·14	+ ·059
2	−2·073	− 4·73	+ 9·805
3	+ ·279	−10·96	− 3·058
4	− ·131	− 3·61	+ ·473
21	− ·273	− ·14	+ ·038

$\Sigma [urf] k_r = + 7·317$
$u_f = 13·67$ $K = ·68$

23 conditions—N. closure

r	k_r	$[urf]$	$[urf]k_r$
1	+1·089	+ 4·73	+ 5·151
2	− ·259	− ·14	+ ·036
3	− ·007	+ 3·61	− ·025
4	+ ·387	−10·96	− 4·242
21	+2·048	+ 4·73	+ 9·687

$\Sigma [urf] k_r = + 10·607$
$u_f = 13·67$ $K = ·47$

26 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·081	− ·14	+ ·011
2	−1·144	− 4·73	+ 5·411
3	+ ·407	−10·96	− 4·461
4	− ·003	− 3·61	+ ·011
21	− ·229	− ·14	+ ·032
24	−2·247	− 4·73	+10·628

$\Sigma [urf] k_r = + 11·632$
$u_f = 13·67$ $K = ·39$

G_1

20 conditions—S. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·103	+ ·35	− ·036
3	− ·021	+ ·15	− ·003
4	− ·081	− 1·81	+ ·147
5	+ ·013	+ ·25	+ ·003
7	+ ·072	+ ·05	+ ·004
8	+ ·053	− 1·46	− ·077
9	+ ·452	+ 2·40	+ 1·085
11	− ·122	− ·55	+ ·067
12	− ·103	− 5·50	+ ·567
17	+ ·425	+ ·70	+ ·298
19	− ·149	− ·63	+ ·094
20	+ ·105	− 1·17	− ·123

$\Sigma [urf] k_r = + 2·026$
$u_f = 3·70$ $K = ·67$

G_1—(*Continued*)

23 conditions—A. closure

r	k_r	$[urf]$	$[urf]k_r$
2	− ·125	+ ·35	− ·044
3	+ ·092	+ 1·81	+ ·167
4	− ·018	+ ·15	− ·003
6	+ ·033	+ ·25	+ ·008
7	− ·059	+ 1·46	− ·086
8	+ ·078	+ ·05	+ ·004
10	+ ·435	+ 2·40	+ 1·044
11	+ ·116	+ 5·50	+ ·638
12	− ·118	− ·55	+ ·065
18	+ ·365	+ ·70	+ ·256
19	− ·089	+ 1·17	− ·104
20	− ·164	− ·63	+ ·105

$\Sigma [urf] k_r = + 2·048$
$u_f = 3·70$ $K = ·67$

20 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·066	− ·17	+ ·011
2	− ·095	− ·02	+ ·002
3	+ ·076	− ·13	− ·010
4	+ ·050	+ ·87	+ ·044
5	− ·297	− ·34	+ ·100
6	+ ·491	− ·04	− ·020
7	− ·281	− ·30	+ ·084
8	− ·340	+ 1·95	− ·663
9	−2·002	−10·75	+21·522
10	−1·706	− 2·55	+ 4·350
11	+ ·587	+ 3·32	+ 1·949
12	+ ·607	+25·28	+15·345
17	− ·272	− 2·63	+ ·715
18	−3·529	− 3·57	+12·599
19	+1·226	− 2·79	− 3·421
20	+ ·202	+ 7·65	+ 1·545

$\Sigma [urf] k_r = + 54·152$
$u_f = 87·31$ $K = ·62$

23 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·482	− ·17	+ ·082
2	+ ·018	− ·02	·000
3	+ ·025	− ·13	− ·003
4	+ ·036	+ ·87	+ ·031
5	− ·075	− ·34	+ ·026
6	+ ·180	− ·04	− ·007
7	− ·193	− ·30	+ ·058
8	− ·297	+ 1·95	− ·579
9	−1·502	−10·75	+16·147
10	−1·358	− 2·55	+ 3·463
11	+ ·361	+ 3·32	+ 1·199
12	+ ·576	+25·28	+14·561
17	+1·076	− 2·63	− 2·830
18	−2·866	− 3·57	+10·232
19	+ ·678	− 2·79	− 1·892
20	+ ·388	+ 7·65	+ 2·968
23	−3·306	− 6·46	+21·357

$\Sigma [urf] k_r = + 64·813$
$u_f = 87·31$ $K = ·51$

23 conditions—N. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·029	+ ·02	− ·001
2	− ·054	− ·17	+ ·008
3	− ·062	− ·87	+ ·054
4	+ ·089	− ·13	− ·012
5	− ·607	+ ·04	− ·024
6	−1·093	− ·34	+ ·372
7	+ ·284	− 1·95	− ·554
8	− ·328	− ·30	+ ·098
9	+1·231	+ 2·55	+ 3·139
10	−2·268	−10·75	+24·381
11	− ·437	−25·28	+11·047
12	+ ·609	+ 3·32	+ 2·022
17	+2·331	+ 3·57	+ 8·322
18	− ·954	− 2·63	+ 2·509
19	+ ·184	− 7·65	− 1·408
20	+ ·941	− 2·79	− 2·625
23	+2·863	+ 5·17	+14·802

$\Sigma [urf] k_r = + 63·130$
$u_f = 87·31$ $K = ·54$

G_1—(*Continued*)

26 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·446	− ·17	+ ·076
2	+ ·125	− ·02	− ·003
3	+ ·050	− ·13	− ·007
4	+ ·049	+ ·87	+ ·043
5	+ ·139	− ·34	− ·047
6	+ ·296	− ·04	− ·012
7	− ·236	− ·30	+ ·071
8	− ·243	+ 1·95	− ·474
9	−1·775	−10·75	+19·081
10	− ·889	− 2·55	+ 2·267
11	+ ·382	+ 3·32	+ 1·268
12	+ ·402	+25·28	+10·163
17	+ ·430	− 2·63	− 1·131
18	−1·575	− 3·57	+ 5·623
19	+ ·493	− 2·79	− 1·375
20	+ ·002	+ 7·65	+ ·015
23	−4·616	− 6·46	+29·819
26	−2·861	− 5·17	+14·791

$\Sigma [urf] k_r = + 80·168$
$u_f = 87·31$ $K = ·29$

J_1

20 conditions—S. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·085	+ ·35	− ·030
3	− ·024	+ ·15	− ·004
4	− ·079	− 1·81	+ ·143
5	− ·077	+ ·25	− ·019
7	+ ·076	+ ·05	+ ·004
8	+ ·041	− 1·46	− ·060
9	+ ·519	+ 2·40	+ 1·246
11	− ·120	− ·55	+ ·066
12	− ·088	− 5·50	+ ·484

$\Sigma [urf] k_r = + 1·830$
$u_f = 3·00$ $K = ·62$

23 conditions—S. closure

r	k_r	$[urf]$	$[urf]k_r$
1	+ ·136	+ ·35	+ ·048
3	− ·002	+ ·15	·000
4	− ·090	− 1·81	+ ·163
5	− ·128	+ ·25	− ·032
7	+ ·063	+ ·05	+ ·003
8	+ ·043	− 1·46	− ·063
9	+ ·500	+ 2·40	+ 1·200
11	− ·092	− ·55	+ ·051
12	− ·084	− 5·50	+ ·462
23	+ ·254	+ ·68	+ ·173

$\Sigma [urf] k_r = + 2·005$
$u_f = 3·00$ $K = ·58$

23 conditions—A. closure

r	k_r	$[urf]$	$[urf]k_r$
2	− ·106	+ ·35	− ·037
3	+ ·089	+ 1·81	+ ·161
4	− ·026	+ ·15	− ·004
6	− ·060	+ ·25	− ·015
7	− ·046	+ 1·46	− ·067
8	+ ·077	+ ·05	+ ·004
10	+ ·506	+ 2·40	+ 1·214
11	+ ·099	+ 5·50	+ ·545
12	+ ·116	− ·55	+ ·064

$\Sigma [urf] k_r = + 1·865$
$u_f = 3·00$ $K = ·61$

26 conditions—S. closure

r	k_r	$[urf]$	$[urf]k_r$
1	+ ·118	+ ·35	+ ·041
3	− ·001	+ ·15	·000
4	− ·098	− 1·81	+ ·177
5	− ·112	+ ·25	− ·028
7	+ ·061	+ ·05	+ ·003
8	+ ·048	− 1·46	− ·070
9	+ ·487	+ 2·40	+ 1·169
11	− ·092	− ·55	+ ·051
12	− ·094	− 5·50	+ ·517
23	+ ·253	+ ·68	+ ·172

$\Sigma [urf] k_r = + 2·032$
$u_f = 3·00$ $K = ·57$

J_1—(*Continued*)

20 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·064	− ·17	+ ·011
2	− ·222	− ·02	+ ·004
3	+ ·088	− ·13	− ·011
4	+ ·060	+ ·87	+ ·052
5	− ·148	− ·34	+ ·050
6	+1·048	− ·04	− ·042
7	− ·360	− ·30	+ ·108
8	− ·332	+ 1·95	− ·647
9	−2·393	−10·75	+25·725
10	−1·896	− 2·55	+ 4·835
11	+ ·652	+ 3·32	+ 2·165
12	+ ·540	+25·28	+13·651

$\Sigma [urf] k_r = + 45·901$
$u_f = 58·33$ $K = ·46$

23 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·443	− ·17	+ ·075
2	− ·126	− ·02	+ ·003
3	+ ·051	− ·13	− ·007
4	+ ·057	+ ·87	+ ·050
5	− ·017	− ·34	+ ·006
6	+ ·878	− ·04	− ·035
7	− ·312	− ·30	+ ·094
8	− ·315	+ 1·95	− ·614
9	−2·167	−10·75	+23·295
10	−1·721	− 2·55	+ 4·389
11	+ ·534	+ 3·32	+ 1·773
12	+ ·523	+25·28	+13·221
23	−1·631	− 3·83	+ 6·247

$\Sigma [urf] k_r = + 48·497$
$u_f = 58·33$ $K = ·41$

23 conditions—N. closure

r	k_r	$[urf]$	$[urf]k_r$
1	+ ·071	+ ·02	+ ·001
2	− ·038	− ·17	+ ·006
3	− ·074	− ·87	+ ·064
4	+ ·090	− ·13	− ·012
5	−1·070	+ ·04	− ·043
6	− ·100	− ·34	+ ·034
7	+ ·320	− 1·95	− ·624
8	− ·377	− ·30	+ ·113
9	+1·746	+ 2·55	+ 4·452
10	−2·467	−10·75	+26·520
11	− ·494	−25·28	+12·488
12	+ ·657	+ 3·32	+ 2·181
23	+ ·819	+ 1·60	+ 1·310

$\Sigma [urf] k_r = + 46·490$
$u_f = 58·33$ $K = ·45$

26 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·414	− ·17	+ ·070
2	+ ·020	− ·02	·000
3	+ ·053	− ·13	− ·007
4	+ ·068	+ ·87	+ ·059
5	+ ·041	− ·34	− ·014
6	+ ·901	− ·04	− ·036
7	− ·325	− ·30	+ ·098
8	− ·305	+ 1·95	− ·595
9	−2·243	−10·75	+24·112
10	−1·575	− 2·55	+ 4·016
11	+ ·539	+ 3·32	+ 1·789
12	+ ·473	+25·28	+11·957
23	−1·629	− 3·83	+ 6·239
26	− ·819	− 1·60	+ 1·310

$\Sigma [urf] k_r = + 48·998$
$u_f = 58·33$ $K = ·40$

N_1

20 conditions—S. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·092	+ ·35	− ·032
3	− ·017	+ ·15	− ·003
4	− ·079	− 1·81	+ ·143
5	− ·042	+ ·25	− ·010
7	+ ·048	+ ·05	+ ·002
8	+ ·037	− 1·46	− ·054
9	+ ·295	+ 1·72	+ ·507
11	− ·060	+ ·39	− ·023
12	− ·094	− 4·82	+ ·453

$\Sigma [urf] k_r = + ·983$
$u_f = 2·32$ $K = ·76$

N_1—(*Continued*)

23 conditions—A. closure

r	k_r	$[urf]$	$[urf]k_r$
2	− ·114	+ ·35	− ·040
3	+ ·089	+ 1·81	+ ·161
4	− ·019	+ ·15	− ·003
6	− ·021	+ ·25	− ·005
7	− ·043	+ 1·46	− ·063
8	+ ·049	+ ·05	+ ·002
10	+ ·278	+ 1·72	+ ·478
11	+ ·107	+ 4·82	+ ·516
12	− ·057	+ ·39	− ·022

$\Sigma [urf] k_r = + 1·024$
$u_f = 2·32$ $K = ·75$

20 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·080	− ·17	+ ·014
2	− ·082	− ·02	+ ·002
3	+ ·053	− ·13	− ·007
4	+ ·042	+ ·87	+ ·037
5	− ·061	− ·34	− ·021
6	+ ·303	− ·04	− ·012
7	− ·202	− ·30	+ ·061
8	− ·247	+ 1·95	− ·482
9	−1·270	− 6·92	+ 8·788
10	− ·943	− ·95	+ ·896
11	+ ·271	− 1·03	− ·279
12	+ ·435	+19·22	+ 8·361

$\Sigma [urf] k_r = + 17·358$
$u_f = 32·28$ $K = ·68$

23 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·420	− ·17	+ ·071
2	− ·017	− ·02	·000
3	+ ·030	− ·13	− ·004
4	+ ·038	+ ·87	+ ·033
5	− ·107	− ·34	+ ·036
6	+ ·395	− ·04	− ·016
7	− ·229	− ·30	+ ·069
8	− ·279	+ 1·95	− ·544
9	−1·528	− 6·92	+10·574
10	−1·076	− ·95	+ 1·022
11	+ ·348	− 1·03	− ·358
12	+ ·446	+19·22	+ 8·572

$\Sigma [urf] k_r = + 19·455$
$u_f = 32·28$ $K = ·68$

23 conditions—N. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·088	+ ·02	− ·002
2	− ·033	− ·17	+ ·006
3	− ·063	− ·87	+ ·055
4	+ ·061	− ·13	− ·008
5	− ·229	+ ·04	− ·009
6	− ·147	− ·34	+ ·050
7	+ ·274	− 1·95	− ·534
8	− ·194	− ·30	+ ·058
9	+1·038	+ ·95	+ ·986
10	−1·188	− 6·92	+ 8·221
11	− ·490	−19·22	+ 9·418
12	+ ·264	− 1·03	− ·272

$\Sigma [urf] k_r = + 17·969$
$u_f = 32·28$ $K = ·67$

26 conditions—E. closure

r	k_r	$[urf]$	$[urf]k_r$
1	− ·378	− ·17	+ ·063
2	+ ·160	− ·02	− ·003
3	+ ·037	− ·13	− ·005
4	+ ·059	+ ·87	+ ·051
5	− ·189	− ·34	+ ·064
6	+ ·345	− ·04	− ·014
7	− ·220	− ·30	+ ·066
8	− ·306	+ 1·95	− ·597
9	−1·448	− 6·92	+10·020
10	−1·173	− ·95	+ 1·114
11	+ ·340	− 1·03	− ·350
12	+ ·497	+19·22	+ 9·552

$\Sigma [urf] k_r = + 19·958$
$u_f = 32·28$ $K = ·62$

The results obtained in tables *LXXX* to *LXXXIX* are now collected in the following table showing values of u_f and u_F for Side, Azimuth, Easting and Northing closures of several points of N.W.Q., the latter for 20, 23 and 26 conditions.

TABLE XC.

	C_1 or Dehra base				U_1				G_1 or Chach base				J_1				N_1 or Karachi base			
		20	23	26		20	23	26		20	23	26		20	23	26		20	23	26
	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F
Side	2·19	·77	0	0	4·91	1·19	·49	·46	3·70	1·67	0	0	3·00	1·17	1·00	·97	2·32	1·34	0	0
Azimuth	2·19	·77	·76	0	4·91	1·19	2·42	·46	3·70	1·67	1·67	0	3·00	1·17	1·14	·97	2·32	1·34	1·30	0
Easting	13·67	6·44	6·35	2·04	93·29	19·98	19·72	14·02	87·31	33·16	22·50	7·14	58·33	12·43	9·83	9·33	32·28	14·92	12·83	12·32
Northing	13·67	6·44	3·06	2·04	93·29	19·98	8·25	14·02	87·31	33·16	24·18	7·14	58·33	12·43	11·84	9·33	32·28	14·92	14·31	12·32

These values all seem reasonable. Rather unexpected results are 2·42 and 8·25 for 23 conditions for U_1.

It appears that the greatest probable error of the adjustment of Easting or Northing is $4\sqrt{33} \doteqdot 23$ feet: in terms of deflection this is negligible and of the order of probable error of latitude (astronomic) result.

As regards azimuths for 20 or 23 conditions the worst case is $1 \cdot 6 \sqrt{2 \cdot 4}$ *i.e.* probable error of $2'' \cdot 4$. Error of $7''$ is in this case possible and liable to occur.

In N. E. Quadrilateral where triangulation is not so good there will be greater errors. Closed on Laplace stations, however, errors are probably reduced to $1 \cdot 6 \sqrt{\cdot 5}$ and $1 \cdot 6 \sqrt{1 \cdot 0}$ *i. e.* probable error to 1·6 and possible to $5''$.

In the above the "possible error" is regarded as three times the probable error.

As further discussion of these results is at present impossible, for reasons explained in the preface the chapter is concluded with a tabular statement of the probable errors of log. side to 7th place of decimals, azimuth in seconds and easting and northing in feet; these are obtained as explained in § 20.

TABLE XCI

	C_1 or Dehra base				U_1				G_1 or Chach base				J_1				N_1 or Karachi base			
		20	23	26		20	23	26		20	23	26		20	23	26		20	23	26
	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F	u_f	u_F	u_F	u_F
Side	49	29	0	0	74	36	23	23	64	43	0	0	57	36	34	33	50	39	0	0
Azimuth	2″·33	1″·39	1″·37	0	3″·50	1″·72	2″·46	1″·07	3″·02	2″·03	2″·03	0	2″·72	1″·70	1″·69	1″·54	2″·39	1″·83	1″·60	0
	feet	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*	*feet*
Easting	14·91	10·24	10·16	5·76	38·93	18·01	17·89	15·07	37·64	23·21	19·10	10·76	30·79	14·23	12·63	12·29	22·89	15·56	14·43	14·15
Northing	14·91	10·24	7·05	5·76	38·93	18·01	11·57	15·07	37·64	23·21	19·83	10·76	30·79	14·23	13·86	12·29	22·89	15·56	15·23	14·15

CHAPTER IX.

Deflections of the Plumb-line and values of "g" derived from observations of the Survey of India.

1. The first use of the tables derived in the earlier chapters will now be made use of to display in convenient form all the data of plumb-line deflections available up to the time of writing (May 1917). As regards the deflections in meridian no comment is necessary. The results of observation are immediately available. With the deflections in prime vertical the case is different. It has been stated already that the triangulation of India was not adjusted on the longitude arcs, which have been subsequently observed or reduced. The triangulation accordingly is burdened with an accumulation of error in azimuth which may be largely reduced by adjustment on the longitude arcs. For this purpose it is not essential for the present purpose to reopen the adjustment of the whole triangulation except as regards the azimuth, and the process followed will be substantially that followed by Colonel Sir Sidney Burrard in Appendix 5, G.T.S. Volume XVIII. The numerical results will however be slightly different owing to the improved methods of computing the effect of a change in azimuth at the origin which have been developed in Chapters I—III; the differences will depend mainly on the taking into account of the effect of a change on azimuth at the origin on longitudes of points considerably removed from the origin.

When the triangulation of India was adjusted General Walker decided to adopt a value of the fundamental azimuth (of Surantal from Kalianpur) which differed from the observed value by a small amount (*vide* Chapter I, § 4) and this of course implied a deflection in prime vertical at Kalianpur. This has given rise to a little confusion as regards the longitudes of India. The astronomic longitude of Kalianpur has been determined with reference to Greenwich, but no account of the implied deflection in prime vertical has been hitherto considered.* Colonel Sir Sidney Burrard in adjusting the azimuth observations eliminated the effect of this oversight by returning to an observed value of the fundamental azimuth. The deflection in meridian remained in terms of the Everest spheroid with Walker's initial azimuth. In dealing with the deflections as a whole it will accordingly be better to keep the azimuths in the same terms as the latitudes, and to recognise that a deflection in prime vertical at Kalianpur is thereby implied. After the adjustments on the longitude arcs have been performed, the results of both azimuth and latitude deflections will be in common terms of Everest spheroid and Walker's origin.

Quantities for correcting all the deflections to refer to any other spheroid and origin are given in table XCV in which all the results are exhibited, as well as the deflections corrected to the special case of Helmert's spheroid and the latest observed value of latitude and azimuth at Kalianpur, **as derived from observations at a group of stations surrounding Kalianpur.**†

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* *Vide*, p. xv G.T.S. Vol. XVII, Survey of India.

† *Vide*, pp. 7,9 Professional Paper No. 5, Survey of India.

2. As the correction for azimuths has already been treated by Colonel Sir Sidney Burrard *loc. cit.* it will not be necessary to state afresh the various practical difficulties which arose owing to longitude stations not being in general identical with azimuth stations. The observation results exhibited by him will be taken unaltered, and immediately applied to Laplace's equation. This equation has been given in somewhat amplified form in (3) of Chapter V. There is now no occasion to consider observation errors of astronomic azimuths or their determination. The accumulated error of geodetic longitude determination is certainly small compared with that of geodetic azimuth and so will be neglected. The equation may accordingly be written

$$(A - G - \delta G) \operatorname{cosec} \lambda - (A_0 - G_0) \operatorname{cosec} \lambda_0 = \mathit{A} - \mathit{G} \quad \ldots \ldots \ldots \quad (1)$$

where the notation has been changed in conformity with the usual practice and A, *A* and G, *G* signify astronomic and geodetic determinations respectively, roman letters referring to azimuth and italic letters to longitude determinations: δG is the correction necessary to the geodetic value of azimuth: as this is the quantity required it will be convenient to rewrite (1). From Chapter I § 4 it is seen that $A_0 - G_0 = +1 \cdot 29$, $\lambda_0 = 24° \; 7' \; 12''$, $(A_0 - G_0) \operatorname{cosec} \lambda_0 = 3 \cdot 16$. Hence

$$\delta G = A - G - (3'' \cdot 16 + \mathit{A} - \mathit{G}) \sin \lambda \quad \ldots \ldots \ldots \quad (2)$$

which serves to determine δG. **So long as Walker's value of azimuth is adhered to all *geodetic* longitudes of India require a correction of $-3'' \cdot 16$.**

For the Helmert spheroid an additional correction $\delta_2 G$ is necessary. The corresponding equation to (1) is

$$(A - G - \delta G - \delta_1 G - w) \operatorname{cosec} \lambda = \mathit{A} - \mathit{G} - v \quad \ldots \ldots \ldots \quad (3)$$

since G, *G* are changed by v and w respectively and the astronomic and geodetic azimuths at the origin have been made identical. Subtracting (3) from (2) it follows that

$$\delta_2 G = 3 \cdot 1571 \sin \lambda + v \sin \lambda - w \quad \ldots \ldots \ldots \quad (4)$$

The solutions of (2) and (4) and the deduction of δG and $\delta_2 G$ are now shown in table XCIII.

3 Having obtained the values of δG and $\delta_2 G$ at all Laplace stations it is next necessary to find values at the intervening azimuth stations. This is done in table XCIV, interpolating according to the number of removes from terminal stations. The Laplace stations are shown in block type. Azimuth stations between which adjustments have been performed are shown in italics.

4. The precision of deflections in prime vertical so far as is due to the astronomical observation, obtained from azimuth observation, is much less than those in meridian.* As may be seen from results obtained in Chapter VII the probable error of azimuth generated in triangulation is much greater than that in latitude or longitude, all being expressed in seconds. The deflection in prime vertical is derived from the azimuth anomaly by multiplication by $\cot \lambda$—a quantity which ranges from 7 to $1 \cdot 4$ in Indian latitudes—and this further increases the lack of precision. Considerable improvement on the other hand should result by the use of Laplace stations, which has been made. A further source of weakness, of varying amount, is the actual azimuth observation itself. The observation is not nearly so satisfactory as that for latitude and involves graduation error of the instrument which, especially in the older observations, introduces a serious uncertainty. It is desirable then to consider the relative degree of reliability of the azimuth observations. On account of the other sources of error, mainly that of accumulation of error in triangulation it is not useful to do this in very great detail, and it is considered sufficient to work out the probable error from the

* For probable errors in astronomic latitude vide *G.T.S. Vol. XI. pages 882—982* and *G.T.S. Vol. XVIII App. 7 Table III.* These are seldom so great as $0'' \cdot 2$. The worst case (Gogipatri) is $\pm \; 0'' \cdot 68$.

results obtained on the several zeros. This takes no account of errors in star places, a defect which was more serious in the early days of the survey than it is at present. Probably graduation error in the instruments has improved at much the same rate as the error of star place, and a fair estimate of probable error will be obtained by consideration of the probable error due to graduation only. The formula used is

$$p.e. \text{ of mean of observations on } n \text{ zeros } = \cdot 6745 \sqrt{\frac{\Sigma \delta^2}{n(n-1)}} \quad . \quad . \quad . \quad . \quad (5)$$

where δ is the discrepancy of the value derived from any zero from the mean result. The results of the application of this formula are given in table XCIV. Some idea of the probable errors in the geodetic values of latitude, longitude and azimuth is given in Chapter VIII, where the N.W. Quadrilateral is considered in detail.

5. In the table XCV the deflections derived from observations of the Survey of India are given. These have been arranged by degree sheets. On the left hand page of the table the data are expressed in terms of the Everest Spheroid, using the observed value of latitude and the deduced value of azimuth at Kalianpur which General Walker adopted: on the right hand page of the table the quantities are given which must be applied properly to the triangulated values to express in terms of any other spheroid. There are four variables to be considered, giving rise to four cases: these are (1) change of semi-major axis, δa, (2) change of semi-minor axis, δb, (3) change of latitude of origin, u_0 and (4) change of azimuth at origin, w_0. The cases given correspond to $\delta a = 1$ km., $\delta b = 1$ km., $u_0 = 1''$, $w_0 = 1''$, and to obtain the general case these must be combined as follows:—

$$\delta a \times \text{case I} + \delta b \times \text{case II} + u_0 \times \text{case III} + w_0 \times \text{case IV}$$

in which δa, δb are expressed in kilometres and u_0, w_0 are expressed in seconds. Thus in the case of the Helmert Spheroid in which $a = 6378 \cdot 2$ km. and $1/e = 298 \cdot 3$ and with revised values of latitude and azimuth at Kalianpur as given on p. 2, $\delta a = \cdot 924$, $\delta b = \cdot 743$, $u_0 = \cdot 31$, $w_0 = 1 \cdot 29$. Deflections in terms of this spheroid are given on the right of the right hand page of the table: but those in terms of any other spheroid may be easily found by making use of different values of δa, etc. It is clear in the notation of this work that the correction to latitude deflection is $-u$, to prime vertical deflection $-v \cos \lambda$ or $-w \cot \lambda$ according as the deflection is derived from longitude or azimuth observations. Values of these quantities have been taken from tables XVII—XX, XXIX—XXXVI.

It frequently happens that latitude observations have been made on a site not quite identical with the triangulation station, but at some (small) distance from it on the prime vertical through it. Similarly the longitude observations done by wire-telegraphy were made at the telegraph offices and not exactly on the station sites. The coordinates of the triangulation stations are generally the quantities given. But at latitude stations the value of latitude given is that of the latitude station, and in longitude stations the longitude of the longitude station. When a change is made to a different spheroid, since the corrections do not satisfy Laplace equation, slight discrepancies occur between deflection derived from longitude and azimuth observations. This point has been explained in Chapter V and has been taken into account in the case of the Helmert spheroid in table XCIII.

The elevation of the referring mark affects the result of azimuth observations (*vide* §6 Chapter V) and accordingly this has been given in the table except for a few cases where the data could not be found.

Longitude arcs by means of wireless telegraphy were observed in collaboration with the expedition of Cav. de Filippi in 1914 between Dehra Dun and eight stations. Their names and the values of Δ are appended. The Dehra Dun observations were made in the transit room adjoining the

Dome Observatory (new) and the longitude (geodetic) of the transit instrument is 7″·18 (equivalent linear measurement being 628·8 feet*) less than that of the Dehra Dun Haig Observatory where all previous longitude observations were made. The geodetic values of the eight stations and the astronomic values of the latter four are not yet available (1917) and deflections cannot be given.

TABLE XCII.

	Skardu	Kargil	Lamayaru	Leh	Depsang	Suget Karaul	Yarkand	Kashgar
A	75° 38′ 22″·80	76° 7′ 40″·65	76° 46′ 32″·01	77° 34′ 53″·78	77° 56′ 49″†	78° 12′ ″†	77° 15′ 55″†	76° 6′ 47″†
G	...	... ...	... ...	... ...	... ...	... ...	... ...	... ...
Lat.	35 18 40 †	34 33 40 †	34 17 1 †	34 10 4 †	35 17 20†	36 20 56†	38 25 1†	39 24 26†

TABLE XCIII.

Azimuth station / Latitude=λ	Azimuth (Everest)	(1) A−G‡	Longitude station / sin λ	Longitude from Kalianpur (Everest)	*A*−*G*‡	(2) (*A*−*G*+3″·16) sin λ	(1)−(2) =δG	(3) 3·1571 × sin λ	(4) v sin λ	(5) w	(Helmert) (3)+(4)−(5)=δ₂G§
Kalianpur H.S. 24° 7′ 11″	A 190° 27′ 6″·39 G 190 27 5·10	+ 1·29	Kalianpur ... ·4086	*A* 0° 0′ 0″ *G* 0 0 0	0	+ 1″·29	0·0	+1·29	0·00	+1·29	0·00
Karachi Observatory S. 24° 49′ 50″	A 221 39 9·5 G 221 39 10·9	− 1·4	Karachi T.O. ... ·4200	*A* − 10 38 24·8 *G* − 10 38 24·3	− 0·5	+ 1·1	− 2·5	+1·33	+ 2·41	+3·62	+ 0·12
Dehra Dun Obs. (old) S. 30° 19′ 57″	A 165 10 58·8 G 165 11 10·7	−11·9	Dehra Dun ·5050	*A* + 0 23 38·9 *G* + 0 24 4·6	−25·7	−11·4	− 0·5	+1·59	− 0·03	+1·25	+ 0·31
Quetta T.O. S. 30° 11′ 57″	A 166 31 12·1 G 166 31 17·0	− 4·9	Quetta T.O. ... ·5030	*A* − 10 38 48·3 *G* − 10 38 45·8	− 2·5	+ 0·3	− 5·2‖	+1·59	+ 3·06	+4·23	+ 0·42
Calcutta Base S., T.S. 22° 36′ 56″	A 177 10 27·3 G 177 10 36·2	− 8·9	Calcutta ... ·3846	*A* + 10 42 0·3 *G* + 10 42 11·3	−11·0	− 3·0	− 5·9	+1·21	− 2·20	−0·90	− 0·09
Orejhar S. 26° 46′ 56″	A 308 36 18·9 G 308 36 23·0	− 4·1	Fyzabad T.O. ... ·4506	*A* + 4 28 50·1 *G* + 4 28 50·6	− 0·5	+ 1·2	− 5·3	+1·42	− 1·08	+0·22	+ 0·12
Jalpaiguri s. 26° 31′ 17″	A 321 33 25·3 G 321 33 30·0	− 4·7	Jalpaiguri ... ·4465	*A* + 11 4 34·8 *G* + 11 4 55·2	−20·4	− 7·7	+ 3·0	+1·41	− 2·68	−1·32	+ 0·05
Nagarkhana H.S. 22° 22′ 56″	A 155 47 13·3 G 155 47 23·5	−10·2	Chittagong T.O. ·3808	*A* + 14 10 47·4 *G* + 14 10 59·1	−11·7	− 3·3	− 6·9	+1·20	− 2·87	−1·71	+ 0·04
Dattaung H.S. 20° 13′ 14″	A 171 27 28·3 G 171 27 38·1	− 9·8	Akyab T.O. ... ·3456	*A* + 15 14 21·0 *G* + 15 14 32·1	−11·1	− 2·7	− 7·1	+1·09	− 2·77	−1·93	+ 0·25
Kyaunggyi S. 18° 49 21″	A 109 26 42·1 G 109 26 58·1	−16·0	Prome ... ·3226	*A* + 17 33 24·6 *G* + 17 33 39·9	−15·3	− 3·9	−12·1	+1·02	− 2·97	−2·27	+ 0·32
Taungzun H.S. 16° 25′ 49″	A 31 16 18·9 G 31 16 32·7	−13·8	Moulmein ... ·2829	*A* + 19 58 5·9 *G* + 19 58 22·5	−16·6	− 3·8	−10·0	+0·89	− 2·92	−2·54	+ 0·51
Bolarum P.W.D. S. 17° 30′ 13″	A 25 57 35·8 G 25 57 36·9	− 1·1	Bolarum ... ·3008	*A* + 0 51 50·3 *G* + 0 51 53·6	− 3·3	0·0	− 1·1	+0·95	− 0·18	+1·10	− 0·33
Vizagapatam Base N., S. 18° 1′ 3″	A 203 44 24·5 G 203 44 25·9	− 1·4	Waltair ... ·3093	*A* + 5 39 42·6 *G* + 5 39 45·8	− 3·2	0·0	− 1·4	+0·98	− 0·95	+0·33	− 0·30
Karaundi H.S. 23° 10′ 40″	A 206 22 35·6 G 206 22 39·6	− 4·0	Jabalpur T.O. ... ·3936	*A* + 2 17 34·8 *G* + 2 17 45·0	−10·2	− 2·8	− 1·2	+1·24	− 0·49	+0·79	− 0·04
Colaba Observatory S. 18° 53′ 47″	A 288 5 27·7 G 288 5 26·7	+ 1·0	Bombay ... ·3238	*A* − 4 50 21·8 *G* − 4 50 28·6	+ 6·8	+ 3·2	− 2·2	+1·02	+ 0·77	+2·05	− 0·26
Deesa T. O. s. 24° 15′ 29″	A 241 16 15·3 G 241 16 19·9	− 4·6	Deesa T.O. ... ·4108	*A* − 5 28 16·4 *G* − 5 28 12·7	− 3·7	− 0·2	− 4·4	+1·30	+ 1·20	+2·46	+ 0·04
Mangalore S. 12° 52′ 15″	A 205 52 50·8 G 205 52 53·6	− 2·8	Mangalore ... ·2227	*A* − 2 48 32·9 *G* − 2 48 35·1	+ 2·2	+ 1·2	− 4·0	+0·70	+ 0·26	+1·55	− 0·59
Bangalore Base S.W., S. 13° 0′ 41″	A 224 31 21·7 G 224 31 27·0	− 5·3	Bangalore ... ·2252	*A* − 0 4 20·3 *G* − 0 4 17·6	− 2·7	+ 0·1	− 5·4	+0·71	− 0·05	+1·22	− 0·56
St. Thomas's Mount S. 13° 0′ 15″	A 12 30 5·3 G 12 30 9·3	− 4·0	Madras ... ·2250	*A* + 2 35 29·6 *G* + 2 35 36·6	− 7·0	− 0·9	− 3·1	+0·71	− 0·35	+0·89	− 0·53
Kudankulam Obs. S. 8° 10′ 22″	A 185 55 18·8 G 185 55 26·5	− 7·7	Nagarkoil ... ·1421	*A* − 0 13 15·8 *G* − 0 13 14·2	− 1·6	+ 0·2	− 7·9	+0·45	− 0·03	+1·19	− 0·77

Obs.=observatory, T.O.=Telegraph office. * *Vide* G.T.S. Vol. XV, p. (5). † Approximate values. ‡ A, *A*=Astronomic values; G, *G*=Geodetic values. § This is the additional correction for Helmert's spheroid. ‖ Derived from unadjusted values of Quetta Secondary Series. This does not enter into the azimuth correction.

TABLE XCIV. (*See Index pp.* 170-172)

Serial Number	Station	Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
1	**Kalianpur** H.S.	0·0	0·0	0·31	{1836 1898}
2	Losalli S.	−0·1	0·0	0·44	1849
3	Salot H.S.	−0·2	0·0	0 27	1849
4	Māta-ka-hūra H.S.	−0·3	0·0	0·34	1849
5	Gurāria H.S.	−0·4	0·0	0 44	1849
6	Rāmpūra H.S.	−0·5	0·0	0 49	1849
7	Aramlia S.	−0·6	0·0	0·32	1850
8	Sānd H.S.	−0·7	0·0	0·90	1850
9	Tiki H.S.	−0·8	0·0	0·38	1851
10	Kānnagar H.S.	−0·9	0·0	0·50	1850
11	Gūru Sikkar H.S.	−1·0	0·0	0·45	1850
12	Birona S.	−1·1	0·0	0 46	1851
13	Khankharia S.	−1·2	0·0	0·33	1851
14	Sarla S.	−1 2	+0·1	0·25	1851
15	Didāwa H.S.	−1·3	+0·1	0·21	1851
16	Virāria H.S.	−1·4	+0·1	0·19	1851
17	Lūnki H.S.	−1·5	+0·1	0·19	1851
18	Kujhra H.S.	−1·6	+0·1	0·29	1851
19	Chānga H.S.	−1·7	+0·1	0·28	1852
20	Mairāb-ka-Shahar T.S.	−1·8	+0·1	0·56	1852
21	Khori T.S.	−1·9	+0·1	0·53	1852
22	Alamkhān T.S.	−2·0	+0·1	0·22	1852
23	Chāti T.S.	−2·1	+0·1	...	1853
24	Kārothol H.S.	−2·2	+0·1	0·31	1853
25	**Karachi Obs.** S.	−2·5	+0·1	0·19	1855
1	**Kalianpur** H.S.	0·0	0·0	0·31	{1836 1898}
26	Pahārgarh H.S.	−0·1	0·0	0·22	1836
27	Kesri H.S.	−0·1	+0·1	0·39	1836
28	Usira H.S.	−0 2	+0·1	0·18	1838
29	Noh T.S.	−0·3	+0·2	0·93	1837
30	Datairi T.S.	−0·3	+0·2	0·40	1836
31	Kaliāna S.	−0·4	+0·3	0 33	1836
32	Banog H.S.	−0·5	+0·3	0·28	{1836 1907}
33	**Dehra Dun Obs. (old)** S.	−0·5	+0·3	0·34	1853
33	**Dehra Dun Obs. (old)** S.	−0·5	+0·3	0·34	1853
34	Rājpur h.s.	−0 5	+0·3	0·26	1914
35	Spur point h.s.	−0·5	+0·3	0·29	1914
36	Jhari pani h.s.	−0·5	+0 3	0·25	1914
37	Mussooree Dome Obs. H.S.	−0·5	+0·3	0·45	1912
38	Nag Tiba H.S.	−0·5	+0·3	0·38	1903
32	Banog H.S.	−0·5	+0·3	0·28	{1836 1907}

Serial Number	Station	Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
25	**Karachi Obs.** S.	−2·5	+0·1	0·19	1855
39	Karachi Base S., S.	−2·4	+0 1	0·30	1853
40	Yūsuf S.	−2·0	+0·2	0 53	1858
41	Bhanar T.S.	−1 9	+0 2	0·27	1859
42	Miāni T.S.	−1·8	+0 2	0·32	1859
43	Dājil S.	−1·6	+0·2	0·22	1860
44	Dera Dīn Panāh S.	−1·4	+0·2	0·25	1859
45	Jharkil T.S.	−1·2	+0·2	0·27	1859
46	Umarkhel H.S.	−1 0	+0·3	0 20	1909
47	Jāoli H.S.	−0·9	+0·3	0·80	1851
48	Medwāni H.S.	−0·7	+0·3	0·50	1853
33	**Dehra Dun Obs. (old)** S.	−0·5	+0·3	0 34	1853
25	**Karachi Obs.** S.	−2·5	+0·1	0 19	1855
49	Andar H.S.	−2·4	+0·1	0·27	1895
50	Piaro H.S.	−2·4	+0·1	0·27	1896
33	**Dehra Dun Obs. (old)** S.	−0·5	+0·3	0 34	1853
49	*Andar* H.S.	−2·4	+0·1	0·27	1895
51	Gandpahar H.S.	−2·1	+0·1	0·13	1906
52	Zawa H.S.	−1·9	+0·1	0·20	1905
53	Mashelak H.S.	−1·9	+0·2	0·34	1908
54	Gundak H.S.	−1·7	+0·2	0·22	1910
55	Salighar H.S.	−1·6	+0·2	0·19	1910
56	Tounsa T.S.	−1·5	+0·2	0·31	1910
44	*Dera Dīn Panāh* S.	−1·4	+0·2	0·25	1859
52	*Zawa* H.S.	−1·9	+0·1	0·20	1905
57	Kisanen Chappar H.S.	−1·9	+0·1	0·18	1907
58	Tuzgi H.S.	−1·9	+0·1	0·36	1907
59	Koh-i-Malik Siah H.S.	−1·9	+0·1	0·30	1907
60	**†Quetta T.O.** S	−5·2	+0·4	0·28	1904
5	*Gurāria* H.S.	−0·4	0·0	0·44	1849
61	Kānkra H.S.	−0·4	0·0	0·34	1862
62	Bānskho H.S.	−0·5	+0·1	0·14	1862
63	Tāsing H.S.	−0·5	+0·1	0·70	1861
64	Rākhi T.S.	−0·6	+0·2	0·19	1860
65	Kheri T.S.	−0·7	+0·2	0·37	1856
66	Bowra T.S.	−0 7	+0·2	0·52	1853
48	*Medwāni* H.S.	−0·7	+0·3	0·50	1853
7	*Aramlia* S.	−0·6	0·0	0·32	1850
67	Rājgarh H.S.	−0·6	0·0	0·56	1863
68	Garinda S.	−0·7	+0·1	0·42	1863
69	Sirsa S.	−0·7	+0·1	0·34	1861
70	Sangatpur T.S.	−0·8	+0·2	0·54	1860
47	*Jāoli* H.S.	−0·9	+0·3	0·80	1851
47	*Jāoli* H.S.	−0·9	+0·3	0·80	1851
71	Murree h.s.	−0·9	+0·3	0·77	1860
72	Ganga Choti H.S.	−0·9	+0·3	0·16	1910
73	Poshkar H.S.	−0 9	+0·3	0·65	1862
74	Gogipatri H.S.	−0·9	+0·3	0·84	1862
75	Rustamgarhi h.s.	−0·9	+0·3	0·67	1862
70	*Sangatpur* T.S.	−0·8	+0·2	0·54	1860

Note.—In the azimuthal observations, Level corrections were introduced from 1863, *vide* G.T.S. Vol. II Appendix 9, p 73 and Diurnal Aberration corrections from 1902 *vide* Handbook of the Trigonometrical Branch 1902, p. 74.

Obs. = observatory, T.O. = Telegraph Office.
S = station, H.S. = hill station, T.S. = tower station of principal triangulation. The same small letters refer to minor triangulation.

* Additional correction for Helmert's spheroid.

† Computed from the unadjusted values of longitude and azimuth of Quetta T.O. station: not used for adjusting any azimuth observations.

TABLE XCIV.—(Continued). (*See Index pp.* 170-172)

Serial Number	Station		Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
11	*Gūru Sikhar*	H.S.	−1·0	0·0	0·45	1850
76	Thob	H.S.	−1·0	0·0	0·38	1873
77	Jambo	H.S.	−1·0	+0·1	0·21	1874
78	Mugrala	H.S.	−1·0	+0·1	0·40	1875
79	Lādimsir	T.S.	−1·0	+0·1	0·45	1862
80	Mandresa	T.S.	−0·9	+0·2	0·17	1862
81	Jhambhera	T.S.	−0·9	+0·2	0·68	1862
82	Akbar	S.	−0·9	+0·2	0·79	1857
47	*Jāuli*	H.S.	−0·9	+0·3	0·80	1851
18	*Rojhra*	H.S.	−1·6	+0·1	0·29	1851
83	Malar	H.S.	−1·5	+0·1	0·32	1877
84	Asu	H.S.	−1·4	+0·1	0·49	1880
85	Vijnot	T.S.	−1·3	+0·1	0·22	1881
86	Dāowāla	T.S.	−1·3	+0·1	0·20	1881
87	Paphra	T.S.	−1·1	+0·1	0·25	1861
79	*Lādimsir*	T.S.	−1·0	+0·1	0·45	1862
1	**Kalianpur**	**H.S.**	0·0	0·0	0·31	{ 1836
88	Budhon	H.S.	−0·4	0·0	0·36	{ 1898
89	Rangir (old)	H.S.	−0·8	0·0	0·64	1864
						1834
90	Amūa	H.S.	−1·4	0·0	0·94	1834
91	Karāra	H.S.	−1·8	0·0	0·69	1842
92	Gurwāni	H.S.	−2·2	0·0	0·64	1845
93	Gora	H.S.	−2·7	0·0	0·52	1845
94	Hurilāong	H.S.	−3·1	−0·1	0·44	1849
95	Chendwār (old)	H.S.	−3·5	−0·1	0·67	1843
96	Pārasnāth	H.S.	−3·7	−0·1	0·35	1850
97	Tilabani	H.S.	−3·9	−0·1	0·76	1845
98	Malūncha	H.S.	−4·3	−0·1	0·57	1844
99	Madhpur	T.S.	−4·9	−0·1	0·49	1868
100	Aknāpur	T.S.	−5·2	−0·1	0·83	1869
101	**Calcutta Base-line S. End**	**T.S.**	−5·9	−0·1	0·37	1845
33	**Dehra Dun Obs. (old)**	**S.**	−0·5	+0·3	0·34	1853
102	Kaliānpur	T.S.	−1·8	+0·2	0·64	1850
103	Ramuapur (old)	T.S.	−2·7	+0·2	0·52	1838
104	Māsi	T.S.	−3·8	+0·2	0·37	1850
105	Bāsadela	T.S.	−4·6	+0·1	0·32	1849
106	**Orejhar**	**S.**	−5·3	+0·1	0·26	1904
88	*Budhon*	H.S.	−0·4	0·0	0·36	1864
107	Gūrmi	T.S.	−0·4	+0·1	0·50	1842
108	Sankrāo	T.S.	−0·4	+0·2	0·39	1843
109	Sirsa	T.S.	−0·5	+0·2	0·70	1843
33	**Dehra Dun Obs. (old)**	**S.**	−0·5	+0·3	0·34	1853
89	*Rangīr (old)*	S.	−0·8	0·0	0·64	1834
110	Mohammadabad	T.S.	−1·4	+0·1	0·44	1840
102	*Kaliānpur*	T.S.	−1·8	+0·2	0·64	1850

Serial Number	Station		Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
90	*Amūa*	H.S.	−1·4	0·0	0·94	1834
111	Nimkār	T.S.	−2·3	+0·1	0·38	1838
103	*Ramuapur (old)*	T.S.	−2·7	+0·2	0·52	1838
91	*Karāra*	H.S.	−1·8	0·0	0·69	1842
112	Pabhosa	H.S.	−2·4	+0·1	1·26	1845
113	Sora	T.S.	−2·9	+0·1	0·44	1845
104	*Māsi*	T.S.	−3·8	+0·2	0·37	1850
92	*Gurwāni*	H.S.	−2·2	0·0	0·64	1845
114	Marār	T.S.	−3·7	+0·1	0·48	1846
115	Bisaul	T.S.	−5·1	+0·1	0·60	1847
106	**Orejhar**	**S.**	−5·3	+0·1	0·26	1904
93	*Gora*	H.S.	−2·7	0·0	0·52	1845
116	Hirdepur	T.S.	−3·0	0·0	0·58	1846
117	Samenda	T.S.	−3·4	0·0	1·13	1846
118	Rājabārī	T.S.	−3·9	+0·1	1·52	1847
105	*Bāsadela*	T.S.	−4·6	+0·1	0·32	1849
106	**Orejhar**	**S.**	−5·3	+0·1	0·26	1904
119	Naunangarhi	T.S.	−2·3	+0·1	0·34	1852
120	Chūni	T.S.	+0·9	+0·1	0·69	1846
121	Rāmganj	T.S.	+2·4	+0·1	0·64	1853
122	**Jalpaiguri**	S.	+3·0	+0·1	0·33	1904
94	*Hurīlāong*	H.S.	−3·1	−0·1	0·44	1849
123	Mednipur	T.S.	−2·9	0·0	0·34	1850
124	Jalālpur	T.S.	−2·6	0·0	0·62	1852
119	*Naunangarhi*	T.S.	−2·3	+0·1	0·34	1852
95	*Chendwār (old)*	H.S.	−3·5	−0·1	0·67	1843
125	Pota	T.S.	−2·8	0·0	0·46	1846
119	*Naunangarhi*	T.S.	−2·3	+0·1	0·34	1852
96	*Pārasnāth*	H.S.	−3·7	−0·1	0·35	1850
126	Bichwi	H.S.	−3·2	−0·1	0·80	1851
120	*Chūni*	T.S.	+0·9	+0·1	0·69	1846
98	*Malūncha*	H.S.	−4·3	−0·1	0·57	1844
127	Sirkanda	T.S.	−1·7	0·0	0·43	1846
120	*Chūni*	T.S.	+0·9	+0·1	0·69	1846
101	**Calcutta Base-line S. End**	**T.S.**	−5·9	−0·1	0·37	1845
128	Anandbās	T.S.	−4·2	−0·1	0·45	1846
129	Madhupur	T.S.	−2·9	0·0	0·63	1846
122	**Jalpaiguri**	S.	+3·0	+0·1	0·33	1904
101	**Calcutta Baseline S. End**	**T.S.**	−6·9	−0·1	0·37	1845
130	Daulatpur	T.S.	−6·3	−0·1	0·15	1868
131	Gangapur	T.S.	−6·5	0·0	0·34	1866
132	Lakhinagar	T.S.	−6·5	0·0	0·14	1866
133	Semu Tān	H.S.	−6·8	0·0	0·39	1865
134	**Nagarkhana**	H.S.	−6·9	0·0	1·29	1905

* Additional correction for Helmert's Spheroid.

TABLE XCIV.—(Continued). (*See Index pp.* 170-172)

Serial Number	Station		Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
130	*Daulatpur*	T.S.	− 6·3	−0·1	0·15	1868
135	Tepri	T.S.	− 4·2	−0·1	0·30	1869
136	Aloākāndi	T.S.	− 2·2	0·0	0·51	1873
137	Halkāchar	T.S.	− 1·1	0·0	0·21	1873
138	Alangjāni	T.S.	+ 0·6	0·0	0·79	1874
139	Ataro Bānki	T.S.	+ 1·3	+0·1	0·34	1856
122	**Jalpaiguri**	S.	+ 3·0	+0·1	0·33	1904
133	*Semu Tān*	H.S.	− 6·8	0·0	0·39	1865
140	Dawa	H.S.	− 5·3	0·0	0·26	1864
141	Rangsanobo	H.S.	− 2·7	0·0	0·32	1861
142	Raikusni	H.S.	− 0·4	0·0	0·29	1858
138	*Alangjāni*	T.S.	+ 0·6	0·0	0·79	1874
134	**Nagarkhana**	**H.S.**	− 6·9	0·0	1·29	1905
143	Fi Tān	H.S.	− 6·9	+0·1	0·52	1865
144	**Dattaung**	H.S.	− 7·1	+0·3	0·35	1866
144	**Dattaung**	**H.S.**	− 7·1	+0·3	0·35	1866
145	Betkamauk	H.S.	− 8·0	+0·3	0·41	1916
146	**Kyaunggyi**	S.	−12·1	+0·3	0·34	1904
147	**Taungzun**	**H.S.**	−10·0	+0·5	0·76	1884
148	Southern Moscos	H.S.	−10·0	+0·5	0·62	1877
149	Mergui Base-line E. End	T.S.	−10·0	+0·5	0·18	1882
150	Mergui Base-line W. End	T.S.	−10·0	+0·5	0·32	1882
151	Natkalintaung	H.S.	−10·0	+0·5	0·20	1881
152	Minthangtaung	H.S.	−10·0	+0·5	0·24	1881
146	**Kyaunggyi**	S.	−12·1	+0·3	0·34	1904
153	Myayabeingkyo	H.S.	−11·1	+0·3	0·45	1889
154	Toungoo	S.	−10·3	+0·2	0·23	1890
155	Letpataung	H.S.	− 9·6	+0·2	0·27	1891
156	Taungpila	H.S.	− 8·8	+0·2	0·21	1891
157	Mingun	H.S.	− 8·0	+0·2	0·19	1892
158	Shienmaga	H.S.	− 7·8	+0·2	0·36	1892
159	Male	H.S.	− 7·3	+0·1	0·19	1892
160	Ubyetaung	H.S.	− 6·8	+0·1	0·31	1894
161	Thonbinzin	H.S.	− 6·5	+0·1	0·18	1894
162	Seikpa	H.S.	− 6·3	+0·1	0·24	1895
163	Tamunja	H.S.	− 5·7	+0·1	0·30	1896
164	Thyoliching	H.S.	− 5·5	+0·1	0·19	1898
165	Loijing	H.S.	− 5·0	+0·1	0·17	1899
141	*Rangsanobo*	H.S.	− 2·7	0·0	0·32	1861
144	**Dattaung**	**H.S.**	− 7·1	+0·3	0·35	1866
166	Yeponetaung	H.S.	− 6·9	+0·3	0·60	1916
163	*Tamunja*	H.S.	− 5·7	+0·1	0·30	1896

Serial Number	Station		Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
160	*Ubyetaung*	H.S.	− 6·8	+0·1	0·31	1894
167	Sinpitaung	H.S.	− 6·8	+0·1	0·36	1901
168	Loi Hpa Lang	H.S.	− 6·8	+0·1	0·16	1903
169	Loi Hpatan	H.S.	− 6·8	+0·1	0·20	1907
170	Loi Kiipma	H.S.	− 6·8	+0·1	0·17	1908
171	Loi Hsam Hsum	H.S.	− 6·8	+0·1	0·21	1911
172	Kumtum Bum	H.S.	− 6·8	+0·1	0·22	1910
173	Kumon Bum	H.S.	− 6·8	+0·1	0·17	1911
1	**Kalianpur**	**H.S.**	0·0	0·0	0·31	1836 / 1898
174	Ahmadpur	H.S.	− 0·1	0·0	0·37	1838
175	Bhimbhat	H.S.	− 0·2	−0·1	0·39	1838
176	Nilgarh	H.S.	− 0·3	−0·1	0·23	1839
177	Badgaon	H.S.	− 0·5	−0·1	0·22	1839
178	Sakri	H.S.	− 0·6	−0·2	0·30	1838
179	Somtana	H.S.	− 0·7	−0·2	0·27	1838
180	Dāmargīda Obs.	H.S.	− 0·9	−0·3	0·40	1838
181	**Bolarum P. W. D. Office**	**S.**	− 1·1	−0·3	0·31	1904
181	**Bolarum P. W. D. Office**	**S.**	− 1·1	−0·3	0·31	1904
182	Pirmulo	H.S.	− 1·2	−0·3	0·23	1869
183	Vānākonda	H.S.	− 1·2	−0·3	0·20	1869
184	Singāwāram	H.S.	− 1·3	−0·3	0·19	1871
185	Kālingkonda	H.S.	− 1·3	−0·3	0·24	1872
186	Sānjib	H.S.	− 1·4	−0·3	0·18	1860
187	**Vizagapatam Base-line N. End**	**S.**	− 1·4	−0·3	0·24	1863
101	**Calcutta Base-line S. End**	**T.S.**	− 5·9	−0·1	0·37	1845
188	Patna	T.S.	− 4·6	−0·2	0·19	1852
189	Chandīpur	T.S.	− 4·3	−0·2	0·30	1854
190	Cuttack	H.S.	− 3·4	−0·2	0·20	1854
191	Khundābolo	H.S.	− 3·0	−0·2	0·35	1857
192	Rawal	H.S.	− 1·8	−0·3	0·30	1860
187	**Vizagapatam Base-line N. End**	**S.**	− 1·4	−0·3	0·24	1863
192	*Rawal*	H.S.	− 1·8	−0·3	0·30	1860
193	Deodonger	H.S.	− 2·0	−0·3	0·11	1914
194	Sindur	H.S.	− 2·3	−0·2	0·16	1913
195	Andhari	H.S.	− 2·6	−0·2	0·20	1913
196	Bhursu	H.S.	− 2·9	−0·1	0·17	1912
94	*Hurīlāong*	H.S.	− 3·1	−0·1	0·44	1849
197	**Karaundi**	**H.S.**	− 1·2	0·0	0·33	1865
198	Sarandi Pat	H.S.	− 1·2	−0·1	0·33	1865
199	Bhīmsain	H.S.	− 1·2	−0·1	0·18	1866
200	Diwai	H.S.	− 1·2	−0·2	0·16	1867
201	Burgpaīli	H.S.	− 1·1	−0·2	0·18	1867
181	**Bolarum P.W.D. Office**	**S.**	− 1·1	−0·3	0·31	1904

Additional correction for Helmert's Spheroid.

TABLE XCIV.—(Continued). (*See Index pp.* 170-172)

Serial Number	Station		Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
91	*Karāra*	H.S.	− 1·8	0·0	0·69	1842
202	Pathāidi	T.S.	− 1·7	−0·1	0·39	1871
203	Ramai	H.S.	− 1·6	−0·1	0·50	1872
204	Karia	H.S.	− 1·5	−0·2	0·20	1873
186	*Sānjib*	H.S.	− 1·4	−0·3	0·18	1860
97	*Tilabani*	H.S.	− 3·9	−0·1	0·76	1845
205	Kalsibhanga	T.S.	− 4·4	−0·2	0·32	1849
188	*Patna*	H.S.	− 4·6	−0·2	0·19	1852
181	**Bolarum P.W.D. Office**	**S.**	− 1·1	−0·3	0·31	1904
206	Achola	H.S.	− 1·4	−0·3	0·25	1840
207	Nitali	H.S.	− 1·6	−0·3	0·27	1840
208	Kanheri	H.S.	− 1·7	−0·3	0·32	1837
209	Alsunda	H.S.	− 1·7	−0·3	0 32	1863
210	Khānpisura	H.S.	− 1·8	−0 3	0 42	1846
211	Dhauleshvar	H.S.	− 1·9	−0·3	0·18	1838
212	Māndvi	H.S.	− 2·1	−0·3	0·32	1841
213	Karanja	H.S.	− 2·1	−0·3	...	1839
214	**Colaba Obs.**	**S.**	− 2·2	−0 3	...	1839
215	**Deesa T.O.**	**S.**	− 4·4	0·0	0 26	1904
216	Sonāda	T.S.	− 3·8	−0·1	0·39	1851
217	Patangdi	H.S.	− 3·3	−0·2	0 30	1861
218	Sāler	H.S.	− 2·7	−0·2	0 60	1845
219	Pārnera	H.S.	− 2·6	−0·3	0 47	1843
220	Kalsubai	H.S.	− 2·4	−0·3	0·64	1842
214	**Colaba Obs.**	**S.**	− 2·2	−0·3	...	1839
25	**Karachi Obs.**	**S.**	− 2·5	+0·1	0·19	1855
221	Hāthria	H.S.	− 3·1	0 0	0·54	1856
222	Dungarpur	H.S.	− 3·4	−0·1	0·31	1852
223	Ingrodi	T.S.	− 3·6	−0·1	0·32	1852
216	*Sonāda*	T.S.	− 3·8	−0·1	0·39	1851
222	*Dungarpur*	H.S.	− 3·4	−0·1	0·31	1852
224	Kunkāvāv	T.S.	− 3·4	−0·1	0 27	1853
7	*Aramlia*	S.	− 0·6	0 0	0·32	1850
225	Indrāwan	T.S.	− 1·0	−0·1	0·29	1847
226	Valvādi	H.S.	− 1·4	−0·2	0 56	1846
210	*Khānpisura*	H.S.	− 1·8	−0·3	0·42	1846
181	**Bolarum P.W.D. office**	**S.**	− 1·1	−0·3	0·31	1904
227	Kodangal	S.	− 2·2	−0·4	0·18	1872
228	Darur	H.S.	− 2 7	−0·4	0·91	1871
229	**Bangalore Base-line S.W. End**	**S.**	− 5·4	−0·6	0·15	1870

Serial Number	Station		Corrections: Everest's Spheroid	Corrections: Helmert's Spheroid*	Probable Error ±	Date of Observation
230	**Mangalore**	**S.**	− 4·0	−0·6	0·58	1873
231	Nughallibētta	H.S.	− 4·7	−0·6	0 23	1871
229	**Bangalore Base-line S.W. End**	**S.**	− 5·4	−0·6	0·15	1870
229	**Bangalore Base-line S.W. End**	**S.**	− 5·4	−0·6	0·15	1870
232	Anandalamalai	H.S.	− 3·9	−0·5	0·10	1866
233	Injambākam	H.S.	− 3·3	−0·5	0·17	1880
234	**St. Thomas's Mount Trestle**	**S.**	− 3·1	−0·5	0 27	1880
234	**St. Thomas's Mount Trestle**	**S.**	−3·1	−0·5	0 27	1880
235	Kistama	H.S.	−2·7	−0·5	0·37	1864
236	Dānapa	H.S.	−2·4	−0·4	0·21	1863
237	Dhūlipalla	S.	−2·3	−0·4	0·17	1868
238	Parampūdi	H.S.	−1·9	−0·4	0·13	1861
187	**Vizagapatam Base-line N. End**	**S.**	−1·4	−0·3	0 24	1863
214	**Colaba Obs.**	**S.**	−2·2	−0·3	...	1839
239	Pāchvad	H.S.	−2·7	−0·4	0·16	1865
240	Karabgati	H S	−3·0	−0·4	0·22	1865
241	Koramūr	H.S.	−3·6	−0·5	0 21	1873
230	**Mangalore**	**S.**	−4·0	−0·6	0·58	1873
214	**Colaba Obs.**	**S.**	−2·2	−0·3	...	1839
242	Mirya	H.S.	−2·5	−0·3	0·89	1844
243	Chaukola	H.S.	−2·6	−0·3	0·83	1843
244	Kumbhari	H.S.	−2·7	−0·4	0 61	1844
240	*Karabgati*	H.S.	−3·0	−0 4	0·22	1865
229	**Bangalore Base-line S.W. End**	**S.**	−5·4	−0·6	0·15	1870
245	Bangalore Base-line N E. End	S.	−5·5	−0·6	0·23	1870
246	Kanjamalai	H.S.	−6·3	−0·7	0·21	1869
247	Pachapālaiyam	S.	−6·7	−0·7	0·26	1870
248	Kutipārai	S.	−7·2	−0·7	0 33	1873
249	Rādhāpuram	S.	−7·8	−0·8	0·14	1869
250	**Kudankulam Obs.**	**S.**	−7·9	−0·8	0·21	1869
234	**St. Thomas's Mount Trestle**	**S.**	−3·1	−0·5	0·27	1880
251	Kallapat Trestle	S.	−3·9	−0·5	0 17	1879
252	Nayinipiriyān ,,	S.	−4·5	−0·6	0·24	1870
253	Pātharankota	S.	−5·1	−0·6	0·14	1877
254	Manēgandi	S.	−6·0	−0·7	0·25	1876
255	Rāmnad	S.	−6·4	−0·7	0·39	1875
250	**Kudankulam Obs.**	**S.**	−7·9	−0·8	0·21	1869

* Additional correction for Helmert's Spheroid.

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Amūa H.S.	326	90	5
Anandalamalai H.S.	274	232	54
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„ No. V } between Dehra and Rājpur	166		20
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Fi Tān H.S.	418	143	52
Fyzabad Longitude S.	317		19
Gandpahar H.S.	11	51	32
Ganga Choti H.S.	54	72	77
Gangapur T.S.	408	131	48
Garinda S.	90	68	23
Gattinārāyantippa h.s.	247		9
Gaus T.S.	321		20
Ghorārāo H.S.	100		10
Godhna T.S.	152		6
Gogipatri H.S.	454	74	22
Gora H.S.	325	93	5
Gudali H.S.	347		46
Gundak H.S.	6	54	76
Gurāria H.S.	184	5	25
Gūrmi T.S.	204	107	2
Gūru Sikkar H.S.	76	11	25
Gurwāni H.S.	320	92	5
Halda s.	232		8
Halkāchar T.S.	396	137	56
Harnāsa T.S.	108		18
Harpālsid T.S.	173		2
Hātbena H.S.	339		58
Hāthria H.S.	48	221	35
Hatni h.s.	149		6

Obs. = Observatory, Long. = Longitude. * Old Baluchistan Series.

INDEX TO DEFLECTION STATIONS—(*Continued*).

Name of Station		Reference Number Table XCV	Reference Number Table XCIV	No. of Series	Name of Station		Reference Number Table XCV	Reference Number Table XCIV	No. of Series	Name of Station		Reference Number Table XCV	Reference Number Table XCIV	No. of Series
Hirdepur	T.S.	324	116	15	Kheri	T.S.	141	65	33	Majhār	H.S.	205		2
Hōnnavalli	H.S.	137		49	Khimūāna	T.S.	67		23	Mal	H.S.	383		24
Hōnnūr	H.S.	264		9	Khirsar	H.S.	62		62	Malar	H.S.	41	83	64
Hurilāong	H.S.	356	94	5	Khojak	H.S.	4		†	Male	H.S.	423	159	66
Imlia	T.S.	306		12	Khori	T.S.	37	21	25	Malūncha	H.S.	375	98	5
Indrāwan	T.S.	107	225	18	Khujnaur	s.	147		20	Mandāla	s.	235		8
Injambākam	H.S.	350	233	54	Khundābolo	H.S.	381	191	24	Mandresa	T.S.	59	80	45
Inrogdi	T.S.	52	223	29	Kidarkanta	H.S.	157		22	Māndvi	H.S.	118	212	7
Isanpur	H.S.	139		33	Kisanen Chapper	H.S.	3	57	74	Manēgandi	S.	287	254	63
Jabalpur Longitude	s.	227		53	Kistama	H.S.	273	235	46	Mangalore Longitude	S.	134	230	54
Jalālpur	T.S.	352	124	21	Kodangal	S.	240	227	9	Manichauk	T.S.	313		20
Jalpaiguri Longitude	s.	389	122	34	Koh-i-Malik Siah	H S.	1	59	74	Marūr	T.S.	319	114	19
Jambo	H.S.	72	77	62	Koramūr	H.S.	133	241	49	Martaban	h.s.	440		52
Jāoli	H.S.	56	47	22	Kudankulam Obs.	S.	284	250	9	Mashelak	H.S.	5	53	76
Jarūra	T.S.	297		3	Kumbhāri	H.S.	132	244	11	Māsi	T.S.	305	104	20
Jetgarh	H.S.	85		23	Kumon Bum	H.S.	430	173	80	Māta-ka-hūra	H.S.	185	4	25
Jhambhera	T.S.	64	81	45	Kumtum Bum	H.S.	431	172	80	Māvinhūnda	H.S.	125		49
Jharipani (IX)	h.s.	162	36	20	Kundgol	H.S.	136		49	Mednipur	T.S.	354	123	21
Jharkil	T.S.	27	45	32	Kunkāvāv	T.S.	50	224	28	Medwāni	H.S.	138	48	22
Kainath	H.S.	93		26	Kurseong	h.s.	387		20	Mergui Base E.,	T.S.	446	149	52
Kaliāna	S.	153	31	6	Kutipārai	S.	289	248	9	Mergui Base W.,	T.S.	447	150	52
Kaliānpur	H.S.	195	1	6	Kyaunggyi	S.	428	146	52	Miāni	T.S.	23	42	32
Kaliānpur	T.S.	180	102	20	Lachkuwa	h.s	171		6	Mingun	H.S.	425	157	66
Kālingkonda	H.S.	341	185	43	Lādi	H.S.	215		8	Minthangtaung	H.S.	448	152	52
Kallapat	Trestle S.	292	251	63	Lādimslar	T.S.	33	79	45	Mira Donger	H.S.	117		11
Kalsībhānga	T.S.	377	205	17	Lakarwas	H.S.	83		25	Mirya	H.S.	120	242	11
Kalsubai	H.S.	116	220	10	Lakhinagar	T.S.	407	132	48	Mohammadabad	T.S.	210	110	4
Kāmkhera	H.S.	213		25	Lambatach	H.S.	143		22	Mooltan Longitude	S.	32		32
Kānākhera	T.S.	301		3	Letpataung	H.S.	437	155	66	Morali	H.S.	95		26
Kanheri	H.S.	128	208	7	Linganapalle	h.s.	241		9	Moulmein Longitude	S.	441		44
Kanjamalai	H S.	286	246	9	Lingmāra	H.S.	330		53	Mugrala	H.S.	61	78	62
Kankesvar	H.S.	114		11	Lohārgara	T.S.	391		16	Murree	h.s.	55	71	22
Kānkra	H.S.	183	61	33	Loi Hpa Lang	H.S.	433	168	72	Murree Observatory	s.	451		22
Kānnagar	H.S.	81	10	25	Loi Hpatan	H.S.	434	169	72	Mussooree Dome	H.S.	161	37	6
Karabgati	H.S.	126	240	49	Loi Hsam Hsum	H.S.	436	171	72	Myayabeingkyo	H.S.	439	153	66
Karachi Base S,	S.	14	39	32	Loijing	H.S.	413	165	68	Nagārkhāna	H.S.	411	134	52
Karachi Longitude	S.	16		22	Loi Kiipma	H.S.	435	170	72	Nagarkoil Longitude	S.	283		9
Karachi Observatory	S.	17	25	32	Lora	H.S.	327		5	Nag Tiba	H.S.	159	38	73
Karanja	H.S.	113	213	7	Losalli	S.	197	2	25	Nāharmau	H.S.	225		5
Karāra	H.S.	312	91	5	Lūnki	H.S.	42	17	25	Namthabad	s.	261		9
Karaundi	H.S.	226	197	53	Mach	h.s.	8		†	Natkalintaung	H.S.	445	151	52
Kardo	H.S.	92		26	Madhpur	T.S.	376	99	5	Naunangarhi	T.S.	351	119	20
Karia	H.S.	340	204	58	Madhupur	T.S.	400	129	16	Navalūr	H.S.	135		49
Kārothol	H.S.	13	24	25	Madras Observatory	S.	348		54	Nayinipiriyān Trestle	S.	294	252	63
Kātpālaiyam	s.	277		9	Mahabaleswar	H.S.	119		11	Niālamari	H.S.	257		46
Kaulia	H.S.	357		*	Mahadeo Pokra	H.S.	358		*	Nilgarh	H.S.	217	176	8
Kem	H.S.	130		7	Mahar	H.S.	363		14	Nimbāgal	s.	265		9
Kesri	H.S.	189	27	6	Mahesari	T.S.	174		6	Nimkār	T.S.	298	111	3
Khāmor	H.S.	87		23	Mahwari	H.S.	368		5	Nitali	H.S.	230	207	7
Khankharia	S.	47	13	25	Mairāb-ka-Shahar	T.S.	39	20	25	Noh	T.S.	186	29	6
Khānpisura	H.S.	121	210	7	Majala	H.S.	124		49	Nojli	T.S.	151		6

Obs. = Observatory. * Special Triangulation. † Old Baluchistān Triangulation. ‡ Thayetmo and Cape Negrais Series.

INDEX TO DEFLECTION STATIONS—(*Continued*).

Name of Station		Reference Number Table XCV	Reference Number Table XCIV	No. of Series
Nuāon	T.S.	353		21
Nughallibètta	H.S.	260	231	54
Ongole	H.S.	346		46
Orejhār	S.	316	106	19
Oria	h.s.	77		25
Pabhosa	H.S.	311	112	12
Pachapālaiyam	s.	276	247	9
Pāchvad	H.S.	123	239	49
Pahārgarh	H.S.	190	26	6
Pahladpur	T.S.	360		14
Pāldi	H.S.	97		29
Pandalagudi	s.	290		9
Paphra	T.S.	31	87	45
Parampūdi	H.S.	338	238	46
Pārasnāth	H.S.	373	96	5
Parewa	T.S.	308		20
Pariāon	T.S.	310		20
Pārnera	H.S.	98	219	10
Patāngdi	H.S.	99	217	10
Pathāidi	T.S.	333	202	58
Pātharankota	S.	295	253	63
Pathārdi	T.S.	314		20
Patna	T.S.	379	188	24
Pāvāgad	H.S.	101		10
Pāvagada	H.S.	266		9
Pavia	H.S.	302		3
Peddapād	s.	244		9
Peshawar Longitude	S.	18		32
Phallut	h.s.	384		20
Pialmudi	s.	242		9
Piaro	H.S.	10	50	69
Pirmulo	H.S.	249	182	43
Pushkar	H.S.	453	73	36
Pota	T.S.	359	125	14
Potenda	S.	303		3
Prome Longitude	S.	429		52
Punna Observatory	S.	285		9
Quetta Telegraph Office	S.	7	60	*
Rādhāpuram	S.	279	249	9
Raikusni	H.S.	398	142	34
Rājahāri	T.S.	322	118	15
Rājgarh	H.S.	86	67	23
Rājpur	h.s.	164	34	20
Rājuli	H.S.	229		53
Rākhi	T.S.	142	64	33
Ramai	H.S.	334	203	58
Rāmbāgh Observatory	s.	15		32
Rāmganj	T.S.	390	121	20
Rāmgir	H.S.	254		53
Rāmnad	S.	291	255	63
Rāmpūra	H.S.	91	6	25

Name of Station		Reference Number Table XCV	Reference Number Table XCIV	No. of Series
Rām Thal	S.	70		23
Rāmuapur (old)	T.S.	296	103	20
Rangīr (old)	S.	212	89	5
Rāngrai	s.	219		8
Rangsanobo	H.S.	399	141	44
Rānigarh	H.S.	172		20
Ranjītgarh	T.S.	57		23
Rāwal	H.S.	343	192	24
Retkamauk	H.S.	427	145	52
Rewat	H.S.	84		23
Robat	S.	449		†
Rojhra	H.S.	43	18	25
Rustamgarhi	h.s.	452	75	22
Sakri	H.S.	223	178	8
Sāler	H.S.	105	218	10
Salighar	H.S.	20	55	76
Salīmpur	T.S.	198		2
Salot	H.S.	192	3	25
Samdari	H.S.	75		62
Samenda	T.S.	323	117	15
Sānd	H.S.	88	8	25
Sandawat	H.S.	444		52
Sangatpur	T.S.	66	70	23
Sānjib	H.S.	342	186	43
Sankrāo	T.S.	178	108	2
Sarandi Pat	H.S.	328	198	53
Sarey Khan Latitude	S.	329		53
Sarkāra	T.S.	175		2
Sarla	S.	46	14	25
Saugor	H.S.	224		5
Sawaipur	T.S.	68		23
Seikpa	H.S.	416	162	66
Semu Tān	H.S.	410	133	52
Senchal	h.s.	386		20
Shāhpur	T.S.	58		23
Sheinmaga	H.S.	424	158	66
Shorpur	H.S.	148		6
Shūlakarai	s.	278		9
Sidhpur	S.	102		10
Siliguri	s.	388		20
Sindur	H.S.	335	194	85
Singāwāram	H.S.	336	184	43
Sinpitaung	H.S.	432	167	72
Sirkanda	T.S.	366	127	13
Sironj Base, N.E.,	S.	194		6
Sirsa	S.	69	69	23
Sirsa	T.S.	176	109	2
Sitāpār	H.S.	331		53
Somtana	H.S.	234	179	8
Sonāda	T.S.	96	216	29
Sora	T.S.	309	113	12

Name of Station		Reference Number Table XCV	Reference Number Table XCIV	No. of Series
Southern Moscos	H.S.	448	148	52
Spurpoint (VIII)	h.s.	168	35	20
St. Thomas's Mount	Tres. S.	349	284	54
Sultan-ka-Got	T.S.	22		32
Sūrāntal	H.S.	193		6
Takalkhera	s.	218		8
Talegaon	s.	236		8
Tamunja	H.S.	415	163	68
Tanakarakulam	S.	280		9
Tarbhān	S.	104		10
Tāsing	H.S.	181	63	33
Taungpila	H.S.	426	156	66
Taungzun	H.S.	442	147	52
Telu	H.S.	60		62
Teona	H.S.	355		21
Tepri	T.S.	405	135	56
Thīkri	H.S.	109		18
Thob	H.S.	74	76	62
Thonbinzin	H.S.	417	161	66
Thyolichiug	H.S.	414	164	68
Tiki	H.S.	82	9	25
Tilabani	H.S.	374	97	5
Tinsia	H.S.	196		25
Tiruvēndipuram	s.	293		63
Tonglu	h.s.	385		20
Tōnsalgutta	s.	243		9
Toungoo	S.	438	154	66
Tounsa	T.S.	28	56	76
Tuagat	h.s.	240		9
Tuzgi	H.S.	2	58	74
Ubyetaung	H.S.	422	160	66
Umarkhel	H.S.	19	46	82
Usira	H.S.	188	28	6
Utiāmau	T.S.	307		12
Valvādi	H.S.	110	226	18
Vānākonda	H.S.	256	183	43
Vijayāpati	S.	282		9
Vijnot	T.S.	26	85	64
Virāria	H.S.	44	16	25
Vizagapatām Base, N.	S.	344	187	24
Voi	s.	233		8
Waltair Longitude	S.	345		43
Yeponetaung	H.S.	420	166	71
Yērragunta	h.s	259		9
Yēttimalai	S.	275		9
Yūsuf	S.	34	40	82
Zawa	H.S.	450	52	74

Obs. = Observatory. Tres. S. = Trestle S. * Quetta Secondary Series. † Robat Triangulation.

Deflections of the Plumb-line

in terms of

any Spheroid.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID.							Serial No.
			Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A−G) cot λ for azimuth or (A−G) cos λ for longitude observations†	Meridian Deflection†	
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
1	30 C	Koh-i-Malik Siah H.S	5393	G 29 51 31·95	G 60 52 19·71	A 321 52 15·7 G 321 52 0·4	Kachakoh E 0 42	+ 26·7		1
	O	No. 449 *ut infra*								
2	L	Tuzgi H.S.	3131	G 28 53 14·38	G 62 14 58·97	A 216 16 46·3 G 216 16 37·3	Shuri E 0 40	+ 16·3		2
3	34 C	Kisanen Chapper H.S.	4362	G 29 3 54·41	G 64 22 24·93	A 166 46 22·6 G 166 46 15·7	Malik Praji E 1 7	+ 12·4		8
4	J	Khojak H.S.	7851	A 30 51 20·21 G 30 51 24·85	G 66 34 41·08				− 4·6	4
5	J	Mashelak H.S	7941	G 30 13 30·77	G 66 44 46·79	A 241 55 31·4 G 241 55 27·8	Takatu E 1 20	+ 6·2		5
		No. 450 *ut infra*								
6	M	Gundak H.S.	8163	G 31 9 49·49	G 67 23 28·55	A 276 29 27·1 G 276 29 25·2	Basha E 0 56	+ 3·1		6
7	N	Quetta Tel Office S.	5500	A 30 11 55·91 G 30 11 57·37	A 67 0 29·27 G 67 0 31·69	A 166 31 12·1 G 166 31 11·8	Takatu E 3 5	+ 0·5	− 1·5	7
8	O	Mach h.s.	3522	A 29 52 20·46 G 29 52 31·51	G 67 18 39·42				−11·1	8
9	O	Dasti S.	316	A 29 0 27·61 G 29 0 29·93	G 67 53 51·59				− 2·3	9
10	35 J	Piaro H.S.	1438	G 26 3 14·21	G 66 34 7·96	A 159 22 15·3 G 159 22 12·8	Kuliri E 0 16	+ 5·1		10
11	M	Gandpahar H.S.	723	G 27 25 1·26	G 67 30 43·99	A 192 10 55·2 G 192 10 46·4	Kharko D 0 9	+ 17·0		11
12	N	Andar H.S.	4047	G 26 1 22·07	G 67 12 10·58	A 181 7 6·5 G 181 7 1·7	Sulemani D 0 24	+ 9·8		12
13	P	Kārothol H.S.	260	A 24 53 44·78 G 24 53 46·69	G 67 53 32·47	A 121 36 58·0 G 121 36 55·3	Kara E 0 38	+ 5·8	− 1·9	18
14	P	Karachi Base-line S. End S.	46	G 24 52 59·63	G 67 9 24·77	A 205 23 30·5 G 205 23 29·2	Karachi Base-line N. End E 0 10	+ 2·8		14
15	P	Rāmbāgh Obsy. s.	...	A 24 51 20·58 G 24 51 21·44	G 67 0 55·21				− 0·9	15
16	P	Karachi Long. S.		G 24 51 2·44	A 67 0 52·88 G 67 0 53·22			− 0·3		16
17	P	Karachi Obsy. S.	35	A 24 49 50·14 G 24 49 50·25	G 67 1 35·13	A 221 39 9·5 G 221 39 8·4	Mutrani E 0 25	+ 2·4	− 0·1	17
18	38 N	Peshawar Long. S.	...	G 34 0 17·51	A 71 33 14·63 G 71 33 0·27			+ 11·9		18
19	P	Umarkhel H.S.	3036	G 32 25 31·07	G 71 15 20·79	A 73 9 24·0 G 73 9 15·3	Sistarg E 0 18	+ 13·7		19
20	39 A	Saighar H.S.	8284	G 31 29 53·82	G 68 25 5·49	A 1 52 50·1 G 1 52 46·2	Tanispa E 0 33	+ 6·4		20
21	D	Dumb h.s.	183	A 28 15 18·30 G 28 15 21·09	G 68 14 9·96				− 2·8	21
22	D	Sultan-ka-Got T.S.	213	A 28 4 8·05 G 28 4 9·41	G 68 36 32·23				− 1·4	22
23	H	Miāni T.S.	300	G 28 34 15·20	G 69 50 46·91	A 188 2 16·6 G 188 2 4·5	Routi D 0 5	+ 22·2		23
24	H	Dāowāla T.S.	282	G 28 20 12·87	G 69 50 30·68	A 28 49 27·9 G 28 49 21·3	Ghundi D 0 3	+ 12·2		24
25	H	Bhanar T.S.	256	G 28 8 55·00	G 69 17 11·38	A 197 50 8·8 G 197 50 0·7	Khai D 0 5	+ 15·1		25
26	H	Vijnot T.S.	276	G 28 2 3·30	G 69 50 32·77	A 159 35 15·6 G 159 35 10·0	Dewari D 0 5	+ 10·5		26

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I : $\delta a = 1$ km			Case II : $\delta b = 1$ km			Case III : Latitude $u_0 = 1''$			Case IV : Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$.					
	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″		″	
1			+8·80			−0·17			−0·58			+1·77			+10·11	+16·4		1
2			+8·23			−0·30			−0·55			+1·83			+9·56	+ 6·5		2
3			+7·08			−0·25			−0·48			+1·83			+8·57	+ 3·6		3
4	+1·64			−5·19			+0·98			+0·17			−1·78				−2·8	4
5			+5·71			−0·09			−0·38			+1·78			+7·38	− 1·5		5
6			+5·30			0·00			−0·34			+1·74			+7·04	− 4·2		6
7	+1·54	+6·36	+5·58	−4·74	−0·88	−0·09	+0·98	−0·09	−0·37	+0·17	+0·10	+1·78	−1·54	+5·26	+7·27	− 7·5	0·0	7
8	+1·48			−4·52			+0·98			+0·16			−1·44				−9·7	8
9	+1·32			−3·89			+0·98			+0·16			−1·12				−1·2	9
10			+6·30			−0·61			−0·45			+2·05			+7·87	− 3·0		10
11			+5·61			−0·39			−0·39			+1·95			+7·29	+ 9·5		11
12			+5·96			−0·57			−0·41			+2·05			+7·60	+ 2·0		12
13	+0·42		+5·73	−0·68		−0·69	+0·99		−0·41	+0·16		+2·13	+0·40		+7·40	− 1·8	−2·3	13
14			+6·17			−0·57			−0·43			+2·13			+7·76	− 5·2		14
15	+0·44			−0·64			+0·98			+0·17			+0·45				−1·4	15
16		+6·37			−0·90			−0·08			+0·01			+5·21		− 5·5		16
17	+0·44		+6·25	−0·63		−0·78	+0·98		−0·43	+0·17		+2·14	+0·46		+7·83	− 5·6	−0·6	17
18		+3·64			−0·49			−0·06			+0·17			+3·20		+ 8·7		18
19			+3·21			+0·06			−0·20			+1·70			+5·14	+ 8·1		19
20			+4·75			+0·02			−0·31			+1·71			+6·51	− 0·4		20
21	+1·17			−3·33			+0·99			+0·15			−0·89				−1·9	21
22	+1·12			−3·19			+0·99			+0·14			−0·85				−0·5	22
23			+4·22			−0·22			−0·29			+1·89			+6·09	+15·7		23
24			+4·25			−0·22			−0·30			+1·91			+6·14	+ 5·9		24
25			+4·56			−0·26			−0·32			+1·91			+6·38	+ 8·3		25
26			+4·26			−0·24			−0·28			+1·92			+6·15	+ 4·1		26

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE.

Deflections of the Plumb-line

Serial No	Sheet No	Observed at	EVEREST'S SPHEROID.							Serial No.
			Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed Station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
27	39 I	Jharkil T.S.	532	G 31 21 13·65	G 70 59 44·80	A 208 7 9·2 G 208 7 3·6	Kasain D 0 3	+ 9·2		27
28	J	Tounsa T.S.	593	G 30 41 51·59	G 70 39 0·13	A 201 7 57·5 G 201 7 41·4	Langawala D 0 11	+ 27·1		28
29	J	Dera Dīn Panāh S.	490	A 30 33 59·63 G 30 34 1·87	G 70 56 7·29	A 209 21 14·7 G 209 21 7·3	Sakwala D 0 4	+ 12·5	− 2·2	29
30	K	Dājil S.	412	G 29 33 20·87	G 70 22 52·98	A 239 26 6·7 G 239 25 53·0	Dalura D 0 6	+ 24·2		30
31	K	Paphra T.S.	316	G 29 5 49·37	G 70 49 45·82	A 273 23 2·0 G 273 22 56·8	Ohanikhan D 0 4	+ 9·3		31
32	N	Mooltan Long. S.	420	A 30 10 56·15 G 30 10 58·70	A 71 26 22·19 G 71 26 27·39			− 4·5	− 2·6	32
33	O	Lādimsir T.S.	468	A 29 21 39·83 G 29 21 41·58	G 71 59 19·71	A 195 0 23·1 G 195 0 22·1	Gaddan D 0 6	+ 1·8	− 1·8	33
34	40 A	Yūsuf S.	215	G 27 51 8·74	G 68 26 14·75	A 195 51 20·0 G 195 51 16·5	Salar D 0 5	+ 6·6		34
35	D	Alamkhān T.S.	67	A 24 49 30·50 G 24 49 31·23	G 68 43 47·38	A 174 28 43·3 G 174 28 39·0	Hakimani D 0 4	+ 9·3	− 0·7	35
36	D	Chūtli T.S.	72	G 24 46 19·67	G 68 23 40·86	A 141 22 40·1 G 141 22 35·0	R.M.	+ 11·1		36
37	G	Khori T.S.	63	A 25 0 30·60 G 25 0 31·53	G 69 3 5·32	A 247 8 33·5 G 247 8 32·8	Jan Mahamad D 0 5	+ 1·5	− 0·9	37
38	H	Chānga H.S.	349	A 24 58 47·25 G 24 58 47·00	G 69 51 23·30	A 238 0 7·4 G 238 0 9·1	Sandohar E 0 0	− 3·6	+ 0·3	38
39	H	Mairāb-ka-Shahar T.S.	44	G 24 50 10·79	G 69 20 25·56	A 181 11 36·7 G 181 11 34·6	Amisha D 0 5	+ 4·5		39
40	I	Asu H.S.	479	G 27 10 32·14	G 70 10 59·67	A 201 37 32·6 G 201 37 31·7	Kalu D 0 9	+ 1·8		40
41	J	Malar H.S.	328	G 26 2 25·80	G 70 3 36·19	A 161 26 22·4 G 161 26 23·4	Ramsar D 0 6	− 2·0		41
42	L	Lūnki H.S.	588	A 24 58 18·73 G 24 58 23·15	G 70 39 42·32	A 255 9 1·4 G 255 8 58·2	Karebhit D 0 3	+ 6·9	− 4·4	42
43	L	Rojhra H.S.	518	A 24 57 26·09 G 24 57 26·28	G 70 14 17·90	A 254 1 46·8 G 254 1 44·8	Dharindera D 0 3	+ 4·3	− 0·2	43
44	P	Virāria H.S.	460	A 24 56 32·64 G 24 56 36·13	G 71 2 58·81	A 106 12 49·8 G 106 12 46·3	Karebhit D 0 1	+ 7·5	− 3·5	44
45	P	Didāwa H.S.	212	A 24 51 17·32 G 24 51 19·36	G 71 18 57·69	A 72 32 16·7 G 72 32 14·0	Sohagi D 0 2	+ 5·8	− 2·0	45
46	P	Sarla S.	132	G 24 46 44·68	G 71 34 7·48	A 244 27 47·6 G 244 27 43·2	Dawal D 0 3	+ 9·5		46
47	P	Khankharia S.	362	A 24 36 58·17 G 24 36 56·19	G 71 53 8·91	A 182 0 14·8 G 182 0 15·2	Kosia D 0 6	− 0·9	+ 2·0	47
48	41 E	Hāthria H.S.	696	G 23 27 14·85	G 69 2 45·83	A 154 56 32·9 G 154 56 34·6	Sura Gandara E 0 1	− 3·9		48
49	J	Dungarpur H.S.	404	A 22 48 8·85 G 22 48 13·54	G 70 59 39·44	A 199 56 38·7 G 199 56 32·8	Chalarwa D 0 15	+ 14·0	− 4·7	49
50	K	Kunkāvāv T.S.	621	A 21 39 10·31 G 21 39 11·96	G 70 56 8·89	A 161 59 40·0 G 161 59 35·5	Mumaiya D 0 10	+ 11·3	− 1·7	50
51	L	Dangarvadi H.S.	96	A 20 42 52·01 G 20 43 0·53	G 70 56 5·21				− 8·5	51
52	N	Ingrodi T.S.	152	A 22 57 2·50 G 22 57 7·58	G 71 48 34·12	A 198 26 44·5 G 198 26 39·4	Por D 0 8	+ 12·0	− 5·1	52
53	O	Chamardi H.S.	361	A 21 49 23·88 G 21 49 26·65	G 71 55 4·34				− 2·8	53
54	43 F	Ganga Chhoti H.S.	9989	G 34 4 31·37	G 73 44 52·15	A 174 38 11·1 G 174 38 24·0	Kafirkhan E 1 7	− 19·1		54

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: δa=1 km			Case II: δb=1 km			Case III: Latitude $u_0=1''$			Case IV: Azimuth $w_0=1''$			a=6378200 metres, $1/\epsilon$=298·3					
	u	$v\cos\lambda$	$w\cot\lambda$	u	$v\cos\lambda$	$w\cot\lambda$	u	$v\cos\lambda$	$w\cot\lambda$	u	$v\cos\lambda$	$w\cot\lambda$	u	$v\cos\lambda$	$w\cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
27			+3·92			0·00			−0·23			+1·74			+5·79	+ 3·1		27
28			+3·65			−0·03			−0·24			+1·77			+5·56	+ 21·2		28
29	+1·43		+3·52	−4·99		−0·03	+0·99		−0·24	+0·11		+1·78	−1·94		+5·46	+ 6·7	− 0·3	29
30			+3·86			−0·05			−0·26			+1·83			+5·81	+ 18·1		30
31			+3·67			−0·14			−0·25			+1·87			+5·62	+ 3·5		31
32	+1·37	+2·17		−4·72	−0·30		+0·99	−0·04		+0·10	+0·06		−1·80	+1·86		− 6·4	− 0·8	32
33	+1·38		+3·02	−4·14		−0·11	+1·00		−0·20	+0·09		+1·85	−1·37		+5·04	− 3·4	− 0·4	33
34			+5·05			−0·30			−0·34			+1·93			+6·83	− 0·6		34
35	+0·37		+5·25	−0·61		−0·65	+0·99		−0·37	+0·14		+2·15	+0·37		+7·03	+ 2·1	− 1·1	35
36			+5·45			−0·68			−0·38			+2·15			+7·18	+ 3·7		36
37	+0·40		+5·03	−0·77		−0·61	+0·99		−0·35	+0·14		+2·14	+0·29		+6·86	− 5·6	− 1·2	37
38	+0·37		+4·57	−0·74		−0·55	+0·99		−0·32	+0·12		+2·14	+0·26		+6·47	− 10·3	0·0	38
39			+4·89			−0·61			−0·35			+2·15			+6·73	− 2·4		39
40			+4·16			−0·31			−0·28			+1·98			+6·07	− 4·5		40
41			+4·38			−0·42			−0·30			+2·06			+6·30	− 8·5		41
42	+0·34		+4·10	−0·73		−0·50	+0·99		−0·29	+0·11		+2·14	+0·22		+6·09	+ 0·6	− 4·6	42
43	+0·35		+4·35	−0·72		−0·53	+0·99		−0·31	+0·12		+2·14	+0·25		+6·28	− 2·2	− 0·5	43
44	+0·32		+3·88	−0·70		−0·48	+0·99		−0·27	+0·11		+2·15	+0·22		+5·92	+ 1·4	− 3·7	44
45	+0·29		+3·73	−0·63		−0·47	+0·99		−0·26	+0·10		+2·16	+0·24		+5·81	− 0·2	− 2·2	45
46			+3·59			−0·46			−0·25			+2·16			+5·69	+ 3·6		46
47	+0·21		+3·42	−0·43		−0·45	+1·00		−0·24	+0·09		+2·18	+0·31		+5·57	− 6·5	+ 1·7	47
48			+5·26			−0·82			−0·73			+2·27			+7·06	− 11·0		48
49	−0·29		+4·16	+1·12		−0·75	+0·99		−0·30	+0·11		+2·34	+1·01		+6·21	+ 8·0	− 5·7	49
50	−0·66		+4·35	+1·14		−0·89	+0·99		−0·31	+0·11		+2·46	+0·59		+6·43	+ 5·2	− 2·3	50
51	−0·98			+2·79			+1·00			+0·11			+1·61				−10·1	51
52	−0·27		+4·08	+0·99		−0·63	+1·00		−0·22	+0·08		+2·33	+0·94		+6·23	+ 6·0	− 6·0	52
53	−0·63			+1·99			+1·00			+0·09			+1·35				− 4·2	53
54			+1·95			+0·07			−0·12			+1·62			+3·90	− 23·4		54

* δa=0·924, δb=0·743. u_0=0·31, w_0=1·29. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID.							Serial No.
			Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A−G) cot λ for azimuth or (A−G) cos λ for longitude observations†	Meridian Deflection†	
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
	43 G	No. 451 *ut infra*								
55	G	Murree h.s.	7250	A 33 54 37·35 G 33 54 57·35	G 73 22 50·15				−20·0	55
56	G	Jāoli H.S.	1918	G 33 16 48·85	G 73 10 26·50	A 214 27 23·4 G 214 27 22·2	Nerh E 0 58	+ 1·8		56
	J	No. 452 *ut infra*								
	J	No. 453 *ut infra*								
	K	No. 454 *ut infra*								
57	L	Ranjītgarh T.S.	900	A 32 35 6·52 G 32 35 12·11	G 74 37 12·48				− 5·6	57
58	P	Shāhpur T.S.	830	A 32 1 34·23 G 32 1 33·77	G 75 5 34·90				+ 0·5	58
59	44 C	Mandresa T.S.	512	G 29 55 9·17	G 72 59 28·42	A 298 34 7·1 G 298 34 5·8	Gajlani D 0 1	+ 2·3		59
60	D	Telu H.S.	470	A 28 56 12·41 G 28 56 11·34	G 72 14 8·80				+ 1·1	60
61	D	Mugrala H.S.	517	G 28 30 57·06	G 72 22 17·41	A 171 53 31·2 G 171 53 32·0	Habib D 0 8	− 1·5		61
62	D	Khirsar H.S.	603	A 28 29 43·75 G 28 29 40·91	G 72 39 32·34				+ 2·8	62
63	F	Akbar S.	641	A 30 53 38·53 G 30 53 43·27	G 73 17 13·28	A 216 51 25·8 G 216 51 25·4	Firoz D 0 5	+ 0·7	− 4·7	63
64	F	Jhambhera T.S.	630	G 30 5 59·27	G 73 49 16·30	A 185 27 27·5 G 185 27 29·9	Fatehgarh D 0 8	− 4·1		64
65	I	Amritsar Long. S.	770	A 31 38 2·51 G 31 37 58·72	A 74 52 26·46 G 74 52 23·45			+ 2·6	+ 3·8	65
66	M	Sangatpur T.S.	779	A 31 17 35·42 G 31 17 34·43	G 75 2 19·27	A 61 34 52·8 G 61 34 49·1	Rabza D 0 5	+ 6·1	+ 1·0	66
67	N	Khimūāna T.S.	731	A 30 22 11·74 G 30 22 14·82	G 75 0 42·52				− 3·1	67
68	O	Sawaipur T.S.	697	A 29 39 13·13 G 29 39 13·96	G 75 3 6·12				− 0·8	68
69	O	Sirsa S.	738	G 29 31 35·39	G 75 1 14·76	A 17 11 0·2 G 17 10 58·1	Banka D 0 6	+ 3·7		69
70	P	Rām Thal S.	951	A 28 29 38·81 G 28 29 39·27	G 75 0 10·60				− 0·5	70
71	45 A	Bithnok H.S.	774	A 27 53 24·97 G 27 53 22·03	G 72 39 54·55				+ 2·9	71
72	A	Jambo H.S.	772	A 27 16 31·94 G 27 16 28·88	G 72 31 5·53	A 153 23 42·9 G 153 23 42·2	Sirad D 0 7	+ 1·4	+ 3·1	72
73	B	Chamu H.S.	1065	A 26 39 53·44 G 26 39 52·74	G 72 35 26·28				+ 0·7	73
74	B	Thob H.S.	856	A 26 3 2·90 G 26 3 5·85	G 72 22 22·17	A 322 26 25·5 G 322 26 20·4	Samdari D 0 8	+ 10·4	− 3·0	74
75	C	Samdari H.S.	846	A 25 48 59·58 G 25 48 59·55	G 72 34 20·84				0·0	75
76	D	Gūru Sikkar H.S.	5650	G 24 38 58·39	G 72 46 39·73	A 248 53 38·4 G 248 53 36·1	Belka D 1 2	+ 5·0		76
77	D	Oria h.s.	4200	A 24 37 47·63 G 24 37 50·96	G 72 45 39·23				− 3·3	77
78	D	Birona S.	673	G 24 26 38·64	G 72 13 4·51	A 121 43 10·7 G 121 43 10·9	Sitora D 0 8	− 0·4		78

*A = Astronomical Value.
G = Triangulated or Geodetic Value.

†*Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I : $\delta a = 1$ km			Case II : $\delta b = 1$ km			Case III : Latitude $u_0 = 1''$			Case IV : Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$.					
	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
55	+1·65			−7·15			+1·00			+0·07			−3·39				− 16·6	55
56			+2·25			+0·06			−0·14			+1·66			+4·24	− 2·9		56
57	+1·55			−6·33			+1·00			+0·05			−2·90				− 2·7	57
58	+1·52			−6·12			+1·00			+0·04			−2·78				+ 3·3	58
59			+2·46			−0·06			−0·16			+1·82			+4·53	− 2·5		59
60	+1·16			−3·83			+1·00			+0·09			−1·35				+ 2·5	60
61			+2·87			−0·13			−0·16			+1·90			+4·95	− 6·7		61
62	+1·07			−3·50			+1·00			+0·08			−1·20				+ 4·0	62
63	+1·43		+2·27	−5·21		−0·02	+1·00		−0·15	+0·07		+1·77	−2·06		+4·32	− 3·9	− 2·6	63
64			+2·02			−0·04			−0·13			+1·81			+4·13	− 8·5		64
65	+1·46	+1·67		−5·70	−0·23		+1·00	−0·03		+0·04	+0·13		−2·51	+1·53		+ 1·1	+ 6·3	65
66	+1·43		+1·35	−5·48		0·00	+1·00		−0·15	+0·04		+1·76	−2·39		+3·47	+ 2·3	+ 3·4	66
67	+1·31			−4·85			+1·00			+0·04			−2·03				− 1·1	67
68	+1·22			−3·40			+1·00			+0·04			−1·04				+ 0·2	68
69			+1·40			−0·02			−0·10			+1·85			+3·64	− 0·1		69
70	+1·01			−3·49			+1·00			+0·04			−1·30				+ 0·8	70
71	+0·96			−3·04			+1·00			+0·08			−0·97				+ 3·9	71
72	+0·83		+2·86	−2·57		−0·21	+1·00		−0·20	+0·08		+1·90	−0·72		+4·87	− 3·7	+ 3·8	72
73	+0·70			−2·09			+1·00			+0·08			−0·50				+ 1·2	73
74	+0·57		+3·02	−1·59		−0·37	+1·00		−0·21	+0·08		+2·07	−0·18		+5·12	+ 5·3	− 2·8	74
75	+0·50			−1·41			+1·00			+0·08			−0·18				+ 0·2	75
76			+2·89			−0·38			−0·20			+2·17			+5·13	− 0·1		76
77	+0·19			−0·44			+1·00			+0·08			+0·27				− 3·6	77
78			+3·24			−0·44			−0·23			+2·20			+5·43	− 5·8		78

* $\delta a = 0 \cdot 743$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
79	45 D	Deesa Tel. Office S.	443	A 24 15 21·15 G 24 15 29·35	A 72 11 1·26 G 72 11 4·85	A 241 16 15·3 G 241 16 15·5	Jairāj E 1 21	− 0·4	− 8·2	79
80	D	Ohaniāna H.S.	953	A 24 6 25·39 G 24 6 36·64	G 72 32 19·66				−11·3	80
81	H	Kānnagar H.S.	3607	G 24 58 28·78	G 73 18 59·95	A 266 45 16·1 G 266 45 19·0	Māl Niver E 0 2	− 6·2		81
82	H	Tiki H.S.	2369	A 24 55 34·52 G 24 55 38·24	G 73 50 44·41	A 106 4 27·1 G 106 4 23·4	Māl Niver E 0 57	+ 8·0	− 3·7	82
83	H	Lakarwas H.S.	2574	A 24 31 41·05 G 24 31 47·99	G 73 49 43·23				− 6·9	83
84	J	Rewat H.S.	1542	A 26 53 54·74 G 26 53 53·98	G 74 16 53·79				+ 0·8	84
85	J	Jetgarh H.S.	1967	A 26 18 8·02 G 26 18 6·39	G 74 18 36·91				+ 1·6	85
86	J	Rājgarh H.S.	2618	G 26 17 49·31	G 74 35 44·37	A 156 43 41·0 G 156 43 39·9	Kisanpura D 0 8	+ 2·2		86
87	K	Khāmor H.S.	1393	A 25 45 11·00 G 25 45 15·01	G 74 47 29·19				− 4·0	87
88	L	Sānd H.S.	1910	G 24 43 6·11	G 74 32 58·48	A 284 36 7·8 G 284 36 3·9	Mendki D 0 7	+ 8·5		88
89	L	Aramlia S.	1532	A 24 25 2·66 G 24 25 7·27	G 74 59 5·69	A 244 39 1·5 G 244 38 58·9	Nanka Hūāro E 0 5	+ 5·7	− 4·6	89
90	M	Garinda S.	1204	A 27 55 30·05 G 27 55 30·55	G 75 1 18·47	A 115 55 45·3 G 115 55 42·2	Biramsir D 0 3	+ 5·8	− 0·5	90
91	P	Rāmpūra H.S.	1920	G 24 28 44·16	G 75 26 52·24	A 260 5 35·8 G 260 5 35·0	Nimthūr D 0 16	+ 1·8		91
92	46 A	Kardo H.S.	807	A 23 57 2·27 G 23 57 10·02	G 72 43 52·88				− 7·8	92
93	A	Kainath H.S.	1385	A 23 51 14·99 G 23 51 23·79	G 72 58 51·75				− 8·8	93
94	A	Dhamanva T.S.	397	A 23 32 2·65 G 23 32 8·40	G 72 30 56·82				− 5·8	94
95	A	Morali H.S.	466	A 23 25 17·47 G 23 25 23·18	G 72 57 44·96				− 5·7	95
96	A	Sonāda T.S.	250	A 23 7 15·61 G 23 7 19·89	G 72 46 0·14	A 334 35 18·1 G 334 35 10·2	Mirzāpur D 0 5	+ 18·5	− 4·3	96
97	B	Pāldi H.S.	208	A 22 53 51·60 G 22 53 57·07	G 72 31 30·86				− 5·5	97
98	D	Pārnera H.S.	614	A 20 32 49·83 G 20 32 56·85	G 72 56 56·42	A 349 0 27·3 G 349 0 13·6	Gambhirgad E 0 17	+ 36·5	− 7·0	98
99	F	Patāngdi H.S.	922	G 22 52 15·70	G 73 53 22·34	A 16 47 30·9 G 16 47 26·6	Bhor D 0 1	+ 10·2		99
100	F	Ghorārāo H.S.	323	A 22 52 8·05 G 22 52 11·17	G 73 21 25·45				− 3·1	100
101	F	Pāvāgad H.S.	2721	A 22 27 39·95 G 22 27 44·33	G 73 31 1·07				− 4·4	101
102	F	Sidhpur S.	169	A 22 4 11·77 G 22 4 15·21	G 73 28 59·81				− 3·4	102
103	G	Alamvādi H.S.	848	A 21 34 30·45 G 21 34 34·13	G 73 30 9·20				− 3·7	103
104	G	Tarbhān S.	140	A 21 0 28·36 G 21 0 34·13	G 73 3 49·79				− 5·8	104
105	H	Sāler H.S.	5140	G 20 43 18·44	G 73 56 21·93	A 151 26 55·7 G 151 26 50·4	Dopāri D 1 29	+ 14·0		105
106	M	Deo Dongri H.S.	1727	A 23 26 43·17 G 23 26 47·79	G 75 32 16·99				− 4·6	106

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: δa=1 km			Case II: δb=1 km			Case III: Latitude $u_0=1''$			Case IV: Azimuth $w_0=1''$			a=6378200 metres, $1/e$=298·3.					
	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
79	+0·11	+3·28	+3·27	−0·13	−0·47	−0·46	+1·00	−0·04	−0·23	+0·09	0·00	+2·21	+0·43	+2·67	+5·46	− 5·9	− 8·6	79
80	+0·07			−0·04			+1·00			+0·09			+0·46				− 11·8	80
81			+2·55			−0·27			−0·18			+2·16			+4·89	− 11·1		81
82	+0·25		+2·24	−0·69		−0·29	+1·00		−0·16	+0·06		+2·15	+0·11		+4·57	+ 3·4	− 3·8	82
83	+0·15			−0·35			+1·00			+0·06			+0·26				− 7·2	83
84	+0·72			−2·27			+1·00			+0·06			−0·64				+ 1·4	84
85	+0·59			−1·80			+1·00			+0·05			−0·42				+ 2·0	85
86			+1·73			−0·17			−0·12			+2·06			+4·09	− 1·9		86
87	+0·45			−1·36			+1·00			+0·05			−0·24				− 3·8	87
88			+1·85			−0·24			−0·13			+2·18			+4·30	+ 4·2		88
89	+0·10		+1·50	−0·26		−0·22	+1·00		−0·11	+0·05		+2·20	+0·26		+4·12	+ 1·6	− 4·9	89
90	+0·92		+1·46	−3·07		−0·07	+1·00		−0·10	+0·06		+1·95	−1·04		+3·78	+ 1·8	+ 0·5	90
91			+1·32			−0·18			−0·09			+2·20			+3·90	− 2·1		91
92	+0·01			+0·13			+1·00			+0·08			+0·52				− 8·3	92
93	−0·02			+0·22			+1·00			+0·07			+0·54				− 9·3	93
94	−0·11			+0·49			+1·00			+0·08			+0·68				− 6 5	94
95	−0·15			+0·59			+1·00			+0·08			+0·70				− 6·4	95
96	−0·24		+3·03	+0·85		−0·52	+1·00		−0·26	+0·08		+2·32	+0·82		+5·32	+ 13·4	− 5·1	96
97	−0·30			+1·04			+1·00			+0·08			+0·91				− 6·4	97
98	−1·09		+3·17	+3·25		−0·75	+1·00		−0·24	+0·08		+2·59	+1·81		+5·63	+ 31·7	− 8·8	98
99			+2·35			−0·42			−0·17			+2·34			+4·74	+ 6·0		99
100	−0·32			+1·07			+1·00			+0·07			+0·90				− 4·0	100
101	−0·46			+1·43			+1·00			+0·07			+1·04				− 5·4	101
102	−0·58			+1·78			+1·00			+0·07			+1·18				− 4·6	102
103	−0·48			+2·23			+1·00			+0·07			+1·61				− 5·3	103
104	−0·59			+2·73			+1·00			+0·07			+1·88				− 7·7	104
105			+2·49			−0·74			−0·18			+2·57			+5·03	+ 9·5		105
106	−0·18			+0·68			+1·00			+0·04			+0·69				− 5·3	106

* δa=0·924, δb=0·743, u_0=0·31, w_0=1·29. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
107	46N	Indrāwan T.S.	1834	G 22 48 48·54	G 75 10 56·60	A 273 34 2·7 G 273 34 1·9	Harnāsa D 0 9	+ 1·9		107
108	N	Harnāsa T.S.	1818	A 22 47 26·71 G 22 47 29·91	G 75 33 10·15				− 3·2	108
109	N	Thīkri H.S.	854	A 22 1 3·92 G 22 1 2·77	G 75 24 49·98				+ 1·2	109
110	P	Valvādi H.S.	1128	A 20 44 21·27 G 20 44 27·73	G 75 11 7·12	A 166 52 6·2 G 166 51 59·8	Ajnād D 0 7	+ 16·9	− 6·5	110
111	47B	Bombay, Colaba Long S.	75	A 18 53 39·15 G 18 53 49·48	A 72 48 55·82 G 72 48 49·10			+ 6·4	−10·3	111
112	B	Bombay, Colaba Obsy. S.	63	G 18 53 46·51	G 72 48 47·31	A 288 5 27·7 G 288 5 24·5	Karanja E 1 7	+ 9·3		112
113	B	Karanja H.S.	997	A 18 51 13·79 G 18 51 24·99	G 72 56 21·88	A 173 10 2·5 G 173 9 56·1	Trombay D 0 4	+ 18·7	−11·2	113
114	B	Kankesvar H.S.	1260	A 18 44 17·89 G 18 44 28·16	G 72 55 34·09				−10·3	114
115	B	Alibagh Obsy. S.	10	A 18 38 26·35 G 18 38 36·69	G 72 52 12·42				−10·3	115
116	E	Kalsubai H.S.	5400	A 19 35 57·89 G 19 36 1·76	G 73 42 35·26	A 73 2 14·5 G 73 2 11·8	Kāmandrug D 1 1	+ 7·6	− 3·9	116
117	F	Mira Donger H.S.	1863	A 18 40 55·97 G 18 41 1·68	G 73 9 48·88				− 5·7	117
118	F	Māndvi H.S.	4121	A 18 37 47·94 G 18 37 51·11	G 73 32 21·71	A 271 15 3·9 G 271 15 7·8	Dighi D 0 56	− 11·6	− 3·2	118
119	G	Mahabaleswar H.S.	4719	A 17 55 9·91 G 17 55 15·55	G 73 40 17·41				− 5·6	119
120	G	Mirya H.S.	473	A 17 1 29·65 G 17 1 35·92	G 73 15 39·43	A 167 2 11·4 G 167 2 7·4	Adhūr D 0 12	+ 13·1	− 6·3	120
121	J	Khānpisura H.S.	2751	A 18 45 22·60 G 18 45 30·65	G 74 47 49·81	A 191 14 39·3 G 191 14 43·9	Agargaon D 0 2	− 13·5	− 8·1	121
122	J	Dhaulesvar H.S.	2939	A 18 25 42·84 G 18 25 41·64	G 74 9 48·48	A 198 21 22·6 G 198 21 24·4	Bābulsār D 0 31	− 5·4	+ 1·2	122
123	K	Pāchvad H.S.	3138	G 17 31 1·97	G 74 39 43·71	A 331 12 27·4 G 331 12 29·6	Palsi D 0 13	− 7·0		123
124	L	Majala H.S.	2613	A 16 46 55·45 G 16 46 56·82	G 74 26 55·57				− 1·4	124
125	L	Māvinhūnda H.S.	2582	A 16 25 4·47 G 16 25 4·19	G 74 47 40·38				+ 0·3	125
126	L	Karabgati H.S.	2544	G 16 7 34·87	G 74 47 56·35	A 179 9 24·9 G 179 9 26·9	Māvinhūnda D 0 7	− 6·9		126
127	M	Dhaigaon S.	1553	A 19 30 30·82 G 19 30 35·04	G 75 12 43·81				− 4·2	127
128	N	Kanheri H.S.	2610	A 18 29 21·84 G 18 29 30·75	G 75 43 16·69	A 311 59 50·6 G 311 59 53·3	Garh Dāud D 0 16	− 8·1	− 8·9	128
129	N	Alsunda H.S.	2165	G 18 26 52·37	G 75 0 35·11	A 227 31 58·4 G 227 32 1·8	Sautāra D 0 4	− 10·2		129
130	N	Kem H.S.	1951	A 18 10 45·68 G 18 10 48·90	G 75 18 23·92				− 3·2	130
131	48 E	Chaukola H.S.	2794	A 15 55 24·94 G 15 55 31·44	G 73 59 21·13	A 166 14 13·4 G 166 14 13·2	Valvan D 0 5	+ 0·7	− 6·5	131
132	I	Kumbhāri H.S.	2898	A 15 9 4·31 G 15 9 1·80	G 74 17 47·20	A 154 15 36·5 G 154 15 33·4	Salili D 0 30	+ 11·4	+ 2·5	132
133	J	Koramūr H.S.	2525	A 14 8 1·71 G 14 8 6·59	G 74 58 24·07	A 235 28 6·8 G 235 28 9·7	Hōnnavalli E 0 3	− 11·5	− 4·9	133
134	L	Mangalore Long S.	186	A 12 52 17·76 G 12 52 14·76	A 74 50 44·70 G 74 50 42·71	A 205 52 50·8 G 205 52 49·6	Mijār E 0 17	+ 5·2	+ 3·0	134

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: $\delta a = 1$ km			Case II: $\delta b = 1$ km			Case III: Latitude $u_0 = 1''$			Case IV: Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/e = 298 \cdot 3$.					
	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
107			+1·55			−0·31			−0·12			+2·35			+4·20	−2·1		107
108	−0·38			+1·15			+1·00			+0·04			+0·85				−4·1	108
109	−0·63			+1·83			+1·00			+0·04			+1·14				+0·1	109
110	−1·06		+1·66	+2·99		−0·39	+1·00		−0·12	+0·04		+2·57	+1·61		+4·53	+12·9	−8·1	110
111	−1·70	+2·89		+4·70	−0·41		+1·00	−0·03		+0·08	−0·09		+2·33	+2·24		+4·2	−12·6	111
112			+3·48			−0·99			−0·32			+2·81			+6·00	+4·2		112
113	−1·72		+3·40	+4·74		−0·97	+1·00		−0·31	+0·08		+2·81	+2·34		+5·95	+13·6	−13·5	113
114	−1·66			+4·85			+1·00			+0·08			+2·47				−12·8	114
115	−1·80			+4·95			+1·00			+0·08			+2·42				−12·7	115
116	−1·45		+2·76	+4·04		−0·73	+1·00		−0·21	+0·06		+2·72	+2·05		+5·44	+3·0	−6·0	116
117	−1·79			+4·91			+1·00			+0·07			+2·39				−8·1	117
118	−1·82		+2·83	+4·64		−0·82	+1·00		−0·21	+0·07		+2·69	+2·16		+5·41	−16·1	−5·4	118
119	−2·10			+5·65			+1·00			+0·06			+2·64				−8·2	119
120	−2·46		+3·44	+6·52		−1·17	+1·00		−0·26	+0·07		+3·11	+2·97		+6·24	+7·8	−9 3	120
121	−1·79		+2·07	+4·84		−0·60	+1·00		−0·16	+0·05		+2·84	+2·32		+5·08	−17·7	−10·4	121
122	−1·91		+2·55	+5·16		−0·76	+1·00		−0·19	+0·06		+2·88	+2·45		+5·45	−10·0	−1·3	122
123			+2·29			−0·75			−0·17			+3·03			+5·40	−11·1		128
124	−2·59			+6·77			+1·00			+0·05			+3·01				−4·4	124
125	−2·75			+7·13			+1·00			+0·05			+3·13				−2·8	125
126			+2·36			−0·92			−0·18			+3·27			+5·65	−11·2		126
127	−1·57			+4·29			+1·00			+0·04			+2·10				−6·3	127
128	−1·92		+1·42	+5·11		−0·20	+1·00		−0·11	+0·03		+2·88	+2·38		+4·84	−12·0	−11·3	128
129			+1·94			−0·36			−0·15			+2·88			+5·27	−14·6		129
130	−2·03			+5·40			+1·00			+0·04			+2·69				−5 9	130
131	−2·95		+3·04	+7·62		−1·15	+1·00		−0·24	+0·06		+3·32	+3·33		+7·08	−5·3	−9·8	131
132	−3·30		+2·91	+8·41		−1·17	+1·00		−0·23	+0·05		+3·49	+3·58		+6·25	+6·6	−1·1	132
133	−3·77		+2·47	+9 45		−1·09	+1·00		−0·19	+0·04		+3·74	+3·91		+6·24	−15·7	−8·8	133
134	−4·36	+1·67	+2·80	+10·77	−0·21	−1·36	+1·00	−0·01	−0·22	+0·05	−0·19	+4·09	+4·34	+1·13	+6·79	+1·0	−1·3	134

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
135	48 M	Navalūr H.S.	2445	A 15 25 28·48 G 15 25 31·17	G 75 3 15·42				− 2·7	135
136	M	Kundgol H.S.	2145	A 15 15 14·46 G 15 15 15·28	G 75 14 46·64				− 0·8	136
137	N	Hōnnavalli H.S.	2777	A 14 16 30·76 G 14 16 32·46	G 75 10 58·48				− 1·7	137
138	53 A	Medwāni H.S.	1935	G 31 17 40·45	G 76 11 58·72	A 64 43 36·5 G 64 43 41·6	Hiu D 0 54	− 8·4		138
139	B	Isanpur H.S.	874	A 30 38 16·03 G 30 38 20·01	G 76 6 40·41				− 4·0	139
140	B	Bowra T.S.	855	G 30 20 50·29	G 76 6 39·00	A 208 37 15·2 G 208 37 12·4	Sudhiwal D 0 4	+ 4·8		140
141	B	Kheri T.S.	822	G 30 5 9·30	G 76 5 54·58	A 212 55 16·6 G 212 55 17·2	Khānpur D 0 3	− 1·0		141
142	C	Rākhi T.S.	785	A 29 17 20·76 G 29 17 21·28	G 76 6 47·49	A 208 30 58·2 G 208 30 55·5	Barowdha D 0 5	+ 4·8	− 0·5	142
143	E	Lambatach H.S.	10474	A 31 0 34·38 G 31 1 8·46	G 76 54 2·93				−34·1	143
144	F	Bajamara H.S.	9681	A 30 45 27·79 G 30 45 56·20	G 77 54 0·73				−28·4	144
145	F	Amsot H.S.	3140	A 30 22 16·02 G 30 22 44·86	G 77 41 14·77				−28·8	145
146	F	Dehra Dun Base-line E. End S.	1967	A 30 16 37·26 G 30 17 7·35	G 77 58 30·74				−30·1	146
147	F	Khujnaur S.	2576	A 30 15 56·70 G 30 16 23·63	G 77 52 58·67				−26·9	147
148	F	Shorpur H.S.	2916	A 30 13 15·30 G 30 13 44·43	G 77 57 30·61				−29·1	148
149	F	Hatni h.s.	3069	A 30 12 31·93 G 30 13 1·52	G 77 52 19·58				−29·6	149
150	F	Bulāwāla h.s.	2432	A 30 6 22·32 G 30 6 51·29	G 77 59 11·27				−29·0	150
151	G	Nojli T.S.	929	A 29 53 14·12 G 29 53 27·76	G 77 40 24·59				−13·6	151
152	G	Godhna T.S.	901	A 29 37 8·73 G 29 37 18·46	G 77 54 2·98				− 9·7	152
153	G	Kaliāna S.	828	A 29 30 47·98 G 29 30 54·70	G 77 39 6·03	A 164 18 46·4 G 164 18 46·9	Dahera E 0 1	− 0·9	− 6·7	153
154	H	Datairi T.S.	767	A 28 43 58·67 G 28 44 4·49	G 77 38 56·31	A 28 44 34·3 G 28 44 34·2	Bostān D 0 6	+ 0·2	− 5·8	154
155	H	Bostān T.S.	758	A 28 30 54·25 G 28 30 59·64	G 77 30 49·08				− 5·4	155
156	H	Chandaos T.S.	699	A 28 5 0·71 G 28 5 1·59	G 77 51 39·60				− 0·9	156
157	I	Kidarkanta H.S.	12509	A 31 0 51·58 G 31 1 21·71	G 78 10 23·37				−30·1	157
158	J	Bahak H.S.	9715	A 30 44 37·60 G 30 45 5·22	G 78 13 36·98				−27·6	158
159	J	Nag Tiba H.S.	9915	A 30 34 41·05 G 30 35 11·57	G 78 9 9·57	A 32 58 41·6 G 32 58 53·9	Eagle's Nest D 3 12	− 20·8	−30·5	159
160	J	Banog H.S.	7433	A 30 28 4·18 G 30 28 36·91	G 78 0 55·96	A 71 5 55·0 G 71 6 8·7 A 280 22 46·8 G 280 23 0·5	Amsot D 2 28 Top Tibba E 1 7	} − 23·3	−32·7	160
161	J	Mussoorie Dome Obsy. H.S.	6937	A 30 27 4·02 G 30 27 40·55	G 78 4 17·41	A 6 17 20·1 G 6 17 35·1	Cole's Satelite Station D 5 26	− 25·5	−36·5	161

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I : $\delta a = 1$ km			Case II : $\delta b = 1$ km			Case III : Latitude $u_0 = 1''$			Case IV : Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/e = 298 \cdot 3$.					
	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
135	−3·19			+8·13			+1·00			+0·04			+3·46				− 6·2	135
136	−3·27			+8·31			+1·00			+0·04			+3·51				− 4·3	136
137	−3·71			+9·31			+1·00			+0·04			+3·84				− 5·5	137
138			+0·75			0·00			−0·05			+1·76			+2·93	− 11·8		138
139	+1·34			−5·03			+1·00			+0·03			−2·16				− 1·8	139
140			+0·81			−0·02			−0·05			+1·81			+3·05	+ 1·4		140
141			+0·82			−0·02			−0·05			+1·82			+3·08	− 4·4		141
142	+1·15		+0·82	−4·08		−0·03	+1·00		−0·06	+0·03		+1·87	−1·63		+3·13	+ 1·3	+ 1 1	142
143	+1·38			−5·29			+1·00			0·00			−2·35				− 31·7	143
144	+1·35			−5·12			+1·00			0·00			−2·25				− 26·1	144
145	+1·30			−4·85			+1·00			0·00			−2·09				− 26 7	145
146	+1·29			−4·79			+1·00			−0·01			−2·07				− 28·0	146
147	+1·29			−4·78			+1·00			0·00			−2·06				− 24·8	147
148	+1·28			−4·75			+1·00			−0·01			−2·05				− 27·0	148
149	+1·28			−4·74			+1·00			0·00			−2·04				− 27·6	149
150	+1·27			−4·67			+1·00			−0·01			−2·00				− 27·0	150
151	+1·24			−4·51			+1·00			0·00			−1·90				− 11·7	151
152	+1·20			−4·32			+1·00			0·00			−1·81				− 7·9	152
153	+1·18		0·00	−4·24		0·00	+1·00		0·00	0·00		+1·85	−1·5		+2·39	− 3·8	− 4·9	153
154	+1·05		0·00	−3·67		0·00	+1·00		0·00	0·00		+1·90	−1·45		+2·45	− 2·7	− 4·3	154
155	+1·01			−3·51			+1·00			0·00			−1·36				− 4·0	155
156	+0·94			−3·18			+1·00			0·00			−1·21				+ 0·3	156
157	+1·38			−5·30			+1·00			−0·01			−2·35				− 27·7	157
158	+1·35			−5·11			+1·00			−0·01			−2·23				− 25·4	158
159	+1·33		−0·26	−4·99		+0·01	+1·00		+0·02	−0·01		+1·79	−2·18		+2·08	− 23·4	− 28·3	159
160	+1·31		−0·19	−4·92		0·00	+1·00		+0·01	−0·01		+1·80	−2·14		+2·15	− 26·0	− 30·6	160
161	+1·31		−0·22	−4·91		0·00	+1·00		+0·01	−0·01		+1·80	−2·14		+2·12	− 28·1	− 34·4	161

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
162	53 J	Jharipani (IX) h.s.	5150	A 30 24 17·55 G 30 25 10·05	G 78 5 20·92	A 14 24 59·9 G 14 25 18·0	Dalanwāla D 4 38	− 30·8	−52·5	162
163	J	Spur point (VIII) h.s.	3850	A 30 23 44·55 G 30 24 37·72	G 78 5 35·96	A 17 59 32·4 G 17 59 49·1	Dalanwāla D 2 48	− 28·5	−53·2	163
164	J	Rajpur h.s.	3500	A 30 23 9·15 G 30 23 56·83	G 78 5 59·89	A 24 15 4·4 G 24 15 21·8	Dalanwāla D 2 26	− 29·7	−47·7	164
165	J	VI ...	3050	A 30 22 44·90 G 30 23 30·79	G 78 6 2·00				−45·9	165
166	J	V ...	2980	A 30 22 7·46 G 30 22 51·83	G 78 5 21·38				−44·4	166
167	J	IV ...	2780	A 30 21 26·78 G 30 22 8·93	G 78 4 30·87				−42·2	167
168	J	III ...	2660	A 30 21 5·57 G 30 21 46·61	G 78 4 7·39				−41·0	168
169	J	Dehra Dun Obsy. (Old) S.	2289	A 30 19 19·56 G 30 19 57·07	G 78 3 34·70	A 165 10 58·8 G 165 11 10·2	Banog E 5 20	− 19·5	−37·5	169
170	J	Dehra Dun Haig Obsy. S.	2240	A 30 18 51·80 G 30 19 28·73	A 78 2 56·47 G 78 3 22·12			− 22·1	−36·9	170
171	J	Lachkuwa h.s.	2674	A 30 4 5·34 G 30 4 34·24	G 78 1 41·67				−28·9	171
172	J	Rānigarh H.S.	7055	A 30 3 34·80 G 30 4 4·47	G 78 42 54·38				−29·7	172
173	K	Harpālsid T.S.	1000	A 29 39 22·24 G 29 39 50·84	G 78 33 20·81				−28·6	173
174	K	Mahesari T.S.	821	A 29 30 8·18 G 29 30 18·21	G 78 8 51·70				−10·0	174
175	K	Sarkāra T.S.	761	A 29 15 35·09 G 29 15 46·91	G 78 32 20·18				−11·8	175
176	L	Sirsa T.S.	739	A 28 54 30·27 G 28 54 39·64	G 78 32 6·14	A 149 55 17·1 G 149 55 20·5	Milik D 0 4	− 6·2	− 9·4	176
177	L	Bānsgopāl T.S.	677	A 28 33 23·28 G 28 33 28·08	G 78 31 59·71				− 4·8	177
178	L	Sankrāo T.S.	670	A 28 2 28·92 G 28 2 29·00	G 78 32 2·97	A 185 44 20·9 G 185 44 18·8	Sakrora D 0 8	+ 3·9	− 0·1	178
179	O	Birond H.S.	6967	A 29 14 29·73 G 20 15 14·15	G 79 42 57·00				−44·4	179
180	P	Kaliānpur T.S.	629	G 28 35 11·10	G 79 44 33·75	A 185 30 18·4 G 185 30 17·7	Donao D 0 4	+ 1·3		180
181	54 A	Tāsing H.S.	2050	A 27 52 59·49 G 27 52 59·47	G 76 12 11·56	A 77 55 36·5 G 77 55 31·6	Jīlo E 0 13	+ 9·3	0·0	181
182	B	Bānskho H.S.	1870	A 26 50 2·37 G 26 50 7·89	G 76 8 20·42	A 148 40 55·6 G 148 40 51·9	Rāmgarh E 0 2	+ 7·3	− 5·5	182
183	C	Kānkra H.S.	1652	A 25 37 58·75 G 25 37 59·53	G 76 7 27·15	A 145 33 8·7 G 145 33 6·9	Bhojpur D 0 18	+ 3·8	− 0·8	183
184	D	Gurāria H.S.	1360	A 24 25 31·98 G 24 25 32·46	G 76 5 2·16	A 300 41 56·8 G 300 41 56·2	Kūsalpura D 0 4	+ 1·3	− 0·5	184
185	D	Māta-ka-hūra H.S.	1645	G 24 14 10·67	G 76 36 49·20	A 181 31 35·0 G 181 31 34·3	Sartal D 0 15	+ 1·6		185
186	E	Noh T.S.	710	A 27 50 53·13 G 27 50 53·08	G 77 38 45·56	A 50 22 36·5 G 50 22 33·4	Mānpur D 0 6	+ 5·9	+ 0·1	186
187	E	Agra-group W. Point	550	A 27 9 41·43 G 27 9 45·86	G 77 56 25·26				− 4·4	187
188	F	Usira H.S.	810	A 26 57 0·50 G 26 57 6·22	G 77 37 52·58	A 146 55 27·2 G 146 55 25·9	Madhoni D 0 12	+ 2·6	− 5·7	188
189	G	Kesri H.S.	1487	A 25 46 41·57 G 25 46 35·81	G 77 40 49·02	A 206 41 38·8 G 206 41 40·1	Dīn D 0 10	− 2·7	+ 5·8	189

*A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: $\delta a = 1$ km			Case II: $\delta b = 1$ km			Case III: Latitude $u_0 = 1''$			Case IV: Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$					
	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	
162	+1·31		−0·23	−4·88		0·00	+1·00		+0·02	−0·01		+1·80	−2·11		+2·11	− 33·4	−50·4	162
163	+1·31		−0·23	−4·87		0·00	+1·00		+0·02	−0·01		+1·80	−2·11		+2·12	− 33·1	−51·1	163
164	+1·30		−0·23	−4·86		+0·01	+1·00		+0·02	−0·01		+1·80	−2·11		+2·11	− 32·3	−45·6	164
165	+1·30			−4·86			+1·00			−0·01			−2·11				−43·8	165
166	+1·30			−4·85			+1·00			−0·01			−2·10				−42·3	166
167	+1·30			−4·84			+1·00			−0·01			−2·10				−40·1	167
168	+1·30			−4·84			+1·00			−0·01			−2·10				−38·9	168
169	+1·30		−0·21	−4·82		0·00	+1·00		+0·01	−0·01		+1·81	−2·08		+2·13	− 22·1	−35·4	169
170	+1·30	−0·24		−4·81	+0·03		+1·00	0·00		−0·01	+0·11		−2·08	−0·05		− 22·1	−34·8	170
171	+1·26			−4·64			+1·00			−0·01			−1·98				−26·9	171
172	+1·26			−4·64			+1·00			−0·02			−1·98				−27·7	172
173	+1·20			−4·35			+1·00			−0·02			−1·83				−26·8	173
174	+1·18			−4·23			+1·00			−0·01			−1·75				− 8·2	174
175	+1·14			−4·06			+1·00			−0·02			−1·66				−10·1	175
176	+1·09		−0·48	−3·80		+0·02	+1·00		+0·03	−0·02		+1·89	−1·53		+2·02	− 8·6	− 7 9	176
177	+1·02			−3·54			+1·00			−0·01			−1·40				− 3·4	177
178	+0·93		−0·48	−3·15		+0·03	+1·00		+0·03	−0·01		+1·94	−1·19		+2·09	+ 1·4	+ 1·1	178
179	+1·15			−4·05			+1·00			−0·03			−1·68				−42·7	179
180			−1·19			+0·06			+0·08			+1·91			+1·43	− 0·5		180
181	+0·90		+0·80	−3·03		−0·05	+1·00		−0·06	+0·02		+1·95	−1·08		+3·20	+ 5·9	+ 1·1	181
182	+0·69		+0·85	−2·22		−0·07	+1·00		−0·06	+0·03		+2·02	−1·68		+3·33	+ 3·8	− 3·8	182
183	+0·40		+0·89	−1·26		−0·10	+1·00		−0·06	+0·03		+2·11	−0·22		+3·45	+ 0·3	− 0·6	183
184	+0·09		+0·94	−0·26		−0·13	+1·00		−0·07	+0·03		+2·21	−0·23		+3·60	− 2·3	− 0·3	184
185			+0·63			−0·09			−0·04			+2·22			+3·37	− 1·8		185
186	+0·89		+0·01	−3·00		0·00	+1·00		0·00	0·00		+1·95	−1·10		+2·53	+ 3·0	+ 1·2	186
187	+0·75			−2·48			+1·00			−0·01			−0·85				− 3·5	187
188	+0·71		+0·01	−2·31		0·00	+1·00		0·00	0·00		+2·01	−0·76		+2·61	− 0·2	− 4·9	188
189	+0·43		−0·01	−1·37		0·00	+1·00		0·00	0·00		+2·10	−0·31		+2·70	− 5·6	+ 6·1	189

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A – G) cot λ for azimuth or (A – G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
190	54 H	Pahārgarh H.S.	1641	A 24 56 6·47 G 24 56 6·92	G 77 41 46·32	A236 19 20·1 G236 19 17·8	Nīmdānt D 0 4	+ 4·9	– 0·5	190
191	H	[illegible]aiādhari H.S.	1867	A 24 38 18·79 G 24 38 17·59	G 77 39 49·31				+ 1·2	191
192	H	Salot H.S.	1834	G 24 14 52·08	G 77 15 2·41	A175 58 10·5 G175 58 10·7	Hatni D 0 8	– 0·4		192
193	H	Sūrāntal H.S.	1802	A 24 14 21·36 G 24 14 20·42	G 77 40 43·91				+ 0·9	193
194	H	Sironj Base-line N.E. End S.	1481	A 24 8 55·45 G 24 8 53·57	G 77 50 41·14				+ 1·9	194
195	H	Kaliānpur H.S.	1765	A 24 7 10·97 G 24 7 11·26	A 77 39 17·57	A190 27 6·29 G190 27 5·10	Sūrantāl D 0 0	+ 2·7	– 0·3	195
		(Origin)‡	1765	A 27 7 11·57 **G 27 7 11·26**	**A 77 39 17·57**	A 190 27 6·39 **G 190 27 5·10**	**Surantal**	+ 2·9	+ 0·3	
196	H	Tinsia H.S.	1776	A 24 6 29·05 G 24 6 27·97	G 77 18 30·70				+ 1·1	196
197	H	Losalli S.	1749	A 24 6 18·19 G 24 6 19·17	G 77 33 14·11	A149 5 52·1 G149 5 50·4	Rāmpur D 0 2	+ 3·8	– 1·0	197
198	I	Salīmpur T.S.	645	A 27 46 36·23 G 27 46 36·46	G 78 30 48·70				– 0·2	198
199	I	Agra-group N. Point	550	A 27 14 10·31 G 27 14 14·10	G 78 1 4·71				– 3·8	199
200	I	Agra Long. S.	550	A 27 9 34·62 G 27 9 39·93	A 78 1 7·49 G 78 1 1·89			+ 5·0	– 5·3	200
201	I	Agra-group E. Point	550	A 27 9 16·21 G 27 9 21·00	G 78 6 3·64				– 4·8	201
202	I	Agra Parade Point	550	A 27 8 52·18 G 27 8 57·47	G 78 1 9·70				– 5·3	202
203	I	Agra-group S. Point	550	A 27 5 32·95 G 27 5 38·51	G 78 1 2·38				– 5·6	203
204	J	Gūrmi T.S.	575	A 26 36 5·97 G 26 36 3·63	G 78 30 49·82	A155 50 8·0 G155 50 8·8	Panāhat D 0 3	– 1·8	+ 2·3	204
205	J	Majhār H.S.	1028	A 26 6 20·30 G 26 6 17·00	G 78 28 17·73				+ 3·3	205
206	K	Algi H.S.	854	A 25 29 48·16 G 25 29 46·19	G 78 21 30·98				+ 2·0	206
207	L	Andhiāri H.S.	1330	A 24 41 11·31 G 24 41 6·78	G 78 13 48·99				+ 4·5	207
208	L	Bhaorāsa H.S.	1387	A 24 8 5·13 G 24 8 3·74	G 78 0 40·73				+ 1·4	208
209	L	Budhon H.S.	1867	A 24 5 8·99 G 24 5 8·41	G 78 31 11·89	A265 22 28·1 G265 22 27·7	Tinsmal D 0 6	+ 0·9	+ 0·6	209
210	M	Mohammadabad T.S.	565	G 27 18 24·05	G 79 25 39·80	A291 59 0·9 G291 58 51·5	Chandanpur D 0 8	+ 18·2		210
211	P	Dargawa H.S.	1152	A 24 37 17·32 G 24 37 13·21	G 79 1 24·63				+ 4·1	211
212	P	Rangīr (old) S.	1184	A 24 0 19·28 G 24 0 20·37	G 79 25 59·25	A106 1 11·0 G106 1 24·2	Tinsmal E 0 11	– 29·6	– 1·1	212
213	55 E	Kāmkhera H.S.	1780	A 23 59 42·89 G 23 59 44·93	G 77 43 6·85				– 2·0	213
214	E	Ahmadpur H.S.	1713	A 23 36 18·42 G 23 36 20·88	G 77 40 48·26	A185 10 55·0 G185 10 53·8	Kāmkhera D 0 9	+ 2·7	– 2·5	214
215	E	Lādi H.S.	1875	A 23 8 39·10 G 23 8 44·13	G 77 42 30·87				– 5·0	215
216	F	Bhīmbhat H.S.	2120	G 22 50 2·06	G 77 37 15·53	A194 34 0·7 G194 33 58·6	Lādi D 0 16	+ 5·0		216

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.
‡ Derived from group of stations surrounding Kaliānpur.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I : δa=1 km			Case II : δb=1 km			Case III : Latitude u_0=1″			Case IV : Azimuth w_0=1″			a=6378200 metres, $1/\epsilon$=298·3.					
	u	v cos λ	w cot λ	u	v cos λ	w cot λ	u	v cos λ	w cot λ	u	v cos λ	w cot λ	u	v cos λ	w cot λ	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
190	+0·22		−0·02	−0·69		0·00	+1·00		0·00	0·00		+2·16	0·00		+2·77	+ 2·1	− 0·5	190
191	+0·14			−0·43			+1·00			0·00			+0·12				+ 1·1	191
192			+0·24			−0·04			−0·02			+2·22			+3·05	− 3·5		192
193	+0·03			−0·10			+1·00			0·00			+0·27				+ 0·6	193
194	+0·01			−0·02			+1·00			0·00			+0·30				+ 1·6	194
195	0·00	0·00	0·00	0·00	0·00	0·00	+1·00	0·00	0·00	0·00	0·00	+2·23	+0·30	0·00	+2·88	+ 0·1	− 0·6	195
(Origin)	0·00	0·00	0·00	0·00	0·00	0·00	+1·00	0·00	0·00	0·00	0·00	+2·23	+0·30	0·00	+2·88	0·0	0·0	(Origin)
196	0·00			+0·01			+1·00			+0·01			+0·33				+ 0·8	196
197	−0·01		+0·06	+0·02		−0·01	+1·00		0·00	0·00		+2·24	+0·32		+2·93	+ 0·9	− 1·3	197
198	+0·88			−2·95			+1·00			−0·01			−1·09				+ 0·9	198
199	+0·77			−2·53			+1·00			−0·01			−0·67				− 3·1	199
200	+0·75	−0·22		−2·48	+0·03		+1·00	0·00		−0·01	+0·05		−0·85	−0·11		+ 5·1	− 4·4	200
201	+0·75			−2·47			+1·00			−0·01			−0·85				− 3·9	201
202	+0·75			−2·47			+1·00			−0·01			−0·84				− 4·5	202
203	+0·74			−2·42			+1·00			−0·01			−0·82				− 4·8	203
204	+0·63		−0·49	−2·04		+0·04	+1·00		+0·03	−0·01		+2·03	−0·64		+2·20	− 4·2	+ 2 9	204
205	+0·52			−1·64			+1·00			−0·01			−0·46				+ 3·8	205
206	+0·37			−1·14			+1·00			−0·01			−0·22				+ 2·2	206
207	+0·16			−1·47			+1·00			−0·01			+0·09				+ 4·4	207
208	0·00			−0·01			+1·00			−0·01			+0·29				+ 1·1	208
209	−0·01		−0·52	+0·03		+0·08	+1·00		+0·04	−0·01		+2·24	+0·34		+2·48	− 1·6	+ 0·3	209
210			−0·99			+0·07			+0·07			+1·98			+1·72	+16·3		210
211	+0·14			−0·42			+1·00			−0·02			+0·10				+ 4 0	211
212	−0·02		−1·08	+0·10		+0·16	+1·00		+0·08	−0·03		+2·24	+0·32		+2·04	−31·6	− 1·4	212
213	−0·04			+0·11			+1·00			0·00			+0·36				− 2 4	213
214	−0·15		−0·01	+0·44		0·00	+1·00		0·00	0·00		+2·28	+0·50		+2·93	− 0·2	− 3·0	214
215	−0·29			+0·84			+1·00			0·00			+0·67				− 5·7	215
216			+0·02			−0·01			0·00			+2·35			+3·06	+ 2·1		216

* δa=0·924, δb=0·743, u_0=0·31, w_0=1·29. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A−G) cot λ for azimuth or (A−G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
217	55 G	Nilgarh H.S.	2533	G 21 45 50·12	G 77 39 18·82	A 321 4 43·7 G 321 4 44·2	Sālbaldi E 0 1	− 1·3		217
218	G	Takalkhera s.	1094	A 21 5 50·17 G 21 5 56·76	G 77 38 24·94				− 6·6	218
219	H	Rāngrai s.	1046	A 20 48 7·16 G 20 48 14·68	G 77 35 53·83				− 7·5	219
220	H	Badgaon H.S.	1128	A 20 44 15·54 G 20 44 23·06	G 77 36 31·79	A 183 9 0·9 G 183 8 59·5	Ashti D 0 8	+ 3·7	− 7·5	220
221	H	Dhānura s.	1135	A 20 44 3·35 G 20 44 10·84	G 77 41 43·27				− 7·5	221
222	H	Dotra s.	1140	A 20 41 22·25 G 20 41 28·91	G 77 32 45·66				− 6·7	222
223	H	Sakri H.S.	1810	G 20 0 14·11	G 77 42 7·32	A 175 24 37·7 G 175 24 35·4	Kopdi D 0 20	+ 6·3		223
224	I	Saugor H.S.	2033	A 23 49 48·71 G 23 49 48·07	G 78 46 18·16				+ 0·6	224
225	I	Nāharmau H.S.	1940	A 23 30 13·14 G 23 30 18·15	G 78 49 49·13				− 5·0	225
226	M	Karaundi H.S.	1625	A 23 10 45·07 G 23 10 40·02	G 79 59 43·34	A 206 22 35·6 G 206 22 38·4	Lora D 0 1	− 6·5	+ 5·1	226
227	M	Jabalpur Long. s.	...	G 23 10 10·10	A 79 56 52·42 G 79 57 2·61			− 9·4		227
228	P	Bhīmsain H.S.	1490	A 20 57 28·54 G 20 57 35·96	G 79 46 7·40	A 297 55 2·8 G 297 55 2·3	Partābgarh E 0 0	+ 1·3	− 7·4	228
229	P	Bājuli H.S.	1070	A 20 12 51·25 G 20 12 55·45	G 79 44 49·27				− 4·2	229
230	56B	Nitali H.S.	2289	A 18 17 2·74 G 18 17 7·16	G 76 16 23·32	A 239 23 1·3 G 239 23 5·7	Harangal D 0 10	− 13·3	− 4·4	230
231	B	Achola H.S.	2274	A 18 14 44·87 G 18 14 48·12	G 76 59 20·51	A 272 47 57·4 G 272 47 58·5	Manganāl D 0 11	− 3·3	− 3·3	231
232	E	Halda s.	1335	A 19 9 24·41 G 19 9 29·38	G 77 41 1·39				− 5·0	232
233	E	Voi s.	1439	A 19 7 14·69 G 19 7 19·89	G 77 34 46·88				− 5·2	233
234	E	Somtana H.S.	1714	G 19 5 0·52	G 77 39 16·29	A 186 51 46·9 G 186 51 48·3	Terbāu D 0 5	− 4·0		234
235	E	Mandāla s.	1294	A 19 2 42·84 G 19 2 48·24	G 77 43 35·14				− 5·4	235
236	E	Talegaon s.	1233	A 19 1 21·65 G 19 1 26·64	G 77 37 16·75				− 5·0	236
237	F	Dāmargīda Obsy. S.	1941	A 18 3 14·92 G 18 3 17·35	G 77 40 4·41	A 188 11 59·1 G 188 11 59·8	Burgāpāli D 0 15	− 2·1	− 2·4	237
238	G	Devanūr s.	1593	A 17 10 56·88 G 17 11 0·43	G 77 41 7·41				− 3·6	238
239	G	Akampalle h.s.	1557	A 17 10 50·39 G 17 10 53·96	G 77 34 29·30				− 3 6	239
240	G	Kodangal S.	1906	A 17 7 53·74 G 17 7 57·35	G 77 38 25·73	A 62 29 16·3 G 62 29 17·8	Nēlagat E 0 1	− 4·9	− 3·6	240
241	G	Linganapalle h.s.	1815	A 17 7 13·40 G 17 7 16·66	G 77 42 28·61				− 3·3	241
242	G	Pialmudi s.	1869	A 17 4 1·06 G 17 4 6·05	G 77 36 22·06				− 5·0	242
243	H	Tōnsalgutta s.	1133	A 16 18 2·36 G 16 18 6·91	G 77 34 49·44				− 4·6	243
244	H	Pēddapād s.	1090	A 16 17 14·13 G 16 17 20·38	G 77 44 30·54				− 6·3	244

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: $\delta a = 1$ km			Case II: $\delta b = 1$ km			Case III: Latitude $u_0 = 1''$			Case IV: Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$.					
	u	v cos λ	w cotλ	u	v cosλ	w cotλ	u	v cosλ	w cotλ	u	v cosλ	w cotλ	u	v cosλ	w cot λ	Deflection in Prime Vertical	Deflec-tion in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
217			0·00			0·00			0·00			+2·46			+3·18	− 4·2		217
218	−0·95			+2·66			+1·00			0·00			+1·41				− 8·0	218
219	−1·05			+2·93			+1·00			0·00			+1·52				− 9·0	219
220	−1·08		+0·03	+2·99		−0·01	+1·00		0·00	0·00		+2·58	+1·53		+3·36	+ 0·6	− 9·0	220
221	−1·08			+3·00			+1·00			0·00			+1·56				− 9·1	221
222	−1·09			+3·04			+1·00			0·00			+1·56				− 8·3	222
223			−0·01			0·00			0·00			+2·67			+3·43	+ 3·4		223
224	−0·08			+0·25			+1·00			−0·02			+0·40				+ 0·2	224
225	−0·18			+0·53			+1·00			−0·02			+0·52				− 5·5	225
226	−0.26		−1·45	+0·81		+0·25	+1·00		+0·07	−0·04		+2·32	+0·62		+1·85	− 8·3	+ 4·5	226
227		−1·38			+0·20			+0·02			−0·02			−1·15		− 8·3		227
228	−0·99		−1·41	+2·79		+0·32	+1·00		+0·10	−0·03		+2·55	+1·43		+2·26	− 0·7	− 8·8	228
229	−1·25			+3·48			+1·00			−0·03			+1·69				− 5·9	229
230	−1·99		+1·03	+5·30		−0·31	+1·00		−0·08	+0·02		+2·91	+2·44		+4·45	− 16·9	− 6·8	230
231	−2·00		+0·52	+5·34		−0·16	+1·00		−0·04	+0·01		+2·92	+2·44		+4·11	− 6·5	− 5·7	231
232	−1·65			+4·47			+1·00			0·00			+2·11				− 7·1	232
233	−1·67			+4·51			+1·00			0·00			+2·12				− 7·3	233
234			+0·01			0·00			0·00			+2·79			+3·61	− 7·0		234
235	−1·70			+4·57			+1·00			0·00			+2·14				− 7·5	235
236	−1·71			+4·60			+1·00			0·00			+2·15				− 7·2	236
237	−2·09		0·00	+5·53		+0·01	+1·00		0·00	0·00		+2·95	+2·49		+3·80	− 5·0	− 4·9	237
238	−2·45			+6·38			+1·00			0·00			+2·79				− 6·4	238
239	−2·45			+6·30			+1·00			0·00			+2·73				− 6·3	239
240	−2·47		+0·01	+6·43		0·00	+1·00		0·00	0·00		+3·10	+2·81		+4·01	− 7·6	− 6·4	240
241	−2·47			+6·44			+1·00			0·00			+2·81				− 6·1	241
242	−2·50			+6·50			+1·00			0·00			+2·83				− 7·8	242
243	−2·89			+7·42			+1·00			0·00			+3·15				− 7·8	243
244	−2·83			+7·27			+1·00			0·00			+3·10				− 9·4	244

* $\delta a = 0·924$, $\delta b = 0·743$, $u_0 = 0·31$, $w_0 = 1·29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				EVEREST'S SPHEROID.						
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
245	56 H	Darur H.S.	1796	G 16 13 35·40	G 77 39 36·51	A 132 35 57·2 G 132 35 59·2	Kōttapalle D 0 15	− 6·9		245
246	H	Tuagat h.s.	1450	A 16 9 46·73 G 16 9 51·66	G 77 34 11·59				− 4·9	246
247	H	Gattinārāyantippa h.s.	1225	A 16 7 48·95 G 16 7 54·81	G 77 45 51·00				− 5·9	247
248	H	Devaragat h.s.	1332	A 16 6 31·98 G 16 6 37·27	G 77 41 26·21				− 5·3	248
249	K	Pirmulo H.S.	2093	A 17 52 58·32 G 17 53 2·81	G 78 35 50·98	A 105 0 48·0 G 105 0 49·0	Narsula D 0 12	− 3·1	− 4·5	249
250	K	Bolarum P.W.D. Office Long. S.	1971	A 17 30 7·36 G 17 30 13·41	A 78 31 7·84 G 78 31 11·12	A 25 57 35·8 G 25 57 35·8	Hyderābād Naubat-pahar D 0 13	0·0	− 6·1	250
251	M	Dīwai H.S.	967	A 19 49 26·87 G 19 49 32·57	G 79 32 28·62	A 154 17 54·2 G 154 17 55·1	Ambāgarh D 0 8	− 2·5	− 5·7	251
252	M	Ankora H.S.	1463	A 19 24 26·63 G 19 24 34·75	G 79 36 27·70				− 8·1	252
253	N	Burgpaili H.S.	983	A 18 54 3·48 G 18 54 7·20	G 79 41 36·96	A 142 8 7·5 G 142 8 8·8	Rechni D 0 8	− 3·8	− 3·7	253
254	N	Rāmgīr H.S.	1772	A 18 35 26·90 G 18 35 26·12	G 79 31 42·36				+ 0·8	254
255	O	Bolīkonda H.S.	1363	A 17 42 29·08 G 17 42 35·82	G 79 47 57·98				− 6·7	255
256	O	Vānākonda H.S.	1664	A 17 36 0·22 G 17 36 6·87	G 79 22 20·70	A 180 4 14·5 G 180 4 15·3	Yarābali D 0 1	− 2·5	− 6·7	256
257	O	Niālamari H.S.	1144	A 17 1 25·93 G 17 1 33·63	G 79 43 29·78				− 7·7	257
258	57 A	Bellary Long. s.	...	G 15 8 33·06	A 76 55 38·89 G 76 55 39·58			− 0·7		258
259	B	Yērragunta h.s.	1698	A 14 48 27·31 G 14 48 23·26	G 76 58 17·56				+ 4·1	259
260	C	Nughallibētta H.S.	3140	G 13 1 32·95	G 76 28 32·46	A 54 31 39·1 G 54 31 41·7	Sātanhalli E 0 8	− 11·2		260
261	E	Namthabad s.	1169	A 15 5 51·75 G 15 5 52·40	G 77 36 26·91				− 0·7	261
262	F	Chikalgurki s.	1516	A 14 59 5·16 G 14 59 4·53	G 77 11 6·39				+ 0·6	262
263	F	Bandūr s.	1447	A 14 57 44·41 G 14 57 42·32	G 77 0 36·91				+ 2·1	263
264	F	Hōnnūr H.S.	1579	A 14 55 22·20 G 14 55 18·96	G 77 6 2·60				+ 3·2	264
265	F	Nimbāgal s.	1565	A 14 51 56·14 G 14 51 52·43	G 77 11 51·78				+ 3·7	265
266	F	Pāvagada H.S.	3022	A 14 6 18·80 G 14 6 15·39	G 77 16 42·43				+ 3·4	266
267	G	Bōmmasandra s.	2005	A 13 59 42·63 G 13 59 36·34	G 77 28 43·96				+ 6·3	267
268	G	Bangalore Base-line N.E. End S.	3016	A 13 4 53·17 G 13 4 56·05	G 77 39 16·23	A 44 32 19·7 G 44 32 19·3	Bangalore Base-line S.W. End E 0 8	+ 1·7	− 2·9	268
269	G	Bangalore Base-line S.W. End S.	3126	A 13 0 36·12 G 13 0 40·91	A 77 34 57·29 G 77 35 0·19	A 224 31 21·7 G 224 31 21·6	Bangalore Base-line N.E. End D 0 13	+ 0·4	− 4·8	269
270	H	Dōddagunta s.	3003	A 12 59 51·52 G 12 59 55·76	G 77 37 33·25				− 4·2	270
271	M	Dānapa H.S.	150	A 15 55 59·69 G 15 56 0·14	G 79 56 7·39	A 265 47 36·0 G 265 47 39·4	Babbēpalle D 0 22	− 11·9	− 0·5	271
272	M	Darutippa S.	195	A 15 0 33·52 G 15 0 36·47	G 79 55 12·92				− 3·0	272

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES. Case I: $\delta a = 1$ km u	$v \cos \lambda$	$w \cot \lambda$	Case II: $\delta b = 1$ km u	$v \cos \lambda$	$w \cot \lambda$	FOR CHANGES OF ORIGIN. Case III: Latitude $u_0 = 1''$ u	$v \cos \lambda$	$w \cot \lambda$	Case IV: Azimuth $w_0 = 1''$ u	$v \cos \lambda$	$w \cot \lambda$	HELMERT'S SPHEROID* $a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$ u	$v \cos \lambda$	$w \cot \lambda$	Deflection in Prime Vertical	Deflection in Meridian	Serial No.
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
245			−0·01			0·00			0·00			+3·27			+4·21	−9·7		245
246	−2·88			+7·39			+1·00			0·00			+3·14				−8·0	246
247	−2·89			+7·43			+1·00			0·00			+3·16				−9·1	247
248	−2·90			+7·45			+1·00			0·00			+3·18				−8·5	248
249	−2·16		−0·71	+5·70		+0·22	+1·00		+0·05	−0·03		+2·97	+2·51		+3·37	−5·6	−7·0	249
250	−2·31	−0·52	−0·66	+6·06	+0·07	+0·22	+1·00	+0·01	+0·05	−0·01	−0·11	+3·04	+2·67	−0·58	+3·49	−2·5	−8·8	250
251	−1·40		−1·30	+3·83		+0·34	+1·00		+0·10	−0·03		+2·69	+1·83		+2·55	−4·5	−7·5	251
252	−1·55			+4·23			+1·00			−0·03			+1·98				−10·1	252
253	−1·74		−1·45	+4·71		+0·42	+1·00		+0·11	−0·03		+2·82	+2·16		+2·63	−5·8	−5·9	253
254	−1·87			+5·05			+1·00			−0·03			+2·29				−1·5	254
255	−2·22			+5·86			+1·00			−0·04			+2·56				−9·3	255
256	−2·27		−1·31	+5·97		+0·43	+1·00		+0·10	−0·03		+3·02	+2·61		+3·04	−4·6	−9·3	256
257	−2·50			+6·53			+1·00			−0·03			+2·81				−10·5	257
258		+0·43			−0·06			0·00			−0·16			+0·16		−0·9		258
259	−3·48			+8·77			+1·00			+0·01			+3·62				+0·5	259
260			+1·17			−0·56			−0·09			+4·05			+5·85	−14·5		260
261	−3·35			+8·47			+1·00			0·00			+3·51				−4·2	261
262	−3·40			+8·59			+1·00			+0·01			+3·56				−3·0	262
263	−3·41			+8·61			+1·00			+0·01			+3·57				−1·5	263
264	−3·43			+8·65			+1·00			+0·01			+3·58				−0·4	264
265	−3·45			+8·71			+1·00			+0·01			+3·60				+0·1	265
266	−3·80			+9·49			+1·00			+0·01			+3·86				−0·5	266
267	−3·85			+9·60			+1·00			0·00			+3·88				+2·4	267
268	−4·28		0·00	+10·55		0·00	+1·00		0·00	0·00		+4·03	+4·19		+5·20	−0·9	−7·1	268
269	−4·32	+0·04	+0·07	+10·63	−0·01	−0·04	+1·00	0·00	−0·01	0·00	−0·19	+4·06	+4·22	−0·21	+5·26	−2·3	−9·0	269
270	−4·32			+10·64			+1·00			0·00			+4·22				−8·4	270
271	−2·97		−1·90	+7·62		+0·72	+1·00		+0·15	−0·04		+3·32	+3·18		+3·12	−13·6	−3·7	271
272	−3·38			+8·55			+1·00			−0·04			+3·50				−6·5	272

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				EVEREST'S SPHEROID.						
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
273	57 N	Kistama H.S.	458	A 14 27 12·28 G 14 27 14·56	G 79 45 18·51	A 180 1 54·1 G 80 1 55·6	Pallakonda E 0 16	− 5·8	− 2·3	273
274	P	Anandalamalai H.S.	923	G 12 55 50·73	G 79 23 46·76	A 171 57 36·3 G 171 57 37·6	Pullur E 0 23	− 5·7		274
275	58 E	Yettimalai S.	617	A 11 3 52·10 G 11 3 50·00	G 77 50 47·10				+ 2·1	275
276	F	Pachapālaiyam s.	970	A 10 59 40·81 G 10 59 39·88	G 77 37 25·80	A 167 34 2·4 G 167 34 1·2	Chennimalai E 0 40	+ 6·2	+ 0·9	276
277	F	Kātpālaiyam s.	878	A 10 56 36·66 G 10 56 35·97	G 77 40 50·63				+ 0·7	277
278	G	Shūlakarai s.	333	A 9 32 15·53 G 9 32 13·28	G 77 56 51·22				+ 2·3	278
279	H	Rādhāpuram S.	167	A 8 17 1·75 G 8 16 59·44	G 77 42 7·71	A 5 55 25·4 G 5 55 24·1	Kudankulam Obsy. D 0 4	+ 8·9	+ 2·3	279
280	H	Tanakarakulam S.	176	A 8 13 57·50 G 8 13 55·39	G 77 38 53·81				+ 2·1	280
281	H	Arasākulam S.	55	A 8 13 41·96 G 8 13 39·52	G 77 44 30·98				+ 2·4	281
282	H	Vijayāpati S.	90	A 8 12 10·67 G 8 12 8·34	G 77 46 35·58				+ 2·3	282
283	H	Nagarkoil Long. S.	110	G 8 11 25·30	A 77 26 1·82 G 77 26 3·56			− 1·7		283
284	H	Kudankulam Obsy. S.	175	A 8 10 23·41 G 8 10 21·55	G 77 41 26·26	A 185 55 18·8 G 185 55 18·6	Rādhāpuram D 0 5	+ 1·4	+ 1·9	284
285	H	Punnæ Obsy. S.	48	A 8 9 29·92 G 8 9 27·79	G 77 37 35·33				+ 2·1	285
286	I	Kanjamalai H.S.	3236	G 11 36 55·92	G 78 3 36·52	A 38 11 59·1 G 38 12 0·1	Morur D 1 3	− 4·9		286
287	K	Manēgandi S.	56	G 9 46 15·13	G 78 55 20·84	A 178 0 47·2 G 178 0 50·3	Manikamkota D 0 1	− 18·0		287
288	K	Black s.	346	A 9 31 4·22 G 9 31 1·30	G 78 2 58·77				+ 2·9	288
289	K	Kutipārai S.	347	A 9 28 47·09 G 9 28 44·87	G 78 0 37·76	A 25 17 6·2 G 25 17 6·8	Koilpati D 0 4	+ 3·6	+ 2·2	289
290	K	Pandalagudi s.	217	A 9 23 30·55 G 9 23 27·69	G 78 5 54·11				+ 2·9	290
291	K	Rāmnad S.	48	G 9 21 51·96	G 78 49 17·66	A 57 57 54·9 G 57 57 56·2	Uttarakoshamangai E 0 0	− 7·9		291
292	M	Kallapat Trestle S.	199	G 11 57 12·30	G 79 33 52·96	A 214 44 19·0 G 214 44 20·0	Pērumukkal D 0 1	− 4·7		292
293	M	Tiruvēndipuram s.	...	A 11 44 43·40 G 11 44 37·64	G 79 42 45·80				+ 5·8	293
294	M	Nayinipiriyān Trestle S.	158	G 11 7 49·06	G 79 20 51·19	A 152 57 0·1 G 152 56 57·4	Kachipērumāl E 0 12	+ 13·7		294
295	N	Pātharankota S.	120	G 10 28 2·31	G 79 12 43·59	A 179 40 40·6 G 179 40 42·9	Kakkrakota D 0 4	− 12·4		295
296	62 D	Rāmnapur (old) T.S.	541	A 28 22 0·10 G 28 22 11·04	G 80 28 38·33	A 302 56 33·7 G 302 56 30·9	Rāmnagar D 0 5	+ 5·2	− 10·9	296
297	63 A	Jarūra T.S.	536	A 27 59 50·22 G 27 59 55·94	G 80 28 10·95				− 5·7	297
298	A	Nimkār T.S.	486	A 27 21 8·16 G 27 21 8·09	G 80 29 3·67	A 178 58 28·0 G 178 58 20·7	Darawal D 0 6	+ 14·1	+ 0·1	298
299	B	Etora T.S.	429	A 26 54 22·63 G 26 54 17·85	G 80 39 38·26				+ 4·8	299
300	B	Dewarsān T.S.	439	A 26 15 58·32 G 26 15 52·89	G 80 18 14·46				+ 5·4	300

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES. Case I : $\delta a=1$ km u	$v \cos\lambda$	$w \cot\lambda$	Case II : $\delta b=1$ km u	$v \cos\lambda$	$w \cot\lambda$	FOR CHANGES OF ORIGIN. Case III : Latitude $u_0=1''$ u	$v \cos\lambda$	$w \cot\lambda$	Case IV : Azimuth $w_0=1''$ u	$v \cos\lambda$	$w \cot\lambda$	HELMERT'S SPHEROID* $a=6378200$ metres, $1/\epsilon=298·3$ u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	Serial No.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
273	−3·63		−1·90	+9·13		+0·82	+1·00		+0·15	−0·03		+3·65	+3·69		+3·61	− 7·5	− 6·0	273
274			−1·74			+0·84			+0·14			+4·08			+4·31	− 7·8		274
275	−5·27			+12·69			+1·00			0·00			+4·87				− 2·8	275
276	−5·31		+0·04	+12·76		−0·02	+1·00		−0·01	0·00		+4·79	+4·89		+6·19	+ 3·6	− 4·0	276
277	−5·33			+12·82			+1·00			0·00			+4·90				− 4·2	277
278	−6·05			+14·34			+1·00			−0·01			+5·36				− 3·1	278
279	−6·71		−0·08	+15·70		+0·06	+1·00		+0·01	0·00		+6·34	+5·78		+8·15	+ 6·2	− 3·5	279
280	−6·74			+15·76			+1·00			0·00			+5·80				− 3·7	280
281	−6·74			+15·76			+1·00			0·00			+5·79				− 3·4	281
282	−6·75			+15·79			+1·00			0·00			+5·80				− 3·5	282
283		+0·13			−0·01			0·00			−0·27			−0·24		− 1·5		283
284	−6·77		−0·06	+15·82		+0·04	+1·00		0·00	0·00		+6·43	+5·81		+8·27	− 1·3	− 3·9	284
285	−6·78			+15·84			+1·00			0·00			+5·82				− 3·7	285
286			−0·45			+0·23			+0·03			+4·53			+5·62	− 7·1		286
287			−1·64			+0·98			+0·13			+5·38			+6·20	− 20·1		287
288	−6·06			+14·36			+1·00			−0·01			+5·37				− 2·5	288
289	−6·08		−0·47	+14·40		+0·29	+1·00		+0·04	−0·01		+5·54	+5·38		+6·94	− 0·9	− 3·2	289
290	−6·13			+14·49			+1·00			−0·01			+5·41				− 2·5	290
291			−1·57			+0·96			+0·13			+5·61			+6·55	− 10·3		291
292			−2·05			+1·06			+0·16			+4·41			+4·64	− 6·9		292
293	−4·93			+11·96			+1·00			−0·03			+4·61				+ 1·2	293
294			−1·94			+1·06			+0·15			+4·73			+5·15	+ 11·5		294
295			−1·88			+1·08			+0·15			+5·02			+5·59	− 14·8		295
296	+1·01		−1·53	−3·40		+0·08	+1·00		+0·10	−0·05		+1·92	−1·34		+1·15	+ 3·6	− 9·6	296
297	+0·94			−3·12			+1·00			−0·05			−1·20				− 4·5	297
298	+0·81		−1·57	−2·62		+0·12	+1·00		+0·11	−0·05		+1·99	−0·95		+1·23	+ 12·5	+ 1·1	298
299	+0·72			−2·28			+1·00			−0·05			−0·78				+ 5·6	299
300	+0·57			−1·77			+1·00			−0·04			−0·54				+ 5·9	300

* $\delta a=0·924$, $\delta b=0·743$, $u_0=0·31$, $w_0=1·29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
301	63 C	Kānākhera T.S.	416	A 25 51 25·97 G 25 51 20·95	G 80 25 31·61				+ 5·0	301
302	C	Pavia H.S.	481	A 25 27 21·18 G 25 27 17·39	G 80 44 12·26				+ 3·8	302
303	D	Potenda S.	993	A 24 37 24·71 G 24 37 23·04	G 80 57 7·18				+ 1·7	303
304	E	Dadaura T.S.	420	A 27 43 3·51 G 27 43 18·33	G 81 42 44·29				−14·8	304
305	E	Māsi T.S.	406	A 27 38 14·79 G 27 38 25·17	G 81 23 8·97	A 153 5 50·5 G 153 5 52·2	Bela D 0 5	− 3·2	−10·4	305
306	E	Imlia T.S.	428	A 27 19 17·83 G 27 19 18·90	G 81 7 37·37				− 1·1	306
307	F	Utiāmau T.S.	386	A 27 0 1·62 G 26 59 57·08	G 81 12 17·24				+ 4·5	307
308	F	Parewa T.S.	380	A 26 38 11·44 G 26 38 4·00	G 81 12 11·14				+ 7·4	308
309	F	Sora T.S.	400	A 26 17 26·39 G 26 17 18·83	G 81 12 23·12	A 239 43 3·6 G 239 42 55·9	Janai D 0 4	+ 15·6	+ 7·6	309
310	G	Pariāou T.S.	346	A 25 50 11·59 G 25 50 5·26	G 81 22 16·31				+ 6·3	310
311	G	Pabhosa H.S.	565	G 25 21 17·32	G 81 19 8·40	A 187 38 4·1 G 187 38 4·7	Karra D 0 14	− 1·3		311
312	H	Karāra H.S.	1966	A 24 4 42·20 G 24 4 42·01	G 81 15 47·29	A 269 18 28·7 G 269 18 34·9	Marwās D 0 16	− 13·9	+ 0·2	312
313	I	Manichauk T.S.	360	A 27 36 28·91 G 27 36 48·14	G 82 5 3·16				−19·2	313
314	I	Pathārdi T.S.	320	A 27 25 56·11 G 27 26 14·77	G 82 45 2·97				−18·7	314
315	I	Bāsadela T.S.	377	A 27 23 50·71 G 27 24 3·24	G 82 16 50·44	A 106 15 8·7 G 106 15 7·9	Saibara D 0 3	+ 1·5	−12·5	315
316	J	Orejhār S.	392	G 26 46 55·54	G 82 12 7·60	A 308 36 18·9 G 308 36 17·7	Bisaul D 0 7	+ 2·4		316
317	J	Fyzabad Long. S.	...	G 26 46 40·66	A 82 8 7·60 G 82 8 8·15			− 0·5		317
318	J	Bisaul T.S.	342	G 26 40 37·38	G 82 20 54·43	A 128 40 15·9 G 128 40 14·8	Orejhār D 0 1	+ 2·2		318
319	K	Marār T.S.	370	G 25 41 17·20	G 82 14 19·00	A 42 20 13·2 G 42 20 13·1	Buria D 0 7	+ 0·2		319
320	L	Gurwāni H.S.	2083	A 24 1 28·93 G 24 1 25·71	G 82 17 28·34	A 210 29 53·8 G 210 29 49·4	Pokra D 0 5	+ 9·9	+ 3·2	320
321	M	Ghaus T.S.	296	A 27 20 48·34 G 27 21 5·08	G 83 5 38·81				−16·7	321
322	N	Rājabāri T.S.	296	G 26 54 3·04	G 83 15 35·49	A 104 47 9·8 G 104 47 9·9	Nandaur D 0 5	− 0·2		322
323	N	Samenda T.S.	285	G 26 0 23·97	G 83 13 30·67	A 304 8 50·2 G 304 8 48·7	Ohit Bisram D 0 6	+ 3·1		323
324	O	Hirdepur T.S.	289	G 25 24 23·05	G 83 14 15·46	A 304 4 33·1 G 304 4 33·8	Barhāni D 0 6	− 1·5		324
325	P	Gora H.S.	1828	G 24 4 55·71	G 83 14 13·47	A 282 48 23·9 G 282 48 27·5	Sewādhi D 0 10	− 8·1		325
326	64 A	Amūa H.S.	2113	A 23 59 57·02 G 23 59 56·24	G 80 29 17·26	A 260 4 21·4 G 260 4 19·0	Lakanpura D 0 19	+ 5·4	+ 0·8	326
327	A	Lora H.S.	1923	A 23 29 46·30 G 23 29 41·53	G 80 9 56·85				+ 4·8	327
328	B	Sarandi Pat H.S.	1627	G 22 13 18·98	G 80 3 5·98	A 159 45 20·8 G 159 45 20·5	Tālla E 0 6	+ 0·7		328

*A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: δa=1 km			Case II: δb=1 km			Case III: Latitude u_0=1″			Case IV: Azimuth w_0=1″			a=6378200 metres, $1/e$=298·3.					
	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	
301	+0·47			−1·44			+1·00			−0·05			−0·38				+ 5·4	301
302	+0·38			−1·11			+1·00			−0·05			−0·24				+ 4·0	302
303	+0·16			−0·43			+1·00			−0·05			+0·08				+ 1·6	303
304	+0·90			−2·91			+1·00			−0·07			−1·11				−13·7	304
305	+0·88		−2·06	−2·85		+0·14	+1·00		+0·14	−0·06		+1·96	−1·07		+1·78	− 5·4	− 9·3	305
306	+0·81			−2·60			+1·00			−0·06			−0·95				− 0·1	306
307	+0·75			−2·35			+1·00			−0·06			−0·82				+ 5·3	307
308	+0·66			−2·06			+1·00			−0·06			−0·68				+ 8·1	308
309	+0·59		−2·02	−1·79		+0·19	+1·00		+0·14	−0·06		+2·06	−0·55		+0·98	+14·4	+ 8·2	309
310	+0·48			−1·43			+1·00			−0·06			−0·38				+ 6·7	310
311			−2·13			+0·25			+0·15			+2·13			+1·01	− 2·5		311
312	+0·02		−2·18	+0·03		+0·32	+1·00		+0·15	−0·06		+2·23	+0·27		+1·16	−15·1	− 0·1	312
313	+0·89			−2·83			+1·00			−0·07			−1·06				−18·1	313
314	+0·87			−2·69			+1·00			−0·08			−1·00				−17·7	314
315	+0·89		−2·57	−2·67		+0·19	+1·00		+0·18	−0·07		+1·98	−0·95		+0·37	+ 0·9	−11·5	315
316			−2·58			+0·22			+0·18			+2·02			+0·44	+ 1·7		316
317		−2·69			+0·38			+0·04			+0·05			−2·13		+ 1·6		317
318			−2·65			+0·23			+0·18			+2·03			+0·40	+ 1·6		318
319			−2·65			+0·29			+0·19			+2·10			+0·53	− 0·5		319
320	+0·02		−2·80	+0·07		+0·41	+1·00		+0·20	−0·08		+2·24	+0·29		+0·66	+ 9·2	+ 2·9	320
321	+0·85			−2·63			+1·00			−0·09			−0·97				−15·7	321
322			−3·14			+0·26			+0·22			+2·01			−0·06	− 0·3		322
323			−3·19			+0·32			+0·22			+2·07			+0·03	+ 3·1		323
324			−3·24			+0·37			+0·23			+2·12			+0·08	− 1·6		324
325			−3·36			+0·49			+0·24			+2·23			+0·21	− 8·3		325
326	−0·01		−1·71	+0·10		+0·25	+1·00		+0·12	−0·05		+2·24	+0·31		+1·54	+ 3·9	+ 0·5	326
327	−0·17			+0·53			+1·00			−0·04			+0·50				+ 4·3	327
328			−1·53			+0·30			+0·11			+2·41			+1·96	− 1·1		328

* δa=0·924, δb=0·743, u_0=0·31, w_0=1·29. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
329	64 B	Sarey Khan Lat. S.	1409	A 22 12 50·66 G 22 12 55·61	G 80 2 49·79				− 5·0	329
330	C	Lingmāra H.S.	1400	A 21 42 55·36 G 21 43 3·07	G 80 7 36·30				− 7·7	330
331	C	Sitāpār H.S.	1237	A 21 24 43·83 G 21 24 50·54	G 80 19 26·36				− 6·7	331
332	J	Dalea H.S.	1622	A 22 19 30·25 G 22 19 33·62	G 82 1 31·25				− 3·4	332
333	K	Pathāidi T.S.	879	A 21 48 43·06 G 21 48 45·96	G 82 16 46·96	A 198 23 42·8 G 198 23 43·3	Konārgarh D 0 5	− 1·2	− 2·9	333
334	L	Ramai H.S.	1313	A 20 56 50·31 G 20 56 51·47	G 82 8 18·55	A 223 15 23·2 G 223 15 23·8	Khalāri E 0 9	− 1·6	− 1·2	334
335	P	Sindur H.S.	2918	G 20 15 33·64	G 83 39 42·83	A 201 20 4·4 G 201 20 10·3	Lakh Parbat D 0 35	− 16·0		335
336	65 C	Singāwāram H.S.	714	A 17 45 8·71 G 17 45 10·38	G 80 56 9·04	A 249 3 4·9 G 249 3 6·5	Nārākonda E 0 29	− 5·0	− 1·7	336
337	D	Dhūlipalla S.	245	A 16 25 53·47 G 16 25 56·75	G 80 5 29·59	A 125 53 37·6 G 125 53 39·9	Kachalboru E 0 47	− 7·8	− 3·3	337
338	G	Parampudi H.S.	684	A 17 12 32·63 G 17 12 38·28	G 81 12 10·06	A 114 12 9·2 G 114 12 13·2	Nāgaldurgam E 0 15	− 12·9	− 5·7	338
339	I	Hātbena H.S.	2600	A 19 51 42·60 G 19 51 42·34	G 82 1 25·96				+ 0·3	339
340	I	Karīa H.S.	2014	A 19 12 2·67 G 19 12 5·98	G 82 7 7·97	A 201 43 17·4 G 201 43 17·9	Motigaon E 0 4	− 1·4	− 3·3	340
341	K	Kālingkonda H.S.	4634	G 17 49 42·44	G 82 18 40·67	A 189 41 25·0 G 189 41 24·8	Kaurālbiding D 0 25	+ 0·6		341
342	K	Sānjib H.S.	2142	A 17 31 12·32 G 17 31 18·68	G 82 41 24·30	A 135 38 16·0 G 135 38 15·9	Dhār E 0 55	+ 0·3	− 6·4	342
343	N	Rāwal H.S.	874	A 18 32 4·73 G 18 32 9·22	G 83 33 11·63	A 317 29 5·0 G 317 29 5·0	Piudi D 0 11	0·0	− 4·5	343
344	N	Vizagapatām Base-line N. End S.	181	A 18 0 56·66 G 18 1 2·93	G 83 13 43·36	A 203 44 24·5 G 203 44 24·5	Bor E 0 12	0·0	− 6·3	344
345	O	Waltair Long. S.	200	A 17 43 20·44 G 17 43 29·31	A 83 19 0·17 G 83 19 3·52			− 3·2	− 8·9	345
346	66 A	Ongole H.S.	250	A 15 29 52·87 G 15 29 56·85	G 80 2 27·72				− 4·0	346
347	B	Gudali H.S.	292	A 14 1 10·65 G 14 1 9·45	G 80 1 13·36				+ 1·2	347
348	C	Madras Observatory Long. S.	54	A 13 4 8·97 G 13 4 4·17	A 80 14 47·06 G 80 14 54·33			− 7·1	+ 4·8	348
349	C	St. Thomas's Mount Trestle S.	250	A 13 0 20·64 G 13 0 14·79	G 80 11 41·38	A 12 30 5·3 G 12 30 6·2	Naumangalam D 0 7	− 3·9	+ 5·9	349
350	D	Injambākam H.S.	29	G 12 54 51·18	G 80 15 11·23	A 99 4 39·1 G 99 4 40·6	Naumangalam E 0·23	− 6·5		350
351	72 B	Naunangarhi T.S.	344	G 26 59 10·19	G 84 23 46·86	A 107 52 43·1 G 107 52 47·8	Bakwa D 0 7	− 9·2		351
352	B	Jalālpur T.S.	232	A 26 3 45·56 G 26 3 39·42	G 84 23 9·46	A 111 52 41·5 G 111 52 40·0	Katwārpur D 0 3	+ 3·1	+ 6·1	352
353	C	Nuāon T.S.	251	A 25 34 45·64 G 25 34 37·94	G 84 14 15·86				+ 7·7	353
354	C	Mednipur T.S.	335	A 25 5 22·35 G 25 5 14·02	G 84 22 6·95	A 215 46 30·0 G 215 46 33·5	Bisunpur D 0 6	− 7·5	+ 8·3	354
355	D	Teona H.S.	740	A 24 34 49·76 G 24 34 38·94	G 84 10 26·42				+ 10·8	355
356	D	Hurilāong H.S.	1378	A 24 2 16·74 G 24 2 5·99	G 84 21 50·58	A 128 18 18·3 G 128 18 24·0	Khaira Pāndu D 0 4	− 12·8	+ 10·8	356

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	For Changes of Axes. Case I: $\delta a = 1$ km u	$v \cos\lambda$	$w \cot\lambda$	Case II: $\delta b = 1$ km u	$v \cos\lambda$	$w \cot\lambda$	For Changes of Origin. Case III: Latitude $u_0 = 1''$ u	$v \cos\lambda$	$w \cot\lambda$	Case IV: Azimuth $w_0 = 1''$ u	$v \cos\lambda$	$w \cot\lambda$	Helmert's Spheroid* $a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$. u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	Serial No.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
329	−0·56			+1·65			+1·00			−0·04			+0·97				− 6·0	329
330	−0·72			+2·10			+1·00			−0·04			+1·15				− 8·9	330
331	−0·83			+2·37			+1·00			−0·04			+1·25				− 8·0	331
332	−0·50			+1·55			+1·00			−0·07			+0·91				− 4·3	332
333	−0·60		−2·98	+2·01		+0·60	+1·00		+0·22	−0·07		+2·45	+1·10		+0·92	− 1·9	− 4·0	333
334	−0·95		−2·98	+2·79		+0·67	+1·00		+0·22	−0·07		+2·55	+1·41		+1·10	− 2·4	− 2·6	334
335			−4·09			+1·00			+0·30			+2·62			+0·44	− 15·9		335
336	−2·19		−2·48	+5·81		+0·79	+1·00		+0·19	−0·05		+2·99	+2·54		+2·21	− 6·3	− 4·2	336
337	−2·75		−1·97	+7·12		+0·71	+1·00		+0·15	−0·04		+3·22	+3·01		+2·92	− 9·3	− 6·3	337
338	−2·40		−2·75	+6·34		+0·93	+1·00		+0·21	−0·06		+3·08	+2·73		+2·19	− 13·8	− 8·4	338
339	−1·35			+3·79			+1·00			−0·07			+1·80				− 1·5	339
340	−1·59		−3·17	+4·42		+0·87	+1·00		+0·24	−0·07		+2·77	+2·03		+1·36	− 2·4	− 5·3	340
341			−3·51			+1·11			+0·27			+2·97			+1·50	0·0		341
342	−2·25		−3·85	+6·03		+1·25	+1·00		+0·29	−0·08		+3·02	+2·61		+1·36	− 0·2	− 9·0	342
343	−1·82		−4·31	+5·04		+1·27	+1·00		+0·33	−0·09		+2·86	+2·26		+0·75	+ 0·1	− 6·8	343
344	−2·03		−4·16	+5·54		+1·29	+1·00		+0·31	−0·09		+2·94	+2·44		+1·00	− 0·1	− 8·7	344
345	−2·15	−3·29		+5·83	+0·46		+1·00	+0·03		−0·09	−0·11		+2·53	−2·83		− 0·4	− 11·4	345
346	−3·16			+8·06			+1·00			−0·04			+3·33				− 7·3	346
347	−3·83			+9·57			+1·00			−0·04			+3·83				− 2·6	347
348	−4·28	−1·54		+10·56	+0·20		+1·00	+0·01		−0·04	−0·19		+4·16	−1·52		− 5·6	+ 0·6	348
349	−4·31		−2·52	+10·63		+1·20	+1·00		+0·20	−0·04		+4·05	+4·18		+3·85	− 5·6	+ 1·7	349
350			−2·60			+1·25			+0·21			+4·08			+3·86	− 8·2		350
351			−3·77			+0·30			+0·26			+2·00			−0·60	− 8·8		351
352	+0·61		−3·84	−1·62		+0·38	+0·99		+0·27	−0·11		+2·06	−0·44		−0·53	+ 3·6	+ 6·5	352
353	+0·48			−1·22			+0·99			−0·11			−0·30				+ 8·0	353
354	+0·36		−3·93	−0·82		+0·49	+0·90		+0·28	−0·11		+2·14	−0·11		−0·42	− 7·1	+ 8·4	354
355	+0·22			−0·40			+0·99			−0·10			+0·09				+ 10·7	355
356	+0·08		−4·04	+0·06		+0·58	+0·99		+0·29	−0·11		+2·22	+0·28		−0·34	− 12·3	+ 10·5	356

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID.							Serial No.
			Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
357	72 E	Kaulia H.S.	7051	A 27 48 25·5 G 27 48 58·6	G 85 14 20·7				−33·1	357
358	E	Mahadeo Pokra H.S.	7095	A 27 40 53·6 G 27 41 31·5	G 85 31 19·9				−37·9	358
359	F	Pota T.S.	222	G 26 22 40·13	G 85 26 20·33	A 180 4 5·0 G 180 4 8·3	Madanpur D 0 4	− 6·7		359
360	F	Pahladpur T.S.	175	A 26 4 27·24 G 26 4 21·01	G 85 27 13·16				+ 6·2	360
361	G	Dūbauli T.S.	189	A 25 40 22·99 G 25 40 16·23	G 85 20 16·53				+ 6·8	361
362	G	Bihar H.S.	391	A 25 12 39·27 G 25 12 26·05	G 85 30 31·33				+13·2	362
363	H	Mahar H.S.	1606	A 24 44 31·12 G 24 44 20·88	G 85 9 55·13				+10·2	363
364	K	Bichwi H.S.	321	G 25 9 49·15	G 86 8 3·99	A 357 49 29·7 G 357 49 32·4	Ekgora E 0 40	− 5·7		364
365	N	Chūni T.S.	197	G 26 11 4·72	G 87 2 52·80	A 185 49 39·4 G 185 49 49·0	Minai D 0 4	− 19·5		365
366	O	Sirkanda T.S.	132	G 25 27 47·43	G 87 8 23·57	A 145 34 17·2 G 145 34 21·8	Pureni D 0 4	− 9·7		366
367	73 A	Bulbul H.S.	3352	A 23 37 53·44 G 23 37 44·63	G 84 26 13·94				+ 8·8	367
368	A	Mahwari H.S.	3153	A 23 26 9·28 G 23 26 4·96	G 84 54 1·75				+ 4·3	368
369	A	Bhursu H.S.	2271	G 23 15 57·13	G 84 44 19·21	A 149 58 57·7 G 149 59 0·8	Bagru E 0 38	− 7·2		369
370	C	Andhari H.S.	1442	G 21 57 39·56	G 84 14 57·17	A 17 40 39·6 G 17 40 43·1	Garpati D 0 1	− 8·7		370
371	E	Chendwār (old) H.S.	2817	A 23 57 16·82 G 23 57 13·75	G 85 26 9·29	A 92 35 20·3 G 92 35 20·5	Kasiātu D 0 16	− 0·5	+ 3·1	371
372	H	Cuttack H.S.	133	A 20 28 52·05 G 20 29 0·68	G 85 52 1·43	A 155 35 54·6 G 155 35 54·3	Kaplās E 1 21	+ 0·8	− 8·6	372
373	I	Pārasnāth H.S.	4481	G 23 57 34·89	G 86 8 10·86	A 145 7 21·0 G 145 7 22·9	Bāmani D 1 2	− 4·3		373
374	I	Tilabani H.S.	1329	G 23 24 59·87	G 86 33 14·64	A 272 58 23·5 G 272 58 23·1	Sūsinia D 0 8	+ 0·9		374
375	M	Malūncha H.S.	970	A 23 54 29·64 G 23 54 29·02	G 87 5 41·86	A 74 46 32·3 G 74 46 35·7	Durgapur D 0 1	− 7·7	+ 0·6	375
376	M	Madhpur T.S.	180	G 23 9 53·06	G 87 44 37·29	A 206 49 9·1 G 206 49 5·5	Parhat D 0 8	+ 8·4		376
377	N	Kalsībhānga T.S.	303	G 22 20 23·80	G 87 8 19·19	A 115 7 20·2 G 115 7 17·7	Kalābani D 0 1	+ 6·1		377
378	O	Dariāpur T.S.	63	A 21 47 28·82 G 21 47 27·95	G 87 52 3·32				+ 0·9	378
379	O	Patna T.S.	80	A 21 47 17·28 G 21 47 20·83	G 87 11 45·53	A 207 38 56·0 G 207 38 58·5	Dāntūn D 0 3	− 6·3	− 3·6	379
380	O	Chandīpur T.S.	53	A 21 26 34·03 G 21 26 36·99	G 87 2 3·66	A 96 49 54·7 G 96 49 55·1	Nilgiri E 0 59	− 1·0	− 3·0	380
381	74 A	Khundābolo H.S.	3115	A 19 51 7·03 G 19 51 12·90	G 84 58 17·43	A 196 41 21·2 G 196 41 22·9	Chiklīkhāi D 0 27	− 4·7	− 5·9	381
382	B	Deodonger H.S.	4534	G 18 54 32·37	G 84 3 35·84	A 146 26 29·1 G 146 26 33·0	Thaladi D 1 21	− 11·4		382
383	B	Mal H.S.	483	A 18 47 6·75 G 18 47 16·97	G 84 30 44·31				−10·2	383
384	78 A	Phallut h.s.	11815	A 27 12 4·30 G 27 12 40·86	G 88 1 0·96				−36·6	384

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES. Case I: $\delta a = 1$ km u	$v \cos\lambda$	$w \cot\lambda$	Case II: $\delta b = 1$ km u	$v \cos\lambda$	$w \cot\lambda$	FOR CHANGES OF ORIGIN. Case III: Latitude $u_0 = 1''$ u	$v \cos\lambda$	$w \cot\lambda$	Case IV: Azimuth $w_0 = 1''$ u	$v \cos\lambda$	$w \cot\lambda$	HELMERT'S SPHEROID* $a = 6378200$ metres, $1/\epsilon = 298{\cdot}3$. u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	Serial No.
	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	
357	+1·01			−2·99			+0·99			−0·12			−1·13				−32·0	357
358	+1·00			−2·90			+0·99			−0·13			−1·08				−36·8	358
359			−4·41			+0·40			+0·30			+2·04			−1·05	− 5·6		359
360	+0·64			−1·63			+0·99			−0·12			−0·47				+ 6·7	360
361	+0·54			−1·30			+0·99			−0·12			−0·32				+ 7·1	361
362	+0·43			−0·92			+0·99			−0·13			−0·14				+13·3	362
363	+0·29			−0·54			+0·99			−0·12			+0·03				+10·2	363
364			−4·94			+0·58			+0·35			+2·12			−1·28	− 4·2		364
365			−5·33			+0·50			+0·37			+2·04			−1·82	− 17·9		365
366			−5·48			+0·60			+0·38			+2·10			−1·79	− 7·9		366
367	−0·04			+0·41			+0·99			−0·11			+0·43				+ 8·4	367
368	−0·08			+0·57			+0·99			−0·12			+0·50				+ 3·8	368
369			−4·36			+0·71			+0·31			+2·29			−0·43	− 6·6		369
370			−4·23			+0·83			+0·31			+2·43			−0·06	− 8·4		370
371	+0·09		−4·69	+0·12		+0·68	+0·99		+0·33	−0·12		+2·23	+0·32		−0·85	+ 0·6	+ 2·8	371
372	−1·02		−5·54	+3·20		+1·30	+0·99		+0·41	−0·13		+2·58	+1·58		−0·69	+ 2·0	−10·2	372
373			−5·10			+0·74			+0·37			+2·23			−1·19	− 2·9		373
374			−5·44			+0·86			+0·39			+2·27			−1·35	+ 2·5		374
375	+0·13		−5·69	+0·15		+0·83	+0·99		+0·41	−0·15		+2·22	+0·35		−1·63	− 5·9	+ 0·2	375
376			−6·21			+1·02			+0·44			+2·28			−1·88	+ 10·5		376
377			−5·99			+1·11			+0·43			+2·37			−1·53	+ 8·1		377
378	−0·49			+2·00			+0·98			−0·16			+1·13				− 0·2	378
379	−0·52		−6·13	+2·00		+1·22	+0·99		+0·45	−0·15		+2·43	+1·12		−1·49	− 4·3	− 4·7	379
380	−0·64		−6·10	+2·32		+1·27	+0·99		+0·45	−0·15		+2·47	+1·24		−1·37	+ 0·9	− 4·2	380
381	−1·28		−5·05	+3·79		+1·29	+0·99		+0·37	−0·12		+2·66	+1·79		−0·16	− 3·9	− 7·7	381
382			−4·60			+1·30			+0·34			+2·80			+0·44	− 10·9		382
383	−1·70			+4·80			+0·99			−0·11			+2·16				−12·4	383
384	+1·00			−2·53			+0·98			−0·17			−0·86				−35·7	384

* $\delta a = 0{\cdot}924$, $\delta b = 0{\cdot}743$, $u_0 = 0{\cdot}31$, $w_0 = 1{\cdot}29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID.							Serial No.
			Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
385	78 A	Tonglu h.s.	10073	A 27 1 11·30 G 27 1 53·54	G 88 5 2·93				−42·2	385
386	B	Senchal h.s.	8600	A 26 58 33·01 G 26 59 8·25	G 88 17 44·78				−35·2	386
387	B	Kurseong h.s.	4428	A 26 51 15·05 G 26 52 5·56	G 88 15 54·68				−50·5	387
388	B	Siliguri s.	401	A 26 41 18·10 G 26 41 40·37	G 88 24 49·54				−22·3	388
389	B	Jalpaiguri Long. s.	280	A 26 31 11·44 G 26 31 17·39	A 88 43 52·42 G 88 44 12·77	A 321 33 25·3 G 321 33 33·0	Dharampur D 0 2	− 15·4	− 6·0	389
390	B	Rāmganj T.S.	249	G 26 18 55·51	G 88 17 30·43	A 218 51 56·2 G 218 52 8·5	Kanchābāri D 0 2	− 24·9		390
391	B	Lohārgara T.S.	205	A 26 2 14·17 G 26 2 12·02	G 88 21 56·60				+ 2·2	391
392	C	Chanduria T.S.	160	A 25 44 31·93 G 25 44 27·47	G 88 22 17·15				+ 4·5	392
393	D	Charaldānga T.S.	149	A 24 52 45·36 G 24 52 43·95	G 88 23 4·21				+ 1·4	393
394	F	Ataro Bānki T.S.	133	G 26 4 50·62	G 89 28 3·10	A 70 52 20·4 G 70 52 32·5	Chandrapur D 0 3	− 24·7		394
395	G	Alangjāni T.S.	143	G 25 59 6·81	G 89 45 41·19	A 293 0 46·2 G 293 0 57·3	Sānding E 0 1	− 22·8		395
396	G	Halkāchar T.S.	103	G 25 9 55·94	G 89 42 48·42	A 145 54 38·0 G 145 54 48·9	Kānchipāra D 0 5	− 23·2		396
397	H	Aloākāndi T.S.	88	G 24 45 29·80	G 89 38 42·19	A 205 17 22·4 G 205 17 28·5	Gaborgram D 0 4	− 13·2		397
398	J	Raikusni H.S.	803	G 26 8 11·37	G 90 39 47·24	A 136 38 12·9 G 136 38 24·2	Bhairaber Chura E 0 34	− 23·0		398
399	O	Rangsanobo H.S.	4455	G 25 15 19·60	G 91 43 20·86	A 125 49 11·9 G 125 49 18·5	Mosingi E 1 21	− 14·0		399
400	79 A	Madhupur T.S.	92	A 23 56 42·82 G 23 56 38·97	G 88 29 7·66	A 172 57 25·5 G 172 57 31·7	Imāmnagar D 0 3	− 14·0	+ 3·9	400
401	A	Anandbās T.S.	67	G 23 21 19·24	G 88 22 40·30	A 6 58 55·2 G 6 58 58·6	Jeodhāra D 0 3	− 7·9		401
402	B	Aknāpur T.S.	98	G 22 54 22·85	G 88 3 6·66	A 147 41 14·5 G 147 41 15·7	Hākistāpur D 0 4	− 2·8		402
403	B	Calcutta Base-line S. End T.S.	13	G 22 36 55·68	G 88 22 54·43	A 177 10 27·3 G 177 10 30·3	Calcutta Base-line N. End D 0 2	− 7·2		403
404	B	Calcutta Long. s.	18	A 22 32 55·58 G 22 32 54·67	A 88 21 17·84 G 88 21 29·10			− 10·4	+ 0·9	404
405	E	Tepri T.S.	67	G 23 57 24·45	G 89 52 11·99	A 156 35 52·8 G 156 35 55·7	Bangaon D 0 4	− 6·5		405
406	E	Daulatpur T.S.	60	G 23 8 43·76	G 89 42 57·76	A 202 38 51·3 G 202 38 49·3	Maheshpur D 0 5	+ 4·7		406
407	I	Lakhinagar T.S.	51	G 23 0 39·73	G 90 45 43·08	A 85 27 44·7 G 85 27 39·6	Kālishpur D 0 4	+ 12·0		407
408	J	Gangapur T.S.	54	G 22 59 34·77	G 90 27 28·63	A 151 19 48·9 G 151 19 49·0	Malgaon D 0 5	− 0·2		408
409	M	Dawa H.S.	205	G 23 45 17·63	G 91 20 16·63	A 173 18 51·3 G 173 18 53·0	Lambusara D 0 5	− 3·9		409
410	N	Semu Tān H.S.	226	G 22 48 38·48	G 91 47 38·49	A 272 20 55·4 G 272 20 57·1	Bhatri Moin E 1 22	− 4·0		410
411	N	Nagārkhāna H.S.	290	A 22 22 57·08 G 22 22 56·40	G 91 48 30·42	A 155 47 13·3 G 155 47 16·6	Chandranath E 0 22	− 8·0	+ 0·7	411
412	N	Chittagong Long. S.	...	G 22 20 18·43	A 91 50 4·91 G 91 50 16·68			− 10·9		412

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I : $\delta a = 1$ km			Case II : $\delta b = 1$ km			Case III : Latitude $u_0 = 1''$			Case IV : Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$					
	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	u	$v \cos \lambda$	$w \cot \lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
385	+0·96			−2·40			+0·98			−0·17			−0·79				− 41·4	385
386	+0·96			−2·36			+0·98			−0·17			−0·78				− 34·4	386
387	+0·94			−2·27			+0·98			−0·17			−0·74				− 49·8	387
388	+0·90			−2·13			+0·98			−0·17			−0·67				− 21·6	388
389	+0·88	−6·63	−6·23	−2·00	+0·93	+0·54	+0·98	+0·09	+0·43	−0·18	+0·03	+2·01	−0·60	−5·37	−2·64	− 13·0	− 5·4	389
390			−6·02			+0·55			+0·42			+2·02			−2·41	− 22·7		390
391	+0·75			−1·61			+0·98			−0·17			−0·43				+ 2·6	391
392	+0·68			−1·37			+0·98			−0·17			−0·32				+ 4·8	392
393	+0·46			−0·67			+0·98			−0·17			0·00				+ 1·4	393
394			−6·71			+0·64			+0·47			+2·03			−2·97	− 21·9		394
395			−6·89			+0·67			+0·48			+2·04			−3·09	− 19·7		395
396			−7·00			+0·81			+0·49			+2·10			−3·01	− 20·2		396
397			−7·04			+0·88			+0·50			+2·13			−2·95	− 10·2		397
398			−7·37			+0·69			+0·51			+2·02			−3·54	− 19·5		398
399			−8·13			+0·91			+0·57			+2·07			−4·00	− 10·0		399
400	+0·20		−6·51	+0·12		+0·94	+0·98		+0·46	−0·17		+2·21	+0·36		−2·32	− 11·7	+ 3·5	400
401			−6·56			+1·04			+0·47			+2·26			−2·22	− 5·5		401
402			−6·44			+1·10			+0·46			+2·31			−2·02	− 0·6		402
403			−6·71			+1·19			+0·49			+2·33			−2·16	− 4·8		403
404	−0·23	−6·41		+1·32	+0·90		+0·98	+0·07		−0·17	−0·03		+0·85	−5·27		− 5·1	0·0	404
405			−7·40			+1·05			+0·52			+2·20			−3·06	− 3·2		405
406			−7·51			+1·23			+0·54			+2·27			−2·93	+ 7·8		406
407			−8·08			+1·34			+0·58			+2·28			−3·36	+ 15·4		407
408			−7·90			+1·32			+0·57			+2·28			−3·20	+ 3·0		408
409			−8·26			+1·19			+0·59			+2·20			−3·73	− 0·2		409
410			−8·86			+1·28			+0·63			+2·29			−4·08	+ 0·1		410
411	−0·10		−9·02	+1·45		+1·32	+0·97		+0·64	−0·22		+2·33	+1·00		−4·15	− 3·8	− 0·3	411
412		−8·49			+1·20			+0·09			−0·04			−6·97		− 3·9		412

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed station	(A−G) cot λ for azimuth or (A−G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
413	83 H	Loijing H.S.	6610	G 24 44 28·17	G 93 43 44·79	A 120 58 12·0 G 120 58 16·0	Pangkibot D 0 48	− 8·7		413
414	K	Thyolichiing H.S.	6566	G 25 0 5·76	G 94 43 46·98	A 113 3 3·9 G 113 3 7·3	Sirohifurar E 1 15	− 7·3		414
415	L	Tamunja H.S.	3387	G 24 39 9·33	G 94 36 48·01	A 116 36 26·7 G 116 36 26·9	Khambiching E 1 34	− 0·4		415
416	P	Seikpa H.S.	3857	G 24 35 37·84	G 95 45 33·72	A 258 32 25·1 G 258 32 37·6	Madhun E 0 17	− 27·3		416
417	P	Thonbinzin H.S.	1932	G 24 14 3·03	G 95 58 8·16	A 277 46 13·1 G 277 46 21·6	Katha E 0 37	− 18·9		417
418	84 C	Fi Tān H.S.	563	G 21 49 20·78	G 92 8 16·02	A 256 23 22·7 G 256 23 29·2	Luraintong E 1 34	− 16·2		418
419	D	Akyab Long. S.	20	A 20 8 14·87 G 20 8 13·10	A 92 53 38·63 G 92 53 49·63			− 10·3	+ 1·8	419
420	H	Yeponetaung H.S.	2819	A 20 14 51·83 G 20 14 55·91	G 93 41 49·34	A 74 17 19·6 G 74 17 23·3	Rongdong D 1 1	− 10·0	− 4·1	420
421	H	Dattaung H.S.	455	G 20 13 14·44	G 93 1 9·09	A 171 27 28·3 G 171 27 31·0	Bengara E 0 37	− 7·3		421
422	M	Ubyetaung H.S.	2766	G 23 40 52·06	G 95 57 42·75	A 303 38 45·7 G 303 38 50·1	Tagaungtaung D 0 17	− 10·0		422
423	M	Male H.S.	848	G 23 2 53·30	G 95 57 18·09	A 316 31 54·4 G 316 32 0·8	Wapyadaung E 1 12	− 15·0		423
424	N	Sheinmaga H.S.	456	G 22 16 33·89	G 95 58 15·79	A 354 23 23·8 G 354 23 31·5	Mingun E 0 31	− 18·8		424
425	N	Mingun H.S.	1343	G 22 3 0·71	G 95 59 41·41	A 174 23 57·7 G 174 24 5·6	Sheinmaga D 0 43	− 19·5		425
426	P	Taungpila H.S.	1012	G 20 41 52·71	G 95 53 4·50	A 240 23 15·5 G 240 23 17·0	Yuba E 0 50	− 4·0		426
427	85 E	Retkamauk H.S.	1582	A 19 47 37·32 G 19 47 38·55	G 93 28 13·32	A 229 44 59·4 G 229 44 59·2	Ingrautaung D 0 14	+ 0·6	− 1·2	427
428	N	Kyaunggyi S.	...	G 18 49 20·95	G 95 12 55·40	A 109 26 42·1 G 109 26 46·0	Prome E 3 45	− 11·4		428
429	N	Prome Long. S.	100	A 18 49 18·62 G 18 49 14·28	A 95 12 42·20 G 95 12 57·44			− 14·4	+ 4·3	429
430	92 G	Kumon Bum H.S.	7970	G 25 38 13·48	G 97 3 34·06	A 308 54 40·0 G 308 54 46·8	Maran Bum D 2 7	− 14·2		430
431	H	Kumtum Bum H.S.	1833	G 24 46 44·32	G 97 9 17·40	A 210 24 26·5 G 210 24 32·9	Maran Bum D 0 17	− 13·9		431
432	93 A	Sinpitaung H.S.	2649	G 23 29 48·36	G 96 45 45·79	A 162 2 20·3 G 162 2 25·8	Taungkalat D 0 32	− 12·7		432
433	E	Loi Hpa Lang H.S.	3591	G 23 14 13·07	G 97 37 24·38	A 188 35 42·8 G 188 35 49·4	Loi Song E 1 11	− 15·4		433
434	J	Loi Hpatan H.S.	6419	G 22 55 35·51	G 98 0 52·58	A 210 28 36·0 G 210 28 43·6	Loi Taow E 0 13	− 18·0		434
435	O	Loi Kiipma H.S	3792	G 22 1 56·87	G 98 4 30·30	A 154 30 44·7 G 154 30 53·0	Loi Hsimu E 0 2	− 20·5		435
436	94 A	LoiHsamHsum H.S.	6472	G 21 41 45·04	G 99 53 57·51	A 115 53 15·6 G 115 53 22·4	R. M.	− 17·1		436
437	B	Letpataung H.S.	3975	G 19 34 7·27	G 96 28 38·76	A 174 46 24·5 G 174 46 31·5	Byingye E 0 36	− 19·7		437
438	B	Toungoo S.	186	G 18 56 1·54	G 96 25 55·21	A 30 46 36·9 G 30 46 42·5	Bhondan E 0 49	− 16·3		438
439	B	Myayabeingkyo H.S.	1411	G 18 21 33·93	G 96 22 53·46	A 169 33 42·4 G 169 33 43·8	Khengdan E 0 32	− 4·2		439
440	H	Martaban h.s.	273	A 16 31 39·6 G 16 31 33·08	G 97 36 59·57				+ 6·5	440

*A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

XCV.

in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES. Case I: $\delta a = 1$ km u	$v\cos\lambda$	$w\cot\lambda$	Case II: $\delta b = 1$ km u	$v\cos\lambda$	$w\cot\lambda$	FOR CHANGES OF ORIGIN. Case III: Latitude $u_0 = 1''$ u	$v\cos\lambda$	$w\cot\lambda$	Case IV: Azimuth $w_0 = 1''$ u	$v\cos\lambda$	$w\cot\lambda$	HELMERT'S SPHEROID* $a = 6378200$ metres. $1/\epsilon = 298{\cdot}3$. u	$v\cos\lambda$	$w\cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	Serial No.
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
413			−9·40			+1·14			+0·66			+2·09			−4·93	− 4·0		413
414			−9·91			+1·14			+0·70			+2·06			−5·43	− 2·1		414
415			−9·93			+1·23			+0·70			+2·09			−5·34	+ 4·7		415
416			−10·00			+1·32			+0·75			+2·08			−5·90	− 21·6		416
417			−10·82			+1·45			+0·76			+2·11			−5·96	− 13·1		417
418			−9·48			+1·33			+0·67			+2·37			−4·49	− 11·9		418
419	−0·79	−9·11		+3·48	+1·29		+0·97	+0·09		−0·24	−0·08		+1·85	−7·52		− 2·8	− 0·1	419
420	−0·69		−11·20	+3·37		+1·70	+0·96		+0·80	−0·25		+2·54	+1·83		−5·57	− 5·2	− 5·9	420
421			−10·77			+1·59			+0·77			+2·55			−5·24	− 2·9		421
422			−11·01			+1·56			+0·78			+2·16			−5·99	− 4·2		422
423			−11·28			+1·65			+0·80			+2·21			−6·09	− 9·1		423
424			−11·63			+1·75			+0·83			+2·29			−6·24	− 13·1		424
425			−11·75			+1·78			+0·84			+2·31			−6·31	− 13·7		425
426			−12·38			+1·98			+0·88			+2·45			−6·54	+ 2·0		426
427	−0·88		−11·29	+3·80		+1·71	+0·96		+0·81	−0·25		+2·60	+1·99		−5·56	+ 5·4	− 3·2	427
428			−13·04			+2·18			+0·94			+2·70			−6·66	− 5·6		428
429	−1·12	−10·48		+4·70	+1·45		+0·95	+9·10		−0·28	−0·11		+2·40	−8·71		− 5·7	+ 1·9	429
430			−11·05			+1·09			+0·77			+1·99			−6·59	− 7·8		430
431			−11·34			+1·35			+0·80			+2·05			−6·58	− 7·5		431
432			−11·55			+1·66			+0·82			+2·16			−6·43	− 6·5		432
433			−12·17			+1·78			+0·87			+2·18			−6·84	− 8·8		433
434			−12·55			+1·87			+0·90			+2·20			−7·09	− 11·1		434
435			−13·01			+2·03			+0·93			+2·28			−7·28	− 13·4		435
436			−14·28			+2·31			+1·02			+2·03			−8·54	− 8·9		436
437			−13·41			+2·27			+0·97			+2·58			−7·07	− 13·2		437
438			−13·79			+2·41			+0·99			+2·66			−7·20	− 9·7		438
439			−14·15			+2·51			+1·02			+2·75			−7·35	+ 2·3		439
440	−1·87			+6·90			+0·94			−0·31			+3·29				+ 3·2	440

* $\delta a = 0{\cdot}924$, $\delta b = 0{\cdot}743$, $u_0 = 0{\cdot}31$, $w_0 = 1{\cdot}29$. *Vide* p. 2.

TABLE

Deflections of the Plumb-line

Serial No.	Sheet No.	Observed at	EVEREST'S SPHEROID. Height in feet	Latitude*	Longitude*	Azimuth*	Name and angular Elevation or Depression of observed Station	(A − G) cot λ for azimuth or (A − G) cos λ for longitude observations†	Meridian Deflection†	Serial No.
				° ′ ″	° ′ ″	° ′ ″	° ′	″	″	
441	94 H	Moulmein Long. S.	90	A 16 30 2·97 G 16 29 54·90	A 97 37 23·41 G 97 37 40 04			− 15·9	+ 8·1	441
442	H	Taungzun H.S.	854	A 16 25 56·2 G 16 25 48·55	G 97 40 18·59	A 31 16 18·9 G 31 16 22·7	Konlah D 0 34	− 12·9	+ 7·7	442
443	95 G	Southern Moscos H.S.	1186	G 13 49 59·67	G 97 55 4·49	A 162 20 54·5 G 162 20 55·6	Middle Moscos D 0 9	− 4·5		443
444	L	Sandawat H.S.	719	A 12 28 0·1 G 12 27 51·87	G 98 40 31·46				+ 8 2	444
445	L	Natkalintaung H.S.	888	G 12 25 33·42	G 98 43 33·26	A 127 46 35·9 G 127 46 37 6	Sandawat D 0 22	− 7 7		445
446	L	Mergui Base-line E. End T.S.	20	A 12 22 29·5 G 12 22 21·17	G 98 46 36·49	A 72 29 47 9 G 72 29 49·3	Mergui Base-line W. End D 0 2	− 6·4	+ 8·3	446
447	L	Mergui Base-line W. End T S.	18	A 12 21 41·3 G 12 21 32 57	G 98 43 59 73	A 252 29 14·0 G 252 29 15·7	Mergui Base-line E. End D 0 2	− 7 8	+ 8·7	447
448	95 L	Minthangtaung H.S.	3850	A 12 19 44·2 G 12 19 35·05	G 98 47 47·88	A 157 5 40 9 G 157 5 43·2	Mergui Base-line E. End D 0 25	− 10·5	+ 9·1	448
		Addenda.								
449	30 C	Robat S.	3095	A 29 49 9·16 G 29 48 58·75	G 60 55 10·90				+ 10·4	449
450	34 L	Zawa H.S	7922	G 28 57 44·43	G 66 35 18·70	A 178 40 55·7 G 178 40 50 0	Zebra E 0 6	+ 10·3		450
451	43 G	Murree Obsy. s.	7458	G 33 54 57·36	G 73 24 25·67	A 227 39 50·4 G 227 39 54·1	Nerh D 1 6	− 5·5		451
452	43 J	Rustamgarhi h.s.	5351	G 34 4 38·70	G 74 49 53·68	A 30 52 6·8 G 30 52 27·1	Gogipatri E 1 24	− 30 0		452
453	43 J	Poshkar H.S.	8323	A 34 2 3·78‡ G 34 1 49·01	G 74 29 51·26	A 318 14 0·7 G 318 13 54·2	Gogipatri D 0 30	+ 9·6	+ 14·8	453
454	43 K	Gogipatri H.S.	7752	A 33 51 46·93‡ G 33 51 43·89	G 74 40 38 61	A 222 17 18·0 G 222 17 12·4	Zebanwan E 0 29	+ 8 3.	+ 3·0	454

* A = Astronomical Value.
G = Triangulated or Geodetic Value.

† *Minus* sign indicates Easterly or Northerly Deflection of Plumb-line.

‡ Observations unsatisfactory in so far that determinations by N. stars differ considerably from those by S. stars, both sets agreeing well among themselves; whatever was the cause it appears probable that the effect would cancel in mean of N. and S. star determinations.

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in terms of any Spheroid.

Serial No.	FOR CHANGES OF AXES.						FOR CHANGES OF ORIGIN.						HELMERT'S SPHEROID*					Serial No.
	Case I: $\delta a = 1$ km			Case II: $\delta b = 1$ km			Case III: Latitude $u_0 = 1''$			Case IV: Azimuth $w_0 = 1''$			$a = 6378200$ metres, $1/\epsilon = 298 \cdot 3$.					
	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	u	$v \cos\lambda$	$w \cot\lambda$	Deflection in Prime Vertical	Deflection in Meridian	
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
441	−1·88	−11·86		+6·93	+1·66		+0·94	+0·10		−0·31	−0·15		+3·31	−9·89		− 6·0	+ 4·8	441
442	−1·90		−16·69	+7·00		+3·38	+0·94		+1·21	−0·31		+3·04	+3·33		−8·61	− 6·0	+ 4·4	442
443			−19·85			+4·56			+1·45			+3·58			−9·88	+ 3·4		443
444	−3·64			+11·05			+0·93			+0·33			+4·72				+ 3·5	444
445			−22·77			+5·76			+1·67			+3·96			−11·13	+ 1·1		445
446	−3·68		−22·91	+11·14		+5·82	+0·93		+1·68	−0·33		+3·98	+4·74		−11·19	+ 2·5	+ 3·6	446
447	−3·69		−22·90	+11·16		+5·81	+0·93		+1·68	−0·33		+3·98	+4·75		−11·18	+ 1·1	+ 3·9	447
448	−3·70		−23·01	+11·19		+5·87	+0·93		+1·69	−0·33		+3·99	+4·76		−11·23	− 1·6	+ 4·3	448
		Addenda.																
449	+1·85			−4·49			+0·96			+0·26			−0·98				+11·4	449
450			+5·92			−0·22			−0·40			+1·85			+7·58	+ 2·5		450
451			+2·12			+0·08			−0·13			+1·63			+4·08	−10·0		451
452			+1·91			+0·05			−0·09			+1·63			+3·87	−34·3		452
453	+1·63		+2·07	−7·22		+0·06	+1·00		−0·10	+0·05		+1·63	−3·48		+4·03	+ 5·2	+18·3	453
454	+1·62		+1·49	−7·12		+0·05	+1·00		−0·09	+0·05		+1·64	−3·42		+3·50	+ 4·4	+ 6·4	454

* $\delta a = 0 \cdot 924$, $\delta b = 0 \cdot 743$, $u_0 = 0 \cdot 31$, $w_0 = 1 \cdot 29$. *Vide* p. 2.

To complete the statement of data concerning gravity values of the residuals, observed *minus* theoretical values are now given. These have been taken from Professional Paper No. 15 and for a full explanation reference should be made thereto. It is only necessary to explain that γ_a, γ_b, γ_c are the theoretical values of gravity based on Helmert's formula :— $\gamma_0 = 978\cdot030\ (1 + 0\cdot005302\ \sin^2\phi - 0\cdot000007\ \sin^2 2\phi)$ and assuming the corrections according to the Free Air, Bouguer and Hayford hypotheses respectively.

TABLE XCVI.

No.	Name	Latitude	Longitude	Height	$g-\gamma_a-\cdot011$	$g-\gamma_b+\cdot030$	$g-\gamma_c-\cdot011$
		° ′	° ′	*feet*	*dynes*	*dynes*	*dynes*
1	Agra	27 10	78 1	585	−·012	+·011	+·006
2	Alīgarh	27 54	78 1	612	−·039	−·019	−·018
3	Allahābād	25 26	81 55	288	−·023	+·008	−·002
4	Amgaon	21 22	80 28	1032	−·015	−·009	−·014
5	Amraoti	20 56	77 46	1123	+·014	+·017	+·015
6	Arrah	25 34	84 39	188	−·067	−·032	−·039
7	Asarori	30 14	77 58	2467	−·061	−·101	...
8	Asirgarh	21 28	76 18	2077	+·046	+·023	+·019
9	Badnūr	21 54	77 54	2103	+·045	+·015	+·027
10	Bangalore	13 1	77 35	3118	+·014	−·050	...
11	Bassein	16 47	94 44	23	+·006	+·046	...
12	Bhopāl	23 16	77 25	1630	+·018	+·004	+·011
13	Bilāspur	22 4	82 12	878	−·006	+·005	+·002
14	Bina	24 11	78 12	1355	+·015	+·010	+·015
15	Buxar	25 35	83 59	207	−·051	−·017	−·025
16	Chātra	24 13	88 23	64	−·025	+·014	−·006
17	Colāba	18 54	72 49	34	+·052	+·092	+·052
18	Cuttack	20 29	85 52	92	−·005	+·033	−·005
19	Daltonganj ...	24 2	84 4	707	−·004	+·013	+·014
20	Damoh	23 50	79 26	1213	−·012	−·012	−·007
21	Darjeeling	27 3	88 16	6966	+·044	−·124	...
22	Dehra Dūn ...	30 19	78 3	2239	−·085	−·115	−·005
23	Dera Ghāzi Khān ...	30 4	70 46	397	−·108	−·080	...
24	Dholpur	26 42	77 55	577	−·030	−·008	−·016
25	Edgar Shaft (Surface) ...	12 56	78 16	2945	+·053	−·005	...
26	Ellichpur	21 18	77 31	1314	+·019	+·016	+·020
27	Fatehpur	30 26	77 44	1434	−·085	−·089	...
28	Ferozepur	30 56	74 37	647	−·004	+·015	...
29	Gaya	24 48	85 0	361	−·031	−·002	−·008
30	Gesupur	28 33	77 42	691	−·031	−·013	−·006
31	Goona	24 39	77 19	1569	+·015	+·003	+·008
32	Gorakhpur	26 45	83 23	257	−·127	−·095	−·081
33	Gwalior	26 14	78 13	658	−·030	−·011	−·018

TABLE XCVI.—(Continued).

No.	Name	Latitude	Longitude	Height	$g-\gamma_a-\cdot011$	$g-\gamma_b+\cdot030$	$g-\gamma_c-\cdot011$
		° ′	° ′	*feet*	*dynes*	*dynes*	*dynes*
34	Hardwār	29 56	78 9	949	−·117	−·106	...
35	Hāthras	27 37	78 3	587	−·020	+·001	000
36	Henzada	17 39	95 27	46	−·031	+·008	...
37	Hoshangābād ...	22 45	77 44	1002	·000	+·007	+·010
38	Jacobābād	28 17	68 27	183	+·003	+·038	+·027
39	Jalgaon	21 0	75 34	760	·000	+·015	+·009
40	Jalpaiguri	26 31	88 44	268	−·124	−·091	−·031
41	Japla	24 32	84 0	474	−·031	−·006	−·009
42	Jhānsi	25 27	78 34	858	−·004	+·008	+·003
43	Jubbulpore ..	23 9	79 59	1467	+·017	+·009	+·019
44	Kaliāna	29 31	77 39	810	−·065	−·051	−·018
45	Kaliānpur	24 7	77 39	1763	+·039	+·021	+·028
46	Kālka	30 50	76 56	2202	−·045	−·074	...
47	Kālsi	30 31	77 50	1684	−·084	−·090	...
48	Katni	23 50	80 26	1254	−·010	−·011	−·004
49	Kesarbāri	26 8	88 31	204	−·071	−·037	...
50	Khandwa	21 50	76 22	1014	+·033	+·010	+·036
51	Khurja	28 14	77 52	649	−·054	−·035	−·030
52	Kisnapur	25 2	88 28	113	+·001	+·038	+·028
53	Kodaikānal ...	10 14	77 28	7665	+·156	−·049	...
54	Kurseong	26 53	88 17	4913	−·011	−·117	...
55	Lalitpur	24 41	78 24	1199	−·016	−·015	−·013
56	Ludhiāna	30 55	75 51	835	−·053	−·040	...
57	Mach	29 52	67 18	3522	−·032	−·103	...
58	Madras	13 4	80 15	20	−·024	+·016	−·064
59	Maihar	24 16	80 48	1161	−·020	−·018	−·014
60	Majhauli Rāj ...	26 18	83 58	219	−·105	−·071	−·068
61	Mandalay	22 0	96 6	244	−·028	+·006	...
62	Maymyo	22 1	96 28	3495	+·050	−·026	...
63	Meiktila	20 51	95 52	799	−·003	+·011	...
64	Meerut	29 0	77 42	734	−·035	−·019	...
65	Mhow	22 33	75 46	1903	−·002	−·025	−·026
66	Mīān Mīr	31 32	74 23	708	−·004	+·013	+·029
67	Moghal Sarai ...	25 17	83 6	257	−·040	−·008	−·016
68	Mogok	22 55	96 30	3685	+·063	−·020	...
69	Mohan	30 11	77 55	1660	−·081	−·093	...
70	Monghyr	25 23	86 28	154	−·067	−·031	−·036
71	Montgomery ...	30 40	73 6	557	−·011	+·011	+·008
72	Mortakka	22 13	76 3	576	−·022	·000	−·006

TABLE XCVI—(Continued).

No.	Name	Latitude	Longitude	Height	$g-\gamma_a-\cdot011$	$g-\gamma_b+\cdot030$	$g-\gamma_c-\cdot011$
		° ′	° ′	*feet*	*dynes*	*dynes*	*dynes*
73	Mukhtiāra ...	22 24	75 59	926	−·039	−·029	−·030
74	Multān	30 11	71 26	404	−·066	−·039	...
75	Mussooree (Camel's Back)	30 28	78 5	6924	+·074	−·093	+·042
76	Mussooree (Dunseverick)	30 27	78 4	7129	+·076	−·098	...
77	Muttra	27 28	77 42	562	−·015	+·007	+·004
78	Muzaffarpur ...	26 7	85 25	179	−·091	−·056	−·053
79	Myingyan	21 29	95 24	248	−·020	+·013	...
80	Mysore	12 19	76 40	2501	+·003	−·040	...
81	Nojli	29 53	77 40	879	−·099	−·087	...
82	Ootacamund ...	11 25	76 42	7395	+·184	−·016	+·001
83	Pathānkot	32 17	75 39	1088	−·175	−·169	−·087
84	Pendra	22 47	82 0	1996	+·010	−·016	−·003
85	Prome	18 50	95 14	101	−·027	+·011	...
86	Pyinmana	19 44	96 12	409	−·014	+·014	...
87	Quetta	30 12	67 1	5520	+·020	−·123	−·004
88	Raipur	21 14	81 41	996	−·013	−·005	−·014
89	Rājpur	30 24	78 6	3321	−·051	−·113	+·015
90	Rāmchāndpur ...	25 41	88 33	132	−·031	+·006	...
91	Rānchi	23 23	85 19	2167	+·040	+·008	+·019
92	Rangoon	16 48	96 9	164	+·010	+·045	...
93	Roorkee	29 52	77 54	867	−·112	−·099	−·055
94	Salem	11 40	78 9	948	−·047	−·038	−·059
95	Sandakphu ...	27 6	88 0	11766	+·178	−·125	+·037
96	Sasarām	24 57	83 59	340	−·025	+·005	−·002
97	Saugor	23 52	78 48	1757	+·010	−·008	·000
98	Seoni	22 5	77 29	2032	+·041	+·014	+·025
99	Shāhpur	22 12	77 54	1286	+·006	+·004	+·012
100	Sibi	29 33	67 53	434	−·137	−·109	−·070
101	Siliguri	26 42	88 25	387	−·160	−·130	−·050
102	Simla	31 6	77 10	7043	+·080	−·100	...
103	Sīpri	25 26	77 39	1533	+·027	+·016	+·018
104	Sultānpur	26 16	82 5	314	−·064	−·034	...
105	Toungoo	18 56	96 27	159	−·011	+·025	...
106	Ujjain	23 11	75 47	1612	−·013	−·026	−·022
107	Umaria	23 32	80 54	1499	+·016	+·007	+·018
108	Yercaud	11 47	78 12	4493	+·072	−·027	−·044

CHAPTER X.

Deflections of the Plumb-line and values of "g" derived in Turkistan (Ferghana) by the Russian observations.

1. The problem of the origin of the Himalayas has already been attacked from the geodetic point of view making use of the deflection results and gravity anomalies obtained in India. All these results relate to points south of the main chain. It is now possible to put certain results obtained by the Russian surveyors into the same terms: and it has accordingly been considered suitable to do this, so that all geodetic data closely related to the Himalayas will be conveniently available in one volume.

2. Values of deflection can be found for Osh base, N.W. end, the starting point of the Russian triangulation which links up with the Indian Pamir triangulation. The Russian triangulation emanates from a base near Osh latitude 40° 31′, longitude 42° 30′ E. of Pulkowa. Latitude and azimuth were observed astronomically while the longitude of Osh was found by electric telegraph from Tashkent, and transferred from Osh to the north-west end of the base by chronometer. Presumably the longitude of Tashkent is in terms of astronomical longitude measured from Pulkowa (Poulcovo) which is $2^h\ 1^m\ 18^s\cdot 57$ of time E. of Greenwich (*vide* Nautical Almanac). This converted becomes 30° 19′ 38″·55 which must be added to the values of longitudes expressed in Russian terms. Calculations of the Russian triangulation were performed on Bessel's spheroid. Suppose the deflections at Osh are ξ, η in prime vertical and meridian (positive if S. or W.). Then the geodetic elements are found by deducting $\eta = -u$, $\xi \sec \lambda = -v$, $\xi \tan \lambda = -w$ from the astronomic values of the latitude, longitude and azimuth respectively. The Astronomic elements of the N.W. end Osh base are:—

Latitude	40° 37′ 16″·67
Longitude	72 56 11·17*
Azimuth	not stated.

From this astronomic origin and a measured base triangulation was carried to a station Kukhtek of the Indian triangulation, the deduced latitude being 37° 17′ 43″·94 and the longitude 74° 59′ 55″·53*.

It is necessary to express these in terms of the geodetic value of Osh and the Helmert spheroid. This can be done approximately by means of the tables already prepared with reference to Kalianpur as origin. Kukhtek is 2° 3′ 44″·36 east of the Osh origin, and as the tables prepared for Kalianpur are shown in absolute longitude the corresponding longitude required is that of Kalianpur increased by this *i.e.* 77° 39′ 17″·57 + 2° 3′ 44″·36 ≑ 79° 43′ 2″ ≑ 79°·7.

* This includes 30° 19′ 38″·55, the difference of longitude of Greenwich and Pulkowa.

3. Suppose corresponding to changes u, v, w at Osh, changes u', v', w' occur at Kukhtek and that both are due to imaginary changes u_0, v_0, w_0 at an origin O on the longitude of Osh and at the latitude of Kalianpur together with a change of axes from those of Bessel to those of Helmert for which $\delta a = +\cdot 803$ and $\delta b = +\cdot 739$ (*vide* Appendix).

From tables XVII-XX a change at O of u_0, v_0 and w_0 causes changes of u_0, $v_0 + \cdot 370\, w_0$, $1\cdot 20\, w_0$ at Osh and from the axes change the further changes (from tables XXIX-XXXIV) at Osh are $+\cdot 803 \times 1\cdot 24 - \cdot 739 \times 10\cdot 56$, $\cdot 803 \times 0 + \cdot 739 \times 0$, $\cdot 803 \times 0 + \cdot 739 \times 0$ *i.e.* $-6\cdot 808$, 0, 0. The total changes at Osh accordingly are $u_0 - 6\cdot 808$, $v_0 + \cdot 370\, w_0$, $+1\cdot 20\, w_0$. The changes at Kukhtek, 2° 3′ 44″ E. of meridian origin and latitude 37°·3 may be found in the same tables under longitude 79°·7.

They are	$u_0 - \cdot 032\, w_0$	$v_0 + \cdot 027\, u_0 + \cdot 284\, w_0$	$w_0 + \cdot 045\, u_0 + 1\cdot 145\, w_0$
	$+\cdot 803 \times 1\cdot 626 - \cdot 739 \times 9\cdot 033$	$-\cdot 803 \times 1\cdot 525 + \cdot 739 \times \cdot 191$	$-\cdot 803 \times \cdot 752 - \cdot 739 \times \cdot 056$
	$1\cdot 306 - 6\cdot 675$	$-1\cdot 225 + \cdot 141$	$-\cdot 604 - \cdot 041$
which reduce to	$u' = u_0 - \cdot 032 w_0 - 5\cdot 369$	$v' = v_0 + \cdot 027 u_0 + \cdot 284 w_0 - 1\cdot 084$	$w' = w_0 + \cdot 045 u_0 + 1\cdot 145 w_0 - \cdot 645$

Now at Osh $\quad u = u_0 - 6\cdot 808,\ v = v_0 + \cdot 370\, w_0,\ w = 1\cdot 2\, w_0$

In terms of vertical deflection $u = -\eta$, $v = -\xi \sec\lambda$, $w = -\xi \tan\lambda$ which determine u_0, v_0, w_0 in terms of ξ and η as follows:—

$$u_0 = -\eta + 6\cdot 808, w_0 = -\frac{1}{1\cdot 2}\xi \tan\lambda, v_0 = -\xi \sec\lambda + \cdot 308\, \xi \tan\lambda \qquad \text{when } \lambda = 40^\circ\ 37'$$

$$u_0 = -\eta + 6\cdot 808, w_0 = -\cdot 715\, \xi, \qquad v_0 = -1\cdot 317\, \xi + \cdot 264\, \xi = -1\cdot 053\, \xi$$

Hence $u' = -\eta + 6\cdot 808 + \cdot 023\xi - 5\cdot 369$, $v' = -1\cdot 053\, \xi + \cdot 027\,(-\eta + 6\cdot 808) - \cdot 203\, \xi - 1\cdot 084$
and $\quad w' = -\cdot 715\xi + \cdot 045\,(-\eta + 6\cdot 808) - \cdot 818\xi - \cdot 645$
$= u' = -\eta + - \cdot 023\xi + 1\cdot 439$, $v' = -1\cdot 256\, \xi - \cdot 027\, \eta - \cdot 900$, $w' = -1\cdot 533\, \xi - \cdot 045\eta - \cdot 339$.

These are the corrections which have to be applied to the Russian values of coordinates of Kukhtek, to bring them into terms of the Indian triangulation.

The Indian values have to be corrected to bring into terms of the Helmert spheroid and the observed values of the elements at Kalianpur, corresponding to $u_0 = 31$, $v_0 = 0$*, $w_0 = 1\cdot 29$, $\delta a = \cdot 924$, $\delta b = \cdot 743$. Denoting the corrections by u'', v'', w'' the corrections at Kukhtek latitude 37°·3 and longitude 75°·0 are:—

$u'' = \cdot 31 \times \cdot 999 + 1\cdot 29 \times \cdot 043$	$v'' = -\cdot 31 \times \cdot 036 + 1\cdot 29 \times \cdot 282$	$w'' = -\cdot 31 \times \cdot 058 + 1\cdot 29 \times 1\cdot 144$
$+ \cdot 924 \times 1\cdot 590 - \cdot 743 \times 9\cdot 032$	$+ \cdot 924 \times 1\cdot 980 - \cdot 743 \times \cdot 247$	$+ \cdot 924 \times \cdot 979 + \cdot 743 \times \cdot 072$
$= \cdot 310 + \cdot 055 + 1\cdot 470 - 6\cdot 711$	$= -\cdot 011 + \cdot 364 + 1\cdot 830 - \cdot 184$	$= -\cdot 018 + 1\cdot 476 + \cdot 905 + \cdot 053$
$= -4\cdot 877$	$= +1\cdot 999$	$= +2\cdot 416$

The values obtained by the Indian triangulation for latitude and longitude of Kukhtek are:—

		37° 17′ 32″·97		75° 0′ 12″·19
	u''	−4 ·88	v''	+ 2 ·00
Values in terms of Helmert spheroid		37 17 28·09		75 0 14·19
Values obtained by Russian triangulation are		37° 17′ 43″·94		74° 59′ 55″·53

and to bring these into accord with the Indian values it is necessary to apply −15″·85 to the latitude and +18″·66 to the longitude.

* This is zero because the deflection at Kalianpur in prime vertical had never been taken account of, although it was implied by the values of azimuths adopted.

The following equations are formed giving the quantities ξ and η at Osh :—

$$-\eta + \cdot 023\,\xi + 1\cdot 439 = -15\cdot 85$$
$$-\cdot 027\,\eta - 1\cdot 256\,\xi - \cdot 900 = +18\cdot 66$$

whence

$$\begin{aligned} \eta &= +\cdot 023\,\xi + 17\cdot 29 \\ 1\cdot 256\,\xi &= -\cdot 027\,\eta - 19\cdot 56 \\ &= -\cdot 001\,\xi - \cdot 467 - 19\cdot 56 \\ \xi &= -\frac{20\cdot 03}{1\cdot 257} = -15\cdot 94 \\ \eta &= 17\cdot 29 - \cdot 37 = +16\cdot 92 \end{aligned} \Bigg\}\text{ at Osh.}$$

The values found by the Russian observers in terms of Tashkent vertical are $\xi = -6\cdot 04$ and $\eta = 23\cdot 43$, *vide* table *XCVIII.*

4. It is possible to bring these results into agreement with the values deduced from the Indian side by supposing that the vertical at Tashkent, latitude 41° 21′ and longitude 69° 18′, is deflected with reference to Kalianpur.

Consider the effect of changes u_0, v_0, w_0 at Tashkent longitude and Kalianpur latitude, and from Bessel to Helmert spheroid for which $\delta a = +\cdot 803$, $\delta b = +\cdot 739$.

From tables *XVII-XX* a change at origin of u_0, v_0, w_0 causes changes of u_0, $v_0 + \cdot 372\,w_0$, $1\cdot 204\,w_0$ at Tashkent and from axes changes the further changes from tables *XXIX-XXXIV* at Tashkent are $-7\cdot 962$, 0, 0. The total changes at Tashkent accordingly are $u_0 - 7\cdot 962$, $v_0 + \cdot 372\,w_0$, $1\cdot 204\,w_0$.

The total changes at Osh λ 40° 37′, L 81° 17′, $\{= 77° 39' + (72° 56' - 69° 18')\}$ calculated from the same tables are :—

$$+\cdot 998u_0 - \cdot 059w_0 - 6\cdot 791 \,\Big|\, v_0 + \cdot 052u_0 + \cdot 357w_0 - 2\cdot 015 \,\Big|\, + \cdot 081u_0 + 1\cdot 197w_0 - 1\cdot 283$$

The following equations are formed :—

$$+\cdot 998u_0 - \cdot 059w_0 - 6\cdot 791 = u = 6\cdot 5 \qquad (1)$$
$$v_0 + \cdot 052u_0 + \cdot 357w_0 - 2\cdot 015 = v = 9\cdot 9 \sec\lambda = 13\cdot 042 \qquad (2)$$
$$+\cdot 081u_0 + 1\cdot 197w_0 - 1\cdot 283 = w = 9\cdot 9 \tan\lambda = 8\cdot 490 \qquad (3)$$

in which the numerical quantities 6·5 and 9·9 are the corrections necessary to ξ and η as determined by the Russian observers, to bring them into agreement with the Kalianpur terms.

Hence

$$\begin{aligned} u_0 &= \cdot 059\,w_0 + 13\cdot 318 && (1) \\ 1\cdot 197\,w_0 &= -\cdot 081\,u_0 + 9\cdot 773 && (3) \\ &= -\cdot 005\,w_0 - 1\cdot 079 + 9\cdot 773 \\ \text{or}\quad 1\cdot 202\,w_0 &= 8\cdot 694 \\ \text{or}\quad w_0 &= 7\cdot 23 \\ \therefore\quad u_0 &= \cdot 427 + 13\cdot 318 = 13\cdot 75 && (1) \\ \text{and}\quad v_0 &= -\cdot 715 - 2\cdot 581 + 2\cdot 015 + 13\cdot 042 = 11\cdot 76 && (2) \end{aligned}$$

5. With these origin changes the corrections at certain degree squares have been computed from the tables and the results are exhibited in table *XCVII.*

TABLE XCVII.

	λ \ L	69°	70°	71°	72°	73°
u v w	40°	7·20 14·41 8·63	7·09 14·05 8·59	6·98 13·70 8·55	6·87 13·34 8·52	6·76 12·99 8·48
u v w	41°	6·78 14·59 8·75	6·67 14·24 8·71	6·56 13·89 8·66	6·45 13·54 8·62	6·34 13·20 8·57

6. The results of the Russian observations* are now given first in terms of Tashkent and Bessel's spheroid and then corrected to the Kalianpur vertical and Helmert's spheroid. The stations are shown in chart No. VI.

TABLE XCVIII.

Serial No.	Station	Latitude	Longitude† in Greenwich Terms	Bessel's Spheroid and Tashkent Vertical		Corrections		Helmert's Spheroid and Kalianpur Vertical	
				Deflection in Prime Vertical	Deflection in Meridian	$-u$	$-w \cot \lambda$	Deflection in Prime Vertical	Deflection in Meridian
	Tashkent	41° 21′	69° 18′	0″·00	0″·00	″	″	− 9″·9	− 6″·5
1	Chodschent ...	40 17	37	+13·06	+ 3·83	−6·94	−10·12	+ 2·9	− 3·1
2	Karatschekum ...	16	70 5	+ 3·71	+22·52	−6·89	−10·09	− 6·4	+15·6
3	Kanybadam ...	19	26	− 4·63	+22·66	−6·83	−10·05	−14·7	+15·8
4	Tschil-Machram ...	33	33	...	+ 2·36	−6·72	...	...	− 4·4
5	Begowat (south) ...	19	43	...	+42·82	−6·80	...	...	+36·0
6	Puntan	44	48	...	− 8·13	−6·62	...	...	−14·8
7	Sary-Kurgan ...	20	71 2	+ 9·47	+30·34	−6·77	−10·03	− 0·6	+23·6
8	Pap	54	4	...	−20·89	−6·53	...	...	−27·4
9	Begowat (north) ...	38	14	...	+ 8·92	−6·60	...	...	+ 2·3
10	Karaül-tjube ...	31	15	+18·14	+16·64	−6·65	− 9·97	+ 8·2	+10·0
11	Warsyk	41 7	16	...	−26·87	−6·40	...	...	−33·3
12	Katput	40 16	20	...	+41·13	−6·76	...	...	+34·4
13	Gurt-tjube	50	28	...	− 4·13	−6·51	...	..	−10·6
14	Kassan	41 15	36	...	−20·30	−6·31	...	...	−26·6
15	Chalmion	40 11	39	...	+49·41	−6·76	...	...	+42·7
16	Namangan	41 0	41	− 2·94	− 9·73	−6·41	− 9·83	−12·8	−16·1
17	Martelan	40 23	47	+ 3·53	+32·56	−6·65	− 9·99	− 6·5	+25·9
18	Bjälaja	41 4	50	...	−13·08	−6·37	...	...	−19·5
19	Kara-tjube	40 37	50	...	+17·17	−6·57	...	...	+10·6
20	Balyktschi	53	52	...	− 1·87	−6·46	...	...	− 8·3
21	Utsch-Kurgan I ...	41 6	72 3	...	−10·63	−6·33	...	...	−17·0
22	Utsch-Kurgan II ...	40 14	4	...	+41·63	−6·70	...	...	+34·9
23	Kuwa	31	4	+ 0·42	+28·40	−6·58	− 9·94	− 9·5	+21·8
24	Tasch-tjube ...	40	10	...	+18·97	−6·51	...	...	+12·5
25	Tschumbagysch ...	54	13	...	− 0·38	−6·41	...	...	− 6·8
26	Isbasken	41 2	21	...	−12·36	−6·34	...	..	−18·7
27	Min-tjube	40 29	22	− 6·84	+33·74	−6·55	− 9·94	−16·8	+27·2
28	Andischan	47	24	...	+12·63	−6·43	...	...	+ 6·2
29	Kisyl-Kurgan ...	20	24	...	+32·39	−6·63	...	...	+25·8
30	Salb-tjube	38	34	...	+25·85	−6·50	...	...	+19·4
31	Massy	41 5	39	...	−15·71	−6·27	...	...	−22·0
32	Chodscha-Syrjan ...	40 48	43	...	+11·49	−6·38	...	...	+ 5·1
33	Tjulka-tjube ...	18	46	...	+32·71	−6·59	...	...	+26·1
34	Osh	31	49	− 6·04	+23·43	−6·50	− 9·90	−15·9	+16·9
35	Chasret Ujunys ...	46	56	...	+13·70	−6·38	...	...	+ 7·3
36	Mady	34	56	...	+22·80	−6·47	...	...	+16·3
37	Dschalabad ...	55	73 1	...	+ 4·38	−6·32	...	...	− 1·9

* *c.f.* Comptes-Rendus de L' Association Geodesique Internationale for 1898 (Annexe A IIc, p. 268).
† Converted from Pulkowa Longitude by applying +30° 20′ (more accurately 30° 19′ 38″·55).

The following description is extracted from *Comptes-Rendus de L' Association Geodesique Internationale* for 1896 (Annexe B XI p. 309):—

"The researches recently completed on the deviation of the plumb-line in Ferghana (Turkistan) are of special interest. This valley lying between 40° 15′ and 41° 15′ in latitude and 39° 30′ and 42° 45′ in longitude, east of Pulkowa is a deep depression the walls of which are pierced in their western part by the narrow bed of the Syr Daria. The bottom of the valley has an approximately elliptic figure with its major axis 250 km. long following the direction of the parallel, and its minor axis 110 km. that of the meridian. On the north the valley is enclosed by chains of mountains of an average height of 2500 to 3500 m. and on the south are the Alai, the Trans Alai, the Pamirs and the Hindu Kush. Such a position leads one to expect considerable deviation particularly in latitude and explains the investigations which have been made in order to verify this supposition. To this end 37 determinations of latitude have been made of points equally distributed over the district and 10 of longitude at points nearly on the same parallel. Taking Bessel's Ellipsoid, and the point Balyktschi as zero, we obtain for the deviation in latitude values for A—G from —25″ on the north up to +51″ on the south of the valley, in longitude the deviation of opposite sign amounts to 25″".

7. A table of gravity residuals in the same district* is also given for the sake of completeness.

TABLE XCIX.

	No.	Station	Latitude	Longitude	Height in Metres	$g_0-\gamma_0$ Cm $10^{-3}\times$
TURKISTAN	1	Pamir Post	38° 10′·0	73° 58′·2	3700	−30
	2	Kala-i-Wanj	38 22·2	71 27·0	1795	−177
	3	Sar-i-pul	38 24·5	70 5·5	1500	−100
	4	Kala-i-Chamb	38 27·3	70 46·5	1345	−152
	5	Rabat Ak Baital ...	38 29·7	73 51·5	4100	+ 74
	6	Rabat Maskol	38 42·0	73 31·7	4200	+169
	7	Kara Kul Lake ...	39 6·4	73 31·2	3920	+ 35
	8	Irkeshtan Fort	39 41·9	73 55·5	2850	− 56
	9	Ak-bossaga	39 48·6	73 13·7	2875	−128
	10	Sufi Kurgan	40 1·5	73 30·0	2115	− 91
	11	Karaul Kishlak... ...	40 2·2	72 6·0	1300	−157
	12	Gultsha	40 19·0	73 25·7	1583	−126
	13	New Marghilan ...	40 23·7	71 46·7	581	−159
	14	Langar	40 24·6	73 5·7	1685	− 67
	15	Osh	40 31·4	72 46·6	1021	−106
	16	Andijan	40 45·8	72 20·6	530	−185
	17	Tashkent	41 19·5	69 17·7	478	− 50
	18	Wysokoji Khojand ...	42 30·9	70 33·9	1060	− 1
	19	Chodient	40 17·1	69 34·7	320	−140
	20	Namangan	40 59·7	71 38·7	440	−178

* *c.f.* C. R. 1911—Volume III pp. 156-158.

APPENDIX.

Various Determinations of the Axes of the Earth.

For convenience of reference the principal values of the elements of the figure of the earth obtained from time to time are given below, expressed in units of 1000 feet and kilometers. It is to be observed that the datum of height in different continents is only in the same terms on the assumption that the geoid is identical with the spheroid. The quantities determined really refer to the several concentric spheroids through these sea level datum points.

1. RADIUS OF THE EARTH CONSIDERED AS A SPHERE.

Reference No.	Authority	Date	Radius in 1,000 feet	Radius in kilometres	Data used
1	Eratosthenes ...	250 B. C.	24370	7428	Arc from Syene (Upper Egypt) to Alexandria.
2	Posidonius ...	80 „	23190	7069	
3	Richard Norwood...	1637 A. D.	21038	6412	Arc from London to York. Mean Lat. 52° 42′ 30″.
4	Jean Picard ...	1669 „	20906	6372	Arc from Paris to Amiens. Mean Lat. 49° 30′.

2. AXES OF THE EARTH CONSIDERED AS A SPHEROID.

Jean Richer (d. 1696) pointed out that the Earth was not a sphere.

Reference No.	Authority	Date	Radius of Curvature in Mean Latitude of Arc: in 1,000 feet	Radius of Curvature in Mean Latitude of Arc: in kilometres	Deduced: a in 1,000 feet	Deduced: $\frac{1}{e}$	Data used	
5	J. and D. Cassini ...	1684-1718	20860·2	6360·8			Arc from Paris to Dunkirk. Mean Lat. 49° 56′ 9″.	The immediate inference was that the degree diminishing with the increasing latitude, the Earth must be a *prolate spheroid*.
6	J. and D. Cassini ...	...	20919·4	6376·1			Arc from Paris to Collioure. Mean Lat. 45° 40′ 42″.	
7	Bouguer and De la Condamine	1735-1751	20795·4	6338·3	20988·5 (=6397·3km)	216·82	Arc in Peru. Mean Lat. 1° 31′ 0″ S.	Bouguer, De la Condamine, Maupertuis, Clairault, De Thury and De Lacaille proved that the Earth was an *oblate and not a prolate spheroid*.
8	Maupertuis and Clairault	1736	21038·4	6412·4			Arc in Finland. Mean Lat. 66° 19′ 35″.	
9	De Lacaille ...	1752	20897·4	6369·4			Arc at Cape of Good Hope. Mean Lat. 33° 18′ 30″ S.	

3. AXES OF THE EARTH CONSIDERED AS A SPHEROID DETERMINED FROM A GROUP OF ARCS, GRAVITY ETC.

Quantities given by authority named are shown in roman figures; deduced quantities are in *italics*.

Reference No.	Authority	Date	Semi Major Axis (=a) in 1,000 feet	Semi Major Axis (=a) in kilometres	Semi Minor Axis (=b) in 1,000 feet	Semi Minor Axis (=b) in kilometres	$\frac{1}{\epsilon}$	Data used etc.
10	Laplace ...	1799	*20919·768*	6376·840	*20852·822*	6355·935	312·20	Arcs in Peru, India, France, England, and Sweden.
11	Everest ...	1830	*,, 22·84095*	*,, 77·276*	*,, 53·28403*	*,, 56·075*	300·8017	Arc from Damargida to Kalianpur Mean Lat. 21° 5′ 13″.
			(,, 22·93180)	...	(,, 53·37458)	...	...	As expressed by Everest in terms of Indian 10-foot bar A (=9·99995658 feet).
12	Airy ...	1830	,, 23·718	*,, 76·542*	,, 53·810	*,, 56·236*	299·33	14 meridian arcs and 4 arcs of parallel.
13	Bessel ...	1841	*,, 23·237 ± ·702*	,, 77·397 ± ·214	*,, 53·296*	,, 56·079	299·15	From 10 meridian arcs.
14	Clarke ...	1857	,, 26·348	*,, 78·345*	,, 55·233	*,, 56·669*	294·26	Arcs: Anglo-Gallic, Russian, Indian, Prussian, Peruvian, Hanoverian, Danish.
15	Pratt ...	1863	,, 26·189	*,, 78·297*	,, 55·316	*,, 56·695*	295·26	Semi axes and ellipticity of the Mean Figure of the Earth. From a comparison of the Anglo-Gallic, Russian and Indian arcs &c.
16	Clarke ...	1866	,, 26·062	*,, 78·258*	,, 55·121	*,, 56·685*	294·98	Arcs: Anglo-Gallic (rejecting 21 lat. stations), 2nd Indian, Russian, Peruvian, Cape.
17	Clarke ...	1880	,, 26·202	*,, 78·301*	,, 54·895	*,, 56·871*	293·47	
18	Clarke-Bessel ...	...	,, 26·202	*,, 78·301*	*,, 56·252*	[*,, 56·980*]	299·15	
19	Clarke-Bessel ...	...	*,, 25·889*	,, 78·190	*,, 55·888*	[*,, 56·869*]	299·15	From Clarke's value of 1866 with an old conversion factor.
20	Darwin ...	1899	...	...	...	...	296·4	From consideration of precession.
21	....	...	,, 25·329	,, 78·085	,, 55·381	,, 56·715	299·15	(1·0001) × Bessel's value: adopted by the Central Bureau, International Geodetic Association.
22	Hayford (C.andG.S.) ...	1906	*,, 26·144 ± ·112*	,, 78·283 ± ·034	*,, 55·885*	,, 56·868	297·8 ± 0·9	
23	Helmert ...	1907	*,, 25·871*	,, 78·200	*,, 55·721*	,, 56·818	298·3	From gravity determinations.
24	Hayford (C.andG.S.) ..	1909	*,, 26·488 ± ·059*	,, 78·388 ± ·018	*,, 56·019*	,, 56·909	297·0 ± 0·5	Adopted for International $\frac{1}{M}$ map.
25	Helmert-Hayford	...	*,, 26·436*	,, 78·372	,, 55·976	,, 56·896	297	Obtained by S. Wallisch taking Helmert's value with weight=unity and the modified Hayford values with weight=4.
26	International Map Committee ...	1909	,, 26·002	,, 78·24	*,, 54·874*	,, 56·56	[*294·2*]	
27	E.W. Brown ...	1914	...	...	...	...	293·7 ± 0·3	From Lunar theory. The value will make the observed motions of perigee and node agree with the theoretical values.
28	Nautical Almanac	1911	...	...	...	...	297	Adopted in the conference of Nautical Almanac directors.
29	Crommelin ...	...	...	...	...	...	294·4 ± 1·5	From Moon's parallax at Greenwich and Cape. Obtained by a hundred pairs of simultaneous observations at the Cape and Greenwich Observatories by a comparison between theoretical and observed values of the Moon's parallax.

4. AXES OF THE EARTH CONSIDERED AS AN ELLIPSOID.

Reference No.	Authority	Date	Equatorial Axes				Polar Axis		Longitude of major axis E. of Greenwich	Data used
			a		*b*		*c*			
			in 1,000 feet	in kilometres	in 1,000 feet	in kilometres	in 1,000 feet	in kilometres		
80	Schubert ...	About 1860	20927·397 (3272671 toises)	6378·665	20925·044 (3272303 toises)	6377·948	20855·759 (3261468 toises)	6356·830	41° 4′	Arcs: French, English, Russian, Indian, Cape, Prussian and Peruvian.
81	Clarke ...	1860	„ 26·629	„ 78·431	„ 25·105	„ 77·966	„ 53·477	„ 56·439	15° 34′	Arcs: French, English, Indian, Russian, Prussian, Peruvian, Cape.

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4. 1° = 57060 toises—Ibid.
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For some other values *see La Figure de La Terre*, Paris, 1901, by Capt. G. Perrier.

Conversion factors used—	*1 metre = 3·28084275 feet.	log	= 0·5159854152
	or 1 foot = 0·30479973 metre.	„	= $\bar{1}$·4840145780
	1 toise = 6·39459252 feet	„	= 0·8058128753

* *Vide "Determination du Rapport du yard au metre"* by M. Benoit, Paris, 1896: also *Text Book of Topographical and Geographical Surveying*, p. 359 by Colonel C. F. Close, C. M. G., R. E.

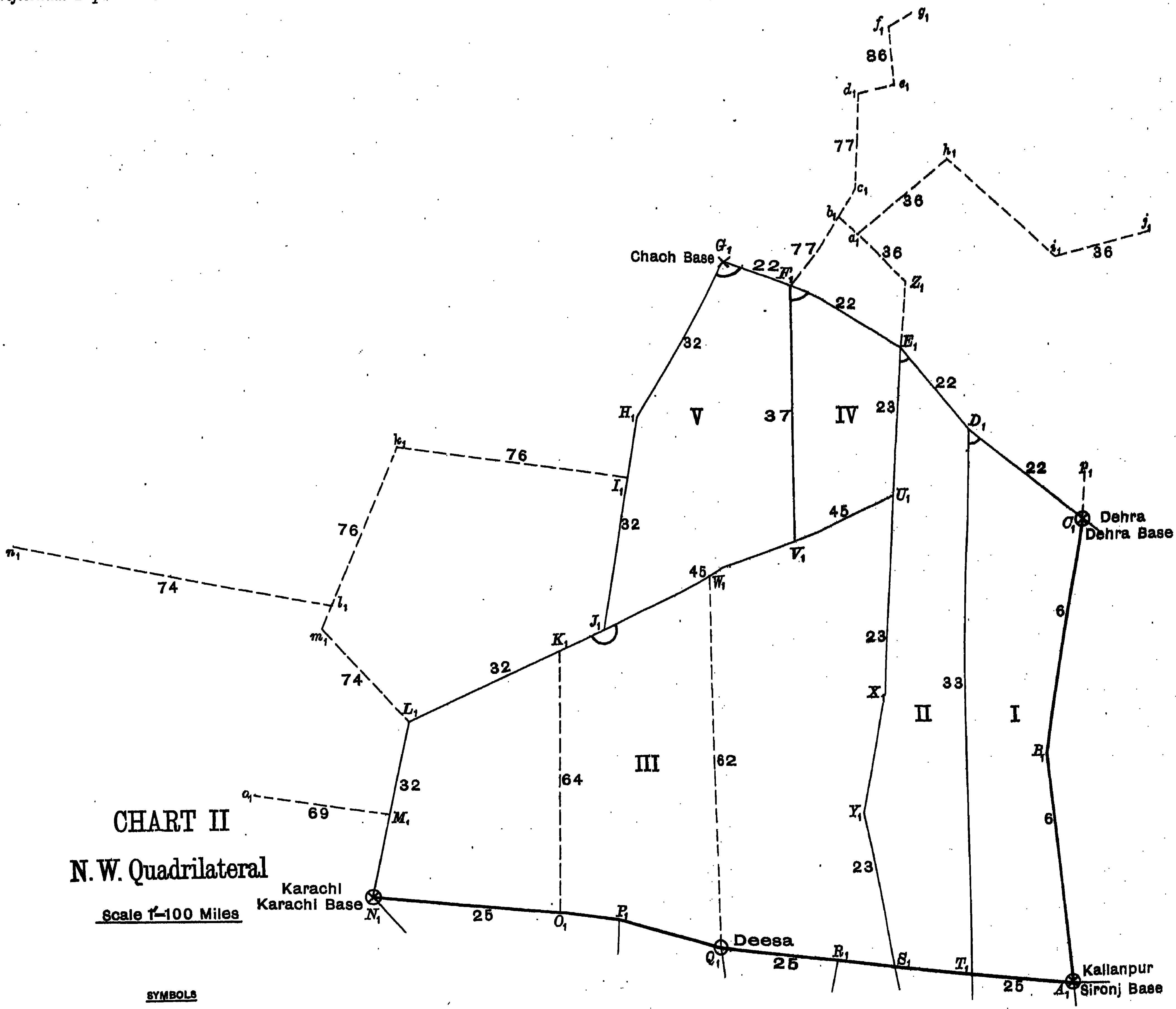

CHART II

N. W. Quadrilateral

Scale 1"=100 Miles

SYMBOLS

Base .... ×

Longitude .... ○

Closing Point ....

Reg. No.7 D.D.1917 (2 D.O.)-8.1r420

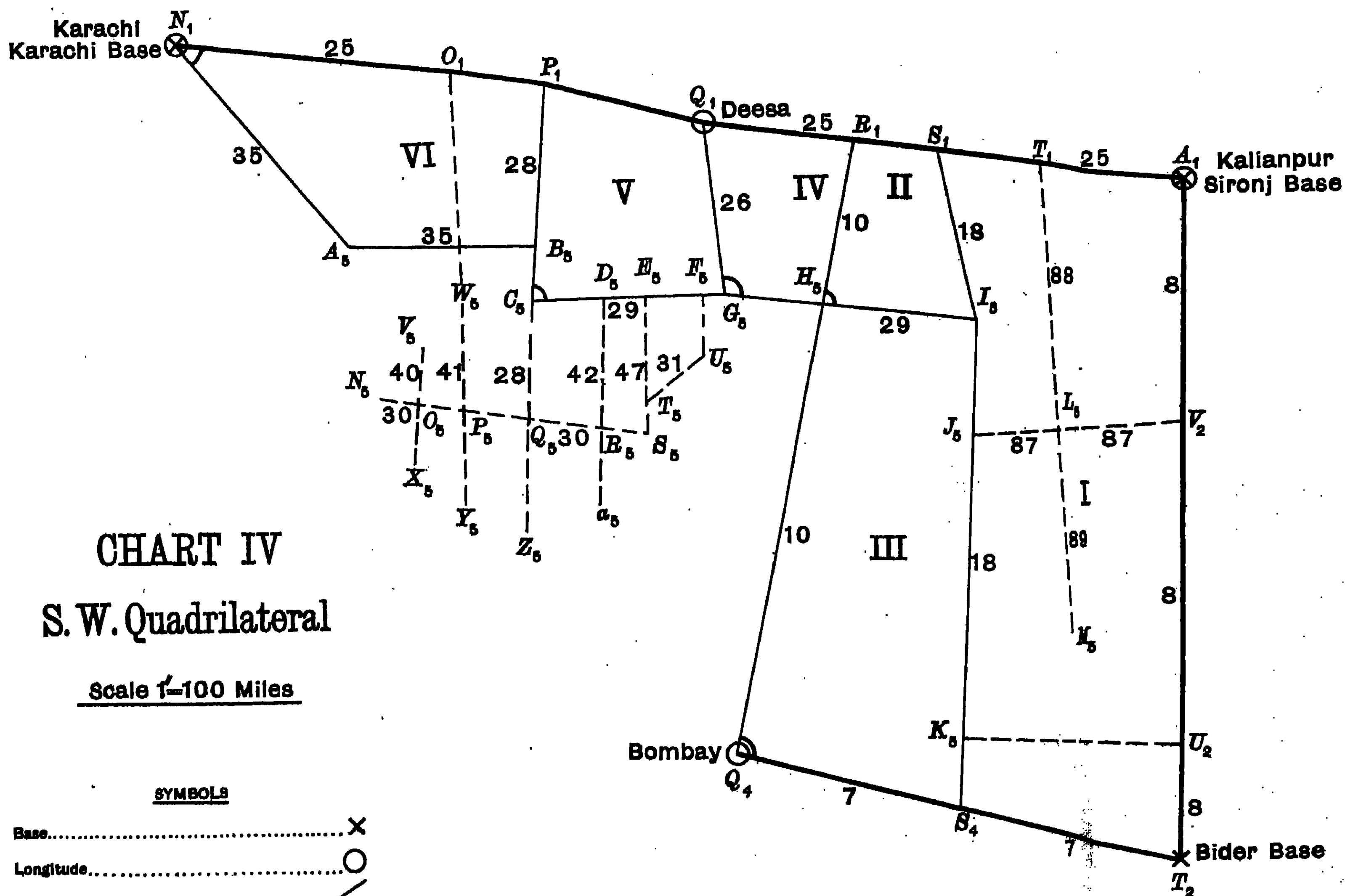

CHART IV

S. W. Quadrilateral

Scale 1"=100 Miles

SYMBOLS

Base...×

Longitude...................................○

Closing Point.........................

7 D.D.1917 (2 D.O.)-S.I-420

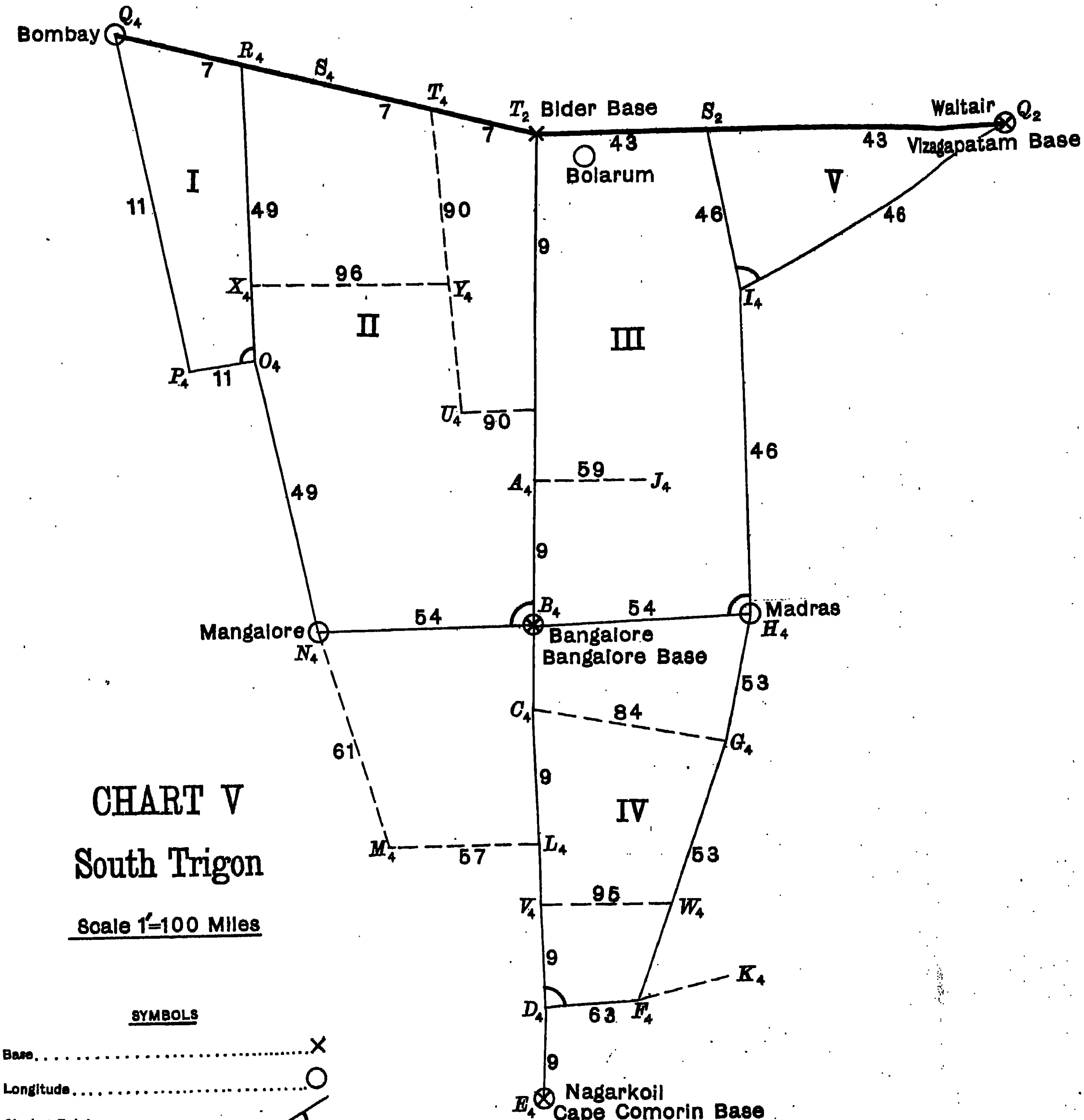

CHART V

South Trigon

Scale 1″=100 Miles

SYMBOLS

Base .... ×

Longitude .... ○

Closing Point ....

Reg. No. 7 D.D. 1917 (2 D.O.)-S.I.-420